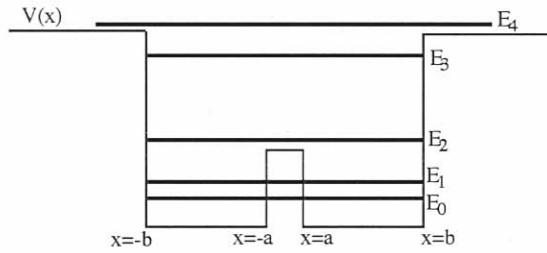
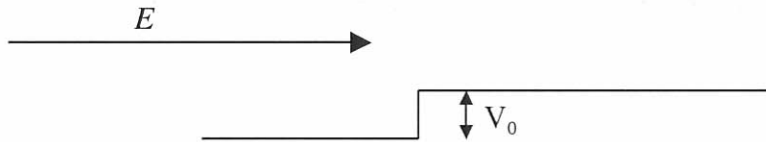


S4. The energy levels  $E_n$  of a symmetric potential well  $V(x)$  are denoted below.



- How many bound states are there?
- Sketch the wave functions for the first three levels ( $n=0,1,2$ ). For each, denote the regions where the particle is classically forbidden.
- Describe the evolution of the wave function if we start in an equal superposition of eigenstates  $n=0$  and  $n=1$ .

S5. A particle with well defined energy  $E$  is scattered from a one dimensional step potential of height  $V_0$ . What is the probability of reflection when  $E < V_0$  and  $E > V_0$ .



**Long Answers: Pick two out of three problems below**

**L1.** A harmonically bound particle experiences a constant force  $F$ .

- Argue that the interaction Hamiltonian associated with this force is  $\hat{H}_{\text{int}} = -F\hat{x}$ .
- Assuming the force is weak compared to the harmonic binding. Show that the lowest nonvanishing perturbation to the energy of the  $n^{\text{th}}$  bound state is

$$\Delta E_n = -\frac{(Fx_0)^2}{\hbar\omega}, \text{ where } x_0 = \sqrt{\frac{\hbar}{2m\omega}}$$

independent of level. Hint: Recall  $\hat{x} = x_0(\hat{a}^\dagger + \hat{a})$ , where  $\hat{a}^\dagger, \hat{a}$  are the creation, annihilation operators.

- This problem can be solved exactly, without resorting to perturbation theory. Solve for the exact energy eigenstates and eigenvalues including the harmonic potential and perturbing force.
- Show that your exact energy spectrum agrees with the perturbation result to the appropriate order in the perturbation's small parameter.