

Department of Physics and Astronomy, University of New Mexico

Thermodynamics and Statistical Mechanics Preliminary Examination

Fall 2008

Instructions:

- The exam consists of 10 short-answer problems (10 points each).
- Where possible, show all work; partial credit will be given if merited.
- Personal notes on two sides of an $8\frac{1}{2}'' \times 11''$ page are allowed.
- Total time: 3 hours.

Unless otherwise noted, commonly used symbols are defined as follows:

P : Pressure

T : Absolute Temperature

R : gas constant; $R = 8.314 \text{ J} \cdot \text{mole}^{-1} \cdot \text{Kelvin}^{-1}$

k : Boltzmann's constant; $k = \frac{R}{6.02 \times 10^{23}}$

$\beta = (kT)^{-1}$

S : Entropy

V : Volume

U : Internal energy

N : Number of particles (Assume $N \gg 1$)

$\hbar = \frac{h}{2\pi}$ ($h = 6.626 \times 10^{-34} \text{ Js}$ is Planck's constant)

1– An ideal monatomic gas occupies a volume of 2 m^3 at a pressure of 4 atm ($1 \text{ atm} \approx 10^5 \text{ Pa}$) and a temperature of 293 K. The gas is compressed to a final pressure of 8 atm.

Compute the final volume, the work done on the gas, the heat released to the surrounding, and the change in the internal energy for a reversible isothermal compression.

2– Two identical objects A and B , are thermally and mechanically isolated from the rest of the world. Their initial temperatures are $T_A > T_B$. Each object has heat capacity C (the same for both objects) which is independent of temperature.

Suppose the objects are used as the high and low temperature heat reservoirs of a heat engine. The engine extracts heat from object A (lowering its temperature), does work on the outside world, and dumps heat to object B (raising its temperature). When the temperatures of A and B are the same, and the heat engine is in the same state as it started, the process is finished. Suppose this heat engine performs the maximum work possible. What are the final temperatures of the objects? How much work does the engine do in this process?

3– The Gibbs free energy of N molecules of a certain gas at temperature T and pressure P is

$$G = NkT \ln P + A + BP + \frac{CP^2}{2} + \frac{DP^3}{3}, \quad (1)$$

where A , B , C , and D are constants. Find the equation of state of the gas.

4– A box with volume V , which is isolated from the rest of the world, is divided into two equal parts by the means of a partition. Initially the left half of the box contains n moles of an ideal gas at temperature T , while the right half is empty.

Suppose that the partition is suddenly lifted. After a sufficiently long time the gas will occupy all of the box. Find the change in the temperature and entropy for this process.

5– For an ideal gas consisting of diatomic molecules the equation of state and the internal energy are given by

$$PV = nRT, \\ U = \frac{5}{2}nRT,$$

respectively (n is the number of moles). Find the specific heat capacity at constant volume c_V and specific heat capacity at constant pressure c_P for this gas.

6– Consider a line of 3 spins. A spin can point up or down. Suppose that when two adjacent spins point in the same direction, they contribute $-\epsilon$ to the energy of the system and when they point in the opposite directions, their contribution is $+\epsilon > 0$. Non-adjacent spins have no interaction.

(a) What are the possible energies of this system and how many states are there with each energy?

(b) If the system is in thermal equilibrium with a heat bath at temperature T , the probability that it has energy E , among the possible values found in part (a), is given by $A e^{-E/kT}$ (where A is a constant). Find A in terms of ϵ , k , and T .

7– Consider a noninteracting Fermi gas consisting of electrons. In such a gas the energy states are occupied according to the following distribution function

$$f(\vec{p}) = \frac{1}{e^{(E-E_F)/kT} + 1},$$

where E_F is the Fermi energy, $E = \sqrt{(pc)^2 + (m_e c^2)^2}$ is the energy, \vec{p} is the momentum, $p \equiv |\vec{p}|$ is the magnitude of the momentum, and m_e is the mass of the electron.

(a) Draw f as a function of E in the limit that $T \rightarrow 0$.

(b) The total number density of electrons in the gas is given by

$$n = \frac{4\pi}{h^3} \int_0^\infty f(\vec{p}) p^2 dp,$$

where h is the Planck's constant. Find n in terms of E_F and h for $T = 0$.

8– Consider a particles of mass m in a box of volume V that is in equilibrium with the surrounding at a temperature T .

(a) Determine the partition function of the particle in terms of β , m , V , and h . Hint: helpful expression:

$$\frac{1}{\sqrt{2\pi}} \int_0^\infty x^2 e^{-x^2/2} dx = 1.$$

(b) Use the partition function and find the average energy of the particle at temperature T .

9– Consider a system of N noninteracting identical bosons of mass m confined to a vessel of fixed volume, in equilibrium with the surroundings at a temperature T . In the limit that $T \rightarrow 0$, all particles will go to the ground state whose energy is $E_0 = mc^2$.

(a) Find the entropy of this system at $T = 0$.

(b) We add one more particle to the system at zero temperature. What is the resultant change in the energy and entropy? Use this to find the chemical potential μ of this system at $T = 0$.

10– Blackbody radiation is a gas consisting of photons at a temperature T . The energy density of photon gas ρ is given by

$$\rho = \frac{1}{\pi^2 c^3} \int_0^\infty \frac{\hbar \omega^3 d\omega}{e^{\hbar\omega/kT} - 1}, \quad (2)$$

(a) Given this expression, show that ρ is proportional to T^4 .

(b) Photon wavelength λ is given by $\lambda = 2\pi c/\omega$. Show that in the limit that $\lambda \gg (hc/kT)$ the spectral density (energy per volume per wavelength) is proportional to λ^{-4} , i.e. the classical limit known as Rayleigh's law is recovered.