PHYC 467: Methods of Theoretical Physics II

Spring 2016

Homework Assignment #3

(Due March 8, 2016)

1- As we saw in the class, the n-th roots of unity form a cyclic group of order n under multiplication. Show that if m is a divisor of n, then the said group has a subgroup of order m. Discuss whether this subgroup is cyclic too.

2- Find the subgroups H, H' of the symmetric group S_4 that leave the polynomials $x_1x_2 + x_3 + x_4$ and $x_1x_2 + x_3x_4$ invariant respectively. Show that H' contains H as a subgroup.

3- Construct the symmetry group of an equilateral triangle (denoted by $C_{3\nu}$ in crystallography). Write down its multiplication table and show that it is isomorphic to the symmetric group S_3. Find the number of inequivalent irreducible representations of this group and their dimensionality.

4- If a set of matrices Γ is a representation of a group G, show that Γ^* (whose matrices are complex conjugates of the corresponding matrices of Γ) is also a representation of G. Show that the same is not true for Γ^{-1} (whose matrices are the inverse of the corresponding matrices of Γ) and $\Gamma^\dagger$ (whose matrices are hermitian conjugates of the corresponding matrices of Γ), unless G is an abelian group.

5- Let $\Gamma^{(i)}$ and $\Gamma^{(j)}$ be two inequivalent irreducible representations of of a group G. Show that the direct product representation $\Gamma^{(i)} \bigotimes \Gamma^{(j)*}$ does not contain the identity representation. Show also that the direct product of an irreducible representation with its own complex conjugate representation contains the identity representation once and only once. (Hint: You may use Schur's lemmas.)