# PHYC 467: Methods of Theoretical Physics II

Spring 2016

Homework Assignment #1

(Due February 2, 2016)

**1-** Show that a second-order Cartesian tensor $T_{ij}$ can be decompsed into three tensors,

$$T_{ij} = I_{ij} + S_{ij} + A_{ij} \, ,$$

where $I_{ij}$ is isotropic, $S_{ij}$ is a traceless symmertric tensor, and $A_{ij}$ is an antisymmetric tensor. Show that these tensors do not mix under an arbitrary change of basis.

**2-** Consider a third-order Cartesian tensor $T_{ijk}$. Find totally symmetric $S_{ijk}$ and totally antisymmetric $A_{ijk}$ tensors obtained from symmetrization and antisymmetrization of the components of $T_{ijk}$ respectively. Write all components of these tensors explicitly. Can $T_{ijk}$ be decomposed into a totally symmetric and a totally antisymmetric tensors in general? Explain.

**3-** A rigid body consists of four particles of masses $m$, $2m$, $3m$, $4m$ that are situated at the points $(a, a, a)$, $(a, -a, -a)$, $(-a, a, -a)$ $(-a, -a, a)$ respectively. What is the moment of inertia tensor of this system? Find the principal moments and the principal axes of inertia.

**4-** In a certain system of units, the electromagnetic the electromagnetic stress tensor $M_{ij}$ is given by

$$M_{ij} = E_i E_j + B_i B_j - \frac{1}{2}\delta_{ij}(E_k E_k + B_k B_k) \, .$$

Knowing that $\vec{E}$ and $\vec{B}$ are vectors, show that $M_{ij}$ is a second-order tensor. Show that $\vec{E} \pm \vec{B}$ are eigenvecotrs of the stress tensor if $|\vec{E}| = |\vec{B}|$, and find the corresponding eigenvalues.