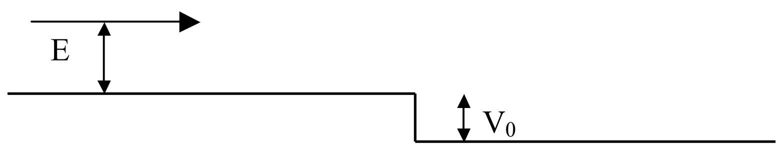
Preliminary Examination: Quantum Mechanics
Department of Physics and Astronomy
University of New Mexico
Spring 2007

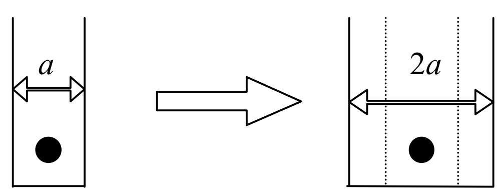
Instructions:
- The exam consists of 10 problems, 10 points each.
- Partial credit will be given if merited.
- Personal notes on two sides of an 8×11 page are allowed.
- Total time is 3 hours.

Problem 1: A nearly monoenergic particle is moving in one dimension where the potential energy make a sharp jump as shown.

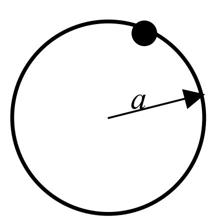


What is the probability for the particle to transmit across the step in according to classical mechanics and according to quantum mechanics?

Problem 2: A particle of mass *m* is trapped in one dimension in a box with hard walls (infinite energy to escape) of width *a*. At time *t*=0, the particle is in the ground state. The width of the box is *suddenly* expanded to 2a. What is the probability of finding the particle in the ground state of the new well (you may leave your answer as an integral)? What is the probability of finding the particle in the first excited state of the new well (give a number).



Problem 3: A particle of mass *m* moves in two dimensions constrained to a ring of radius *a*. The particle is otherwise "free" (no other external forces act on the particle). What are the conserved physical quantities? What are energy eigenvalues and degeneracies of these levels? How are the degeneracies related to the conserved quantities?



Problem 4: A wave packet in a 1D harmonic oscillator is described by the state,

$$|\psi\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle,$$

where α is a complex number and $|n\rangle$ is an eigenstate of the number operator, $\hat{N} \equiv \hat{a}^\dagger \hat{a}$, where $\hat{a}^\dagger, \hat{a}$ are the usual creation and annihilation operators.

(i) Show that this state is normalized?
(ii) Is this an energy eigenstate?
(iii) Show that $\hat{a}|\psi\rangle = \alpha|\psi\rangle$.

Problem 5: Consider a particle with angular momentum *J*=1. Show that,

$$|\psi_a\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}\left(|1\rangle + |-1\rangle\right),\ |\psi_b\rangle = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}\left(|1\rangle + |-1\rangle\right),\ |\psi_c\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle - |-1\rangle\right)$$

are eigenstates of the x-component of angular momentum, $\hat{J}_x$, with the eigenvalues you expect. Here $\{|m\rangle \mid m = -1,0,1\}$ denote the three eigenstates of $\hat{J}_z$, $\hat{J}_z|m\rangle = m|m\rangle$. (Hint, express $\hat{J}_x$ in terms of raising and lowering operators).

Problem 6: Consider again, a particle with angular momentum *J*=1, whose dynamics is governed by a Hamiltonian $\hat{H} = \kappa \hat{J}_z^2$. At time *t*=0, we prepare the particle in the state $|\psi(0)\rangle = |\psi_a\rangle$, as given in Problem 5. Show that at a time $t = \pi/(2\kappa)$,

$$|\psi(t = \pi/(2\kappa))\rangle = \frac{e^{-i\pi/4}|\psi_a\rangle - e^{i\pi/4}|\psi_b\rangle}{\sqrt{2}}.$$

Problem 7: Consider two distinguishable noninteracting particles of mass *m*, each trapped in a separate harmonic well in 1D. The Hamilton is

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2\left(x_1 + a\right)^2 + \frac{1}{2}m\omega^2\left(x_2 - a\right)^2.$$

(i) Show that this Hamiltonian separable in relative coordinate and center of mass.
(ii) Express the total energy eigenvalues of the two-body system in terms energy eigenvalues associated with the motion of center of mass and relative coordinate.

Problem 8: Neglecting the spin of the electron or proton, what is the degeneracy of the first excited state of a hydrogen atom? What are the quantum numbers that uniquely specify each of the substates? Now add electron spin and spin-orbit coupling. What are the good quantum numbers? Sketch an energy level diagram for each of these levels of description. Calculation of the energy level splitting is not necessary.

Problem 9: Consider the Hamiltonian describing two bound states of an atom. Written in bra-ket notation, the unperturbed Hamiltonian in this subspace is,

$$\hat{H}_0 = E_1|\psi_1\rangle\langle\psi_1| + E_2|\psi_2\rangle\langle\psi_2|.$$

Interaction with a weak external field is described by the Hamiltonian

$$\hat{H}_1 = E_{\text{int}}\left(|\psi_1\rangle\langle\psi_2| + |\psi_2\rangle\langle\psi_1|\right),$$

where $\left|\dfrac{E_{\text{int}}}{E_2 - E_1}\right| << 1$. To lowest nonvanishing order in perturbation theory, what are the shifts in the energies of the two bound states?

Problem 10: Given the total Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_1$ in Problem 9, solve for the *exact* energy eigenvalues. Perform a Taylor series expansion on your result to find the perturbation shift in eigenvalues to lowest order in $E_{\text{int}}/(E_2 - E_1)$.