PHYC 467: Methods of Theoretical Physics II

Spring 2016

Homework Assignment #7

(Due May 9, 2016)

1- Consider a sample of size N where the sample elements are independent. Assuing that the parent population has zero mean, show that:

$$V[s^2] = \frac{N-1}{N^3}[(N-1)\mu_4 - (N-3)\mu_2^2] \,.$$

2- Sample values x_i, $i = 1, \ldots, N$, are drawn independently from the distribution $P(x|\tau) = (1/\tau)\exp(-x/\tau)$. Find the maximum likelihood estimate of the parameter $\lambda = 1/\tau$ (denoted by $\hat{\lambda}$) in terms of the measured values, and show that:

$$E[\hat{\lambda}] = \frac{N}{N-1}\lambda \,.$$

3- Consider the linear estimator $\hat{\mu} = \sum_{i=1}^{n} a_i x_i$. Impose the conditions (i) that it is unbiased and (ii) that it is as efficient as possible, and prove that the sample mean is the best linear unbiased estimator of the population mean μ.
[Hint: If the real nubers $a_1, \ldots, a_n$ satisfy the contraint $\sum_{i=1}^{n} a_i = C$, where C is a given constant, show that $\sum_{i=1}^{n} a_i^2$ is minimized when $a_i = C/n$ for all i.]

4- Each of a series of experiments consists of a large, but unknown, number n of trials in each of which the probability of success p is the same but unknow. In the ith epxeeriment, $i = 1, \ldots, N$, the total number of successes is $x_i \gg 1$. Determine the log-likelihood function. Using the Striling's approximation for $\ln(n-x)$, show that:

$$\frac{d\ln(n-x)}{dn} \approx \frac{1}{2(n-x)} + \ln(n-x) \,,$$

and evaluate $\partial({}^nC_x)/\partial n$.

By finding the equations determining the maximum likelihood estimators $\hat{p}$ and $\hat{n}$, show that to order $1/n$ they must satisfy the simultaneous arithmetic and geometric mean constraints:

$$\hat{n}\hat{p} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad , \qquad (1-\hat{p})^N = \prod_{i=1}^{N}\left(1 - \frac{x_i}{\hat{n}}\right) .$$