PHYC 581: High Energy Astrophysics

Fall 2018

Homework Assignment #4

(Due November 12, 2018)

1- Consider a binary system in which the compact star and its companion have masses M_1 and M_2, respectively, and a mass ratio $q \equiv M_2/M_1 < 0.5$.

(a) Derive an expression for the mean density $\bar{\rho}$ of the lobe-filling star (M_2) in terms of the binary period T, expressed in unit of hours.

(b) Assuming that the lobe-filling star is close to the lower main sequence (for which $M/M_\odot \approx R/R_\odot$) and that the solar mean density is $\rho_\odot = 1.4$ g cm^{-3}, show that

$$\bar{\rho} = \left(\frac{M_\odot}{M_2}\right)^2 \,(1.4 \text{ gcm}^{-3})\,.$$

(c) Derive the period-mass relation, and the period-radius relation (with the period always expressed in units of hours).

2- Z Chamaeleontis is a dwarf nova, consisting of an $M_1 = 0.85M_\odot$ white dwarf with radius $R_1 = 0.0095R_\odot$ orbiting a late M-type main sequence secondary star of mass $M_2 = 0.17M_\odot$. The orbital period of T of this binary has been measured at 0.0745 days. Provide a description of this system, including the size of the Roche lobe and the locations of the L_1 Lagrangian point. Is the secondary filling its Roche lobe? (Late M-type main-sequence stars have a radius ranging from 2.5×10^{10} cm for an M_2, down to $\approx 0.8 \times 10^8$ cm for an M_6.)

3- An X-ray binary is known to contain a neutron star (NS) in orbit around a main sequence star with mass $M_2 = 0.5M_\odot$, which is NOT filling its Roche lobe. The orbital period of this system is $T = 5.0$ hrs. However, the NS is emitting X-rays that are believed to be generated at its surface when accreting plasma thermalizes the energy it has gained during its infall onto the NS.

This plasma is apparently a portion of the wind blown off the surface of the companion (not Roche lobe overflow!). But it is clear that the required mass-loss rate $\dot{M}_w$ is much higher than the normal stellar wind for this star. It appears that what drives the mass overflow is the irradiation of the companion's surface by X-rays from the NS (i.e., self-excited accretion process). Our task here is to determine if this model is viable.

(a) Use the data and the relevant binary relations to argue that the companion really does not fill its Roche lobe.

(b) Assume that the secondary's wind is (i) optically thin, (ii) isotropic (the fact that the secondary is rotating means its entire surface is irradiated), and (iii) that the overflowing gas attains a velocity equal to twice the escape velocity at the stellar surface. For a "standard" NS, what is the capture rate $\dot{M}_{acc}$ due to Bondi-Hoyle accretion from this wind in terms of $\dot{M}_w$? Assume the NS is not magnetized.

(c) What are the temperature and luminosity of the emitted X-rays in terms of $\dot{M}_w$? Assume the X-rays are emitted isotropically.

(d) Assume that half of the X-rays impinging on the surface of the companion are absorbed and lead directly to generating $\dot{M}_w$. Is a self-sustained mass transfer as described above feasible?

4- In a related system, let us consider accretion by the black hole candidate in Cygnus X-1 via Bondi-Hoyle capture of the companion's wind. Cygnus X-1 has a binary period of 5.6 days, and the mass function suggests the compact object has a mass $\sim 10M_\odot$ in orbit about an O/B supergiant star. The O/B star loses $1M_\odot$ in 10^6 years, and the wind speed is equal to the escape velocity at its surface.

(a) Estimate the accretion rate onto the black hole $\dot{M}_{bh}$.

(b) Estimate the accretion efficiency η if the observed X-ray luminosity is $L_{X-ray} \approx 5\times10^{37}$ erg s^{-1}. The efficiency may be defined as $\eta \equiv L_{X-ray}/\dot{M}_{bh}c^2$.

5- Consider a $10^8 M_\odot$ Schwarzschild black hole accreting from a disk at a rate of $\dot{M} = 0.1M_\odot$ yr^{-1}. The matter accreting through the disk loses angular momentum via dissipation until it reaches the last stable orbit at $r = 3r_S$, where $r_S \equiv 2GM/c^2$ is the Schwarzschild radius. After that, the matter falls quickly into the black hole without much dissipation, carrying all of its angular momentum with it.

(a) Estimate the rate at which the angular momentum of the black hole increases. The specific angular momentum at $r = 3r_S$ is $l_0 = 2\sqrt{3}GM/c$.

(b) Roughly how long will it take to spin the black hole up to a maximally rotating Kerr black hole? The angular momentum of such a black hole is $J = GM^2/c$.