PHYC 467: Methods of Theoretical Physics II

Spring 2016

Homework Assignment #6

(Due April 29, 2016)

1- The continuous random variables X and Y have a joint probability density function proportional to $xy(x-y)^2$ with $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Find the marginal distributions for X and Y and show that they are negatively correlated with correlation coefficient $-2/3$.

2- A discrete random variable X takes integer values $n = 0,\ 1,\ ...,\ N$ with probabilities p_n. A second random variable Y is defined as $Y = (X-\mu)^2$, where μ is the mean of X. Prove that the covariance of X and Y is given by:

$$\mathrm{Cov}[X,Y] = \sum_{n=0}^{N} n^3 p_n - 3\mu \sum_{n=0}^{N} n^2 p_n + 2\mu^3 \,.$$

Assuming that X takes all of its possible values with the same probability, demonstrate that X and Y can be uncorrelated, even though Y is defined in terms of X.

3- Two continuous random variables X and Y have the following joint probability density function:

$$f(x,y) = A(x^2 + y^2)\,,$$

where A is a constant and $0 \leq x,\ y \leq a$. Show that X and Y are negatively correlated and find the correlation coefficient.

4- Consider that the multinomial probability distribution:

$$M_n(x_1,,...,x_k) = \frac{n!}{x_1!...x_k!}\ p_1^{x_1}...p_k^{x_k}\,.$$

Show that in the limit where $n \to \infty$ and $np_i = \lambda_i$ remains finite, it can be approximated by a multiple Poisson distribution with $k-1$ factors:

$$M_n'(x_1,...,x_{k-1}) = \prod_{i=1}^{k-1} \frac{e^{-\lambda_i}\lambda_i^{x_i}}{x_i!}\,.$$

[Hint: You may use the approximations $n! \approx n^{\epsilon}[(n-\epsilon)!]$ and $(1-a/n)^n \approx e^{-a}$ for large n.]