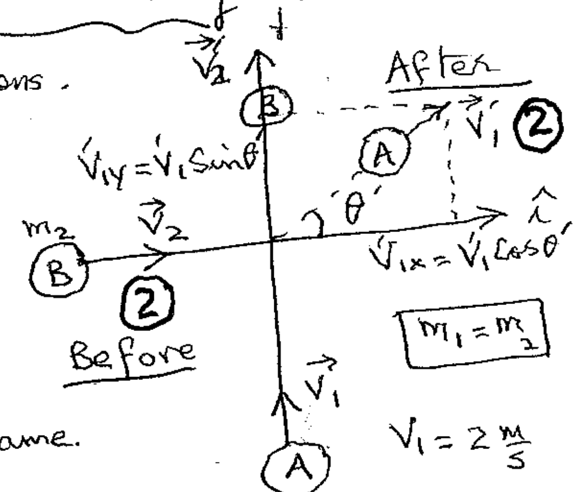
P-151/002 Random Grading

(7-44) This is a case of elastic collisions.

25 So we expect:

- LM must be conserved
- KE must be conserved

Conservation of LM implies that components in x- and y-directions must be conserved as well as the angle between $\vec{p}_2$ and $\vec{p}_1$ must be the same.



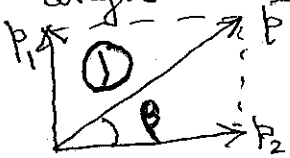
$v_1 = 2\,\frac{m}{s}$

$v_2 = 3.7\,\frac{m}{s}$

$v_1' = ?$

$v_2' = ?$

$\theta = ?$



$$\tan\beta = \frac{p_1}{p_2} = \frac{m_1 v_1}{m_2 v_2} = \frac{2\,\frac{m}{s}}{3.7\,\frac{m}{s}} = 0.541$$

$\Rightarrow \underline{\beta = 28.4^\circ}$ 2

X-direction

$$\cancel{m_2} v_2 = \cancel{m_1} v_1' \cos\theta'$$

$\Rightarrow v_1'\cos\theta' = 3.7\,\frac{m}{s}$ —(1) 3

Y-direction

$$\cancel{m_1} v_1 = \cancel{m_2} v_2' + \cancel{m_1} v_1' \sin\theta'$$

$v_1'\sin\theta' = v_1 - v_2'$ —(2) 3

$\Rightarrow$ KE must be conserved:

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2 \Rightarrow v_1'^2 + v_2'^2 = v_1^2 + v_2^2 = \left(2\,\frac{m}{s}\right)^2 + \left(3.7\,\frac{m}{s}\right)^2 = 17.7\,\frac{m^2}{s^2}$$

$\Rightarrow v_1'^2 + v_2'^2 = 17.7\,\frac{m^2}{s^2}$ —(3) 3

$(1)^2 + (2)^2$ gives

$$v_1'^2 \underbrace{(\cos^2\theta' + \sin^2\theta')}_{1} = \left(3.7\,\frac{m}{s}\right)^2 + \left(2\,\frac{m}{s} - v_2'\right)^2$$

$\Rightarrow v_1'^2 = 13.7\,\frac{m^2}{s^2} + 4\,\frac{m^2}{s^2} - \left(4\,\frac{m}{s}\right)v_2' + v_2'^2$ —(4)

$\Rightarrow v_1'^2 - v_2'^2 = 17.7\,\frac{m^2}{s^2} - \left(4\,\frac{m}{s}\right)v_2'$ 3

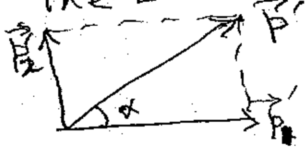
(3) − (4) gives:

$2 v_2'^{\cancel{2}} = \left(4\,\frac{m}{s}\right)\cancel{v_2'}$ $\Rightarrow \underline{v_2' = 2\,\frac{m}{s}}$ 1

From (2) we get:

$v_1'\sin\theta' = 2\,\frac{m}{s} - 2\,\frac{m}{s} = \text{ZERO} \Rightarrow \underline{\theta' = 0}$ 2 $\Rightarrow$ From (1): $\underline{v_1' = 3.7\,\frac{m}{s}}$ 2

The billiard balls simply EXCHANGE 1 velocities after collision



$$\tan\alpha = \frac{p_2'}{p_1'} = \frac{m_2 v_2'}{m_1 v_1'} = \frac{2\,\frac{m}{s}}{3.7\,\frac{m}{s}} = 0.541 = \tan\beta$$

$\Rightarrow \underline{\alpha = \beta}$, as expected

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{7-71} $m = 25\cancel{g}\left(\frac{1\,kg}{1000\,\cancel{g}}\right) = 2.5\times10^{-2}\,kg$

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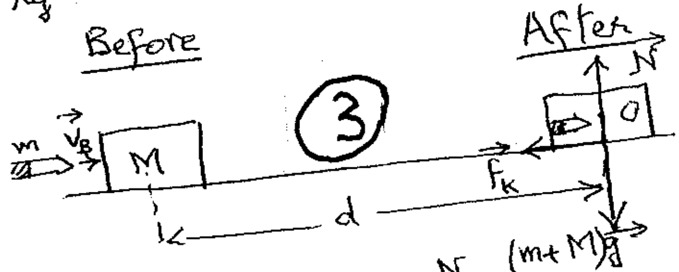
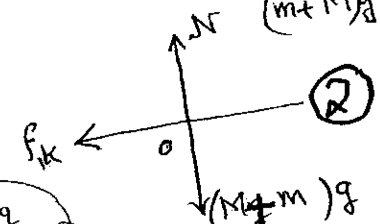
$M = 1.35\,kg$

$\mu_k = 0.25$

$d = 9.5\,m$

$v_B = ?$





From free body diagram,

$\sum F_y = 0 \Rightarrow N = (M+m)g$ 1

By definition,

$f_k = \mu_k N = \mu_k (M+m)g$

$\hookrightarrow = 0.25 \times 1.375\,kg\left(9.8\,\frac{m}{s^2}\right)$

$$\begin{array}{r} 1.35\,kg \\ +\ .025\,kg \\ \hline 1.375\,kg \end{array}$$

$\Rightarrow f_k = \underline{3.37\,N}$ 2

By Conservation of linear momentum,

$m v_B + 0 = (M+m)v' \leftarrow$ velocity after collision

$ = \left(1+\frac{M}{m}\right)v' \approx \frac{M}{m}v'$ —— (1) 1

$\Rightarrow v_B = \frac{M+m}{m}v'$

KE after inelastic collision

$\hookrightarrow = \frac{1}{2}(M+m)v'^2$ This KE is used up in doing work against the force of kinetic friction,

$+f_k d = +\frac{1}{2}(M+m)v'^2 \Rightarrow v'^2 = \frac{2f_k d}{M+m}$

$\Rightarrow v'^2 = \frac{2\times3.37\,N\,(9.5\,m)}{1.375\,kg} = 48\,\frac{m^2}{s^2} \Rightarrow \underline{v' = 6.93\,\frac{m}{s}}$ 3

From (1) we have,

$v_B = \frac{M}{m}v' = \frac{1.35\,\cancel{kg}}{2.5\times10^{-2}\,\cancel{kg}}\left(6.93\,\frac{m}{s}\right)$

$\hookrightarrow = \underline{381.2\,\frac{m}{s}}$ 3