PHYC 467: Methods of Theoretical Physics II

Spring 2016

Homework Assignment #5

(Due April 21, 2016)

1- Points A, B, C, and D are connected by telephone lines joining AB, AC, BC, BD and CD. There is a probability p that any of the links is broken.

(a) Show that the probability that a call can be amfe from A to B is $1 - p^2 - 2p^3 + 3p^4 - p^5$.

(b) Show that the probability that a call can be made from A to D is $1 - 2p^2 - 2p^3 + 5p^4 - 2p^5$.

2- A continuous random variable X has a probability density function $f(x)$. Show that the variable $Y = F(X)$ is uniformly distributed where $F(x)$ is the cumulative probability function corresponding to $f(x)$.

3- A particle is confined to the one-dimensional box $0 \leq x \leq a$. Classically, it can be in any small interval dx with equal probaility. However, quantum mechanics tells us that the probability distribution is proportional to $\sin^2(n\pi x/a)$, where n is an integer. Find the variance in the particle's position in both the classical and quantum-mechanical picture. Show that the latter tends to the former in the limit of large n, in agreement with the correspondence principle.

4- The probability density function for a Gaussian distribution with mean μ and standard deviation σ is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right].$$

Obtain an expression for the k-th central moment of this distribution.