PHYC 581: High Energy Astrophysics

Fall 2018

Homework Assignment #2

(Due October 10, 2018)

1- Recall the relation

$$\frac{dE}{dt} = \vec{F} \cdot \vec{v}$$

in Newtonian mechanics, where E is the total energy of a particle that is moving with velocity $\vec{v}$ and $\vec{F} = d\vec{p}/dt$ is the net force. Show that this relation is also valid in relativistic mechanics.

2- In the quasar $3C279$, EGRET aboard the *Compton* Gamma Ray Observatory observed a non-thermal spectral component extending up to 30 GeV. If the particles producing these γ-rays were for some reason accelerated by an electric field E within 10^{14} cm of the black hole, what would be the value of E?

3- A process analogue to Fermi acceleration involves plasma oscillations instead of magnetized gas clouds or shock waves. This process draws energy from the plasma waves and transfers it to the particles.

(a) Consider a sinusoidal electric field

$$\vec{E}(x,t) = E_0 \cos(kx - \omega t)\hat{z} \, .$$

Let a particle of mass m, and charge e "suddenly" be placed within this field. Show that the kinetic energy imparted to the particle by the wave is

$$\frac{e^2 E_0^2}{2m(kv_x - \omega)^2} \sin^2[(kv_x - \omega)t] \, ,$$

where the particle's velocity vector is written as $\vec{v} = (v_x, v_y, v_z)$.

(b) Assume non-relativistic motion, and let the velocity distribution function $f(v_x)$ be slowly varying near the wave phase velocity ω/k. Show that the power imparted to the particles by the wave is

$$P \approx \frac{\pi e^2 E_0^2 f(\omega/k)}{2mk} \, .$$

(c) How long would it take an electron top reach a velocity $v_z = c/2$ (from $v_z = 0$) if $v_x = \omega/2k$ and $E_0 = 1000$ V/cm? How about the case $v_x = 0.9\omega/k$?

4- Consider a region of uniform electric field $\vec{E} = E\hat{z}$, in which charges e of mass m are injected. The conductivity in this region is σ. If the resistivity of the plasma is due just to particle-particle collisions, estimate the maximum Lorentz factor $\gamma_{\rm max}$ attained by the charges. You may assume that E is so large that the particle velocity quickly reaches c. What is $\gamma_{\rm max}$ for $E = 10^{10}$ statvolt cm^{-1} and $\sigma = 10^6$ mhos?

5- Consider the motion of a point charge in crossed (i.e., perpendicular) uniform fields $\vec{E}$ and $\vec{B}$.

(a) Show that the fields in the frame drifting with velocity $\vec{v} = c\vec{E} \times \vec{B}/B^2$ are

$$\vec{E}' = 0 \quad , \quad \vec{B}' = \vec{B}\left(1 - \frac{E^2}{B^2}\right)^{1/2} ,$$

where B and E denote the magnitude of $\vec{B}$ and $\vec{E}$ respectively. What is the significance of this frame?

(b) Show that in this drifting frame, the particle undergoes circular motion with angular frequency

$$\omega' = \frac{u'_\perp}{R'} ,$$

where R' is the radius of the circle and $u'_\perp$ is the component (perpendicular to the magnetic field) of the particle's velocity relative to the drifting frame. Find R' and ω'.

(c) Consider the special case when a particle starts from rest in the laboratory. Show that each of the cycles in the space is enlarged by a factor γ^2 in the perpendicular direction and by γ^3 in the drift direction as compared with the non-relativistic limit.