PHYC 523: QAuantum Field Theory I

Fall 2016

Homework Assignment #1

(Due September 14, 2016)

1- Consider a massless real scalar field ϕ with the following Lagrangian density λ being a real constant):

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{4!}\lambda \phi^4 \, .$$

Show that the action of this scalar field is invarinat under dilatation transformation (a being an arbitrary real number):

$$x_\mu \to a x_\mu \quad , \quad \phi \to a^{-1}\phi \, .$$

In general, the Noether current corresponding to dilatation invariance of the action is:

$$j^\mu_{\rm D} = T^{\mu\rho} x_\rho + \frac{1}{2}\partial^\mu \phi^2 \, ,$$

where $T^{\mu\rho}$ is the energy momentum tensor. Verify that for the above Lagrangian density this current is divergenceless.

2- In general, there are ambiguities in the form of the energy-momentum tensor $T_{\mu\nu}$. As an example, consider the new definition:

$$\Theta_{\mu\nu} \equiv T_{\mu\nu} + a(\partial_\mu \partial_\nu - g_{\mu\nu}\partial_\rho \partial^\rho)\phi^2 \, ,$$

where a is a dimensionless constant. Show that $\partial_\mu \Theta^{\mu\nu} = 0$.

For the amssless scalar field in problem 1, show that $\Theta^\mu_\mu = 0$ if $a = -1/6$. Using this "new improved energy-momentum tensor", we can define a new dilatation current as follows:

$$j^{\mu\prime}_{\rm D} \equiv x_\rho \Theta^{\mu\rho} \, .$$

Show that this new current is related to the original dilatation current $j^\mu_{\rm D}$ in problem 1 as follows:

$$j^{\mu\prime}_{\rm D} = j^\mu_{\rm D} + \frac{1}{6}\partial_\rho (x^\mu \partial^\rho - x^\rho \partial^\mu)\phi^2 \, ,$$

and it is divergenceless as well. Use this to prove that dilation invariance is equivalent to traceleessness of $\Theta_{\mu\nu}$.

3- Problem **2.1** from Peskin and Shroeder.

4- Problem **2.2** parts **(a)-(c)** from Peskin and Schroeder.