PHYC 581: High Energy Astrophysics

Fall 2018

Homework Assignment #1

(Due September 24, 2018)

1- The use of microcalorimeters for the detection of X-rays constitutes a relatively new technology. The basic idea behind their design is that the absorption of an incoming X-ray produces a measurable change in temperature, whose value is proportional to the energy of incoming photon. But building such devices is not that easy:

(a) To see why, calculate how much a 6 keV X-ray photon would raise the temperature of a penny at room temperature. Modern U.S. pennies consist of zinc with a thin copper plating, with a specific heat capacity $C \approx 0.38$ J g^{-1} K^{-1}. What degree of accuracy would be required to measure this temperature, and is that feasible?

The microcalorimeter detector relies on the temperature change induced within a tiny flake of silicon semiconductor, whose conductivity depends on the amount of impurity added to the silicon and on its temperature. Consider 1 gram of silicon held at 0.1 Kelvin. What must its heat capacity be in order for a 6 keV X-ray photon to cause a temperature rise of about 4 μK? (A fractional change of few parts in 10^5 is measurable with today's technology)

(b) Microcalorimeters can measure the energy of an incoming X-ray to an accuracy of 0.3%, or better. Compare this with a charged-coupled device (a CCD), in which the photon energy is shared among many electrons, each of which carries a typical energy ≈ 3.65 eV. The energy of the absorbed X-ray is determined from the number of liberated CCD electrons, which has an uncertainty due to Poisson distribution. What kind of resolution should one expect from this older type of instrument?

2- In this problem we examine several important characteristics of X-ray detectors.

(a) A crucial consideration for detectors that record the arrival of individual photons is detector *dead time*. If the count rate from a bright X-ray source is high enough, the X-ray detector may miss a legitimate count because it is still processing a previous event. The detector is "dead" for a time Δt after registering a count. Work out a simple equation for the actual count rate R' in terms of the measured count rate R and Δt. Counts that are missed do not themselves cause dead time. If we measure a source to have a count rate of $R = 100$ counts s^{-1} and the dead time is $\Delta t = 1$ ms, what is the actual count rate?

(b) At X-ray energies of 2-10 keV, the brightest steady source in the sky (excluding the Sun) is Sco-X1, with a flux of 3×10^{-7} erg cm^{-2} s^{-1} in this bandpass. Calculate the approximate flux from this source assuming an average photon energy of 4 keV. Use this to estimate the count rates expected for the following two detectors, each consisting of a proportional counter behind a collimator: *Uhuru* (the first X-ray satellite observatory; proportional counter effective area 280 cm^2, which is a third of the geometric area of 840 cm^2 due to the less than 100% efficiency in detecting photons) and the proportional counter array (PCA) of RXTE (effective area 920 cm^2 for one of the five units; the geometric area of the unit is about 1580 cm^2).

Repeat the above calculations for the Crab Supernova remnant (flux 2×10^{-8} erg cm^{-2} s^{-1}), Perseus cluster of galaxies (flux 1×10^{-9} erg cm^{-2} s^{-1}), and quasar 3C273 (flux 8×10^{-11} erg cm^{-2} s^{-1}). These are all among the brightest objects in their class. The dead time for each PCA unit is 8.8 μs. Are dead-time corrections important for the PCA when observing Sco-X1.

(c) Consider a detector with a count rate for the background of B. If photons from a source are detected at the rate R, then the signal (i.e., the total number of counts) detected in time t is $S = Rt$ and noise in this signal is $N = \sqrt{Rt + Bt}$. Using your results from above, calculate how long an integration time the *Uhuru* satellite needed to measure the brightness of 3C273 with $S/N = 100$ (an accuracy of 1%). Make the calculation using both $B = 0$ and with the true value of $B = 10$ counts s^{-1}. Note that the above estimate of the noise assumes that B is known perfectly, which is not the acse in X-ray astronomy because B varies with the position of the satellite in its orbit and with time.

(d) The *Einstein* X-ray Observatory had mirrors with a total collecting area of 350 cm^2. If the Imaging Proportional Counter (IPC) at the focus of the mirrors detects half of the photons incident on the mirrors (due to losses in the mirrors and IPC), calculate the count rate expected for 3C273 using your photon flux from above. If a background of 10 counts s^{-1} is uniformly spread throughout the IPC, which had an approximately square field of view with a side of 75 arcminutes, and all of the counts from 3C273 are concentrated in a spot 1 arcminute (the resolution of the IPC), will the background contribute significant uncertainty to the measurement of the flux from 3C273?

3- The precision with which sources are measured is critically important to how reliably a detection is believed to have been made. In measurements governed by counting statistics, one must define an error very carefully because in principle there is no absolute limit to the range of fluctuations associated with a measured quantity. A measurement can always produce a number greater than a quoted limit if one is willing to wait long enough.

Every instrument produces its own characteristic background noise. Some of it is statistical - e.g., due to random events - but the rest may be systematic, meaning that the instrument does not perform exactly as expected. Though the latter needs to be evaluated case by case, the former can actually be quantified.

(a) For a steady source producing an average number of counts m over a fixed interval, the probability of measuring x events during that interval is given by the *Poisson* distribution

$$P(x) = \frac{m^x e^{-m}}{x!} \, .$$

Show that $\sum_{x=0}^{\infty} P(x) = 1$.

(b) If instead the variable to be measured is continuous (e.g., when the count is so high that its variation may be described differentially), it is more appropriate to use the *normal distribution* to describe the differential probability $dP(x)$ of finding a value x within the interval $(x, x + dx)$:

$$dP(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right] \, ,$$

where σ characterizes the width of the distribution. Show that the full width half maximum (FWHM) of the normal distribution is 2.36σ.

(c) Show that the normal distribution approximates the Poisson distribution very well for large m, if $\sigma = \sqrt{m}$.

(d) Suppose that a CDD is counting photons from a variable source with an average count rate of 100 photons per given exposure time. During a particular observation, however, it measures 130. What is the probability that this higher count rate satisfies an actual brightening of the source, as opposed to a mere statistical fluctuation? Is this significant?

4- How far should we expect X-rays to propagate through the intergalactic medium? Suppose the gas in the intergalactic medium is ionized and has an average electron density $n_e \approx 2 \times 10^{-7}$ cm^{-3}. What is the mean-free-path (in Gpc) to Thomson scattering through this plasma? (The cross section for this process is $\sigma_T = 0.66 \times 10^{-24}$ cm^2.) Should we expect to see X-ray sources at a distance corresponding to a significant fraction of the size of the visible universe ~ 13.5 Glyr?

5- Typically, a neutrino of energy E_ν has a scattering cross section of

$$\sigma \approx 10^{-44} \left(\frac{E_\nu}{m_e c^2}\right)^2 \ \text{cm}^2$$

off nucleons when $E_\nu \ll 1$ GeV.

(a) Find the mean-free-path of a neutrino with energy $E_\nu = 10$ MeV (produced from Boron decay in the *pp* III chain of Hydrogen burning) inside the Sun. The average density of the Sun is $\bar{\rho}_\odot \sim 1$ g cm^{-3}.

(b) Now consider a neutron star that is formed as a result of supernova core collapse. For a neutron star $M_{\rm NS} \approx M_\odot$ and $R_{\rm NS} \sim 10$ km. What is the mean-free-path of a neutrino with $E_\nu = 10$ MeV (the typical energy for supernova neutrinos) inside a neutron star?