

Lab 3: AC Low-pass filters (version 1.3)

WARNING: Use electrical test equipment with care! Always double-check connections before applying power. Look for short circuits, which can quickly destroy expensive equipment.

Partial list of equipment needed:

Digital oscilloscope

Function generator

ELVIS board

Scope probes

Resistors: 200 Ω and 1 k Ω (not critical)

Capacitor: 100 nF (not critical, substitute as needed)

Inductor: 3.9 mH (not critical, substitute as needed)

Introduction to the Elvis board

The National Instruments Elvis board is a powerful platform for circuit development. One of its many functions is measurement of complex impedance, including the pure reactance of capacitors and inductors. Acquire a capacitor with a value around 100 nF and an inductor in the range of 3—5 mH. Turn on the main Elvis board power with the switch located behind it. Turn on the 2nd power switch at upper right. Insert the desired component (capacitor or inductor) in the DMM/Impedance analyzer using pins OUT+ and OUT-. Start the NI ELVISmx Instrument Launcher program on the adjacent PC (found under the National Instruments menu). Select the DMM; an I/O device should be identified (this can take several seconds). Select the appropriate circuit diagram element for capacitor or inductor and push the run button to measure the capacitance or inductance. Record both values. Resistors are measured with the V- Ω and COM connections located on the far left side of the Elvis board.

Frequency response and complex impedance

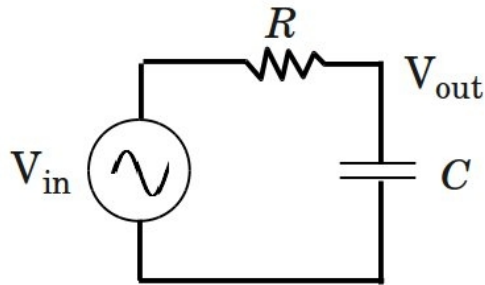
The following measurements will explore the frequency response of an R-C and L-R circuit. They will show the frequency-dependent behavior of a circuit element (capacitor or inductor) and the importance of phase-shift on AC signals. This behavior can be conveniently modeled using complex impedance. For example, assuming a steady-state periodic input such as a sinusoidal wave $V_o \sin(\omega t)$, the impedance of a capacitor (C) in the same circuit can be written: $Z_C = 1/j\omega C$, where $j = \sqrt{-1}$. You will be building a voltage divider network as in the previous experiments, but one of the resistors is replaced by a capacitor or inductor.

First, test your scope probes on CH 1 and CH 2 to make sure they are operating properly. Consult the previous lab instructions if needed. Setup the function generator to produce

a 1 Volt p-p sine wave at 1 kHz.

Note 1: The oscilloscope amplitude may differ from the function generator setting by a factor of 2. This is because the output impedance of the function generator is set to $50\ \Omega$. Change it to high impedance (High Z) using the Utility menu.

Using the same guidelines as in the previous experiments, build the following circuit on the breadboard using the resistor and capacitor measured above (nominally $R = 200\ \Omega$ and $C = 0.1\ \mu\text{F}$):



V_{in} is supplied by the function generator and V_{out} is the voltage across the capacitor. Measure V_{in} on CH 1 of the scope. You can use a scope probe connected to R; clip the reference lead to ground. The less cluttered option is to use a BNC T-connector to route the function generator signal directly to the CH 1 input (i.e. scope probe not needed) and to the circuit with a BNC/EZ-hook cable. Connect a scope probe to CH 2 and clip it across C to measure V_{out} .

Measurement of the signal waveforms

Measure the amplitude and phase of V_{out} relative to the V_{in} as a function of frequency. The amplitude response is simply the ratio of peak-to-peak voltage for the two channels. A frequency range between 1 and 30 kHz should provide enough data to map out the response, depending on the choice of R and C. The absolute frequency start and end points are not critical; take enough data (8--10 points is adequate, more is better) to show the drop off in signal with increasing frequency. Increase the frequency high enough to make V_{out}/V_{in} less than 0.1.

The frequency dependent voltage amplitude change at V_{out} is a straightforward measurement, but determining the voltage phase difference is more challenging. You are displaying two waveforms and can use the adjustable cursors to measure the time delay between key points. This time delay can be directly converted to a phase difference because the period of the waveforms is known. In addition, many modern digital scopes can measure the phase difference directly, displaying the result in degrees of phase. This can be accessed in the measurement menu. If your scope has this capability, perform both measurements and confirm they agree.

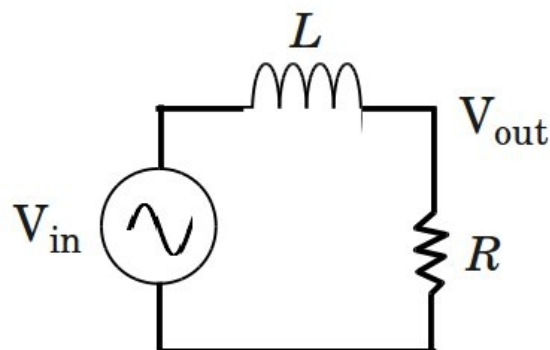
Note 2: When measuring phase, it is *critical* to keep track of what is being measured relative to what. For example, the phase difference $\Delta\phi = -45^\circ$ could be interpreted as $\Delta\phi = +315^\circ$ depending on the perspective. Just as with voltage, phase is always a *relative* measurement. It has a point of reference and it must be correctly accounted for in your analysis.

Note 3: The R-C circuit is one of the most important concepts in electronic instrumentation. An experimentalist must be constantly aware of it because there is always stray capacitance present, i.e. unintentional capacitance that can affect the integrity of a measurement. The cutoff frequency of an R-C circuit is defined as $f_c = \omega_c/2\pi = 1/2\pi RC$. The circuit shown above is a low-pass filter because the output signal is reduced as the frequency increases in the neighborhood of f_c . If the resistor and capacitor are exchanged, the circuit becomes a high-pass filter.

If you look at the input connection of any oscilloscope, the input impedance associated with it will be labeled. A typical scope may have a $1\text{ M}\Omega$ input resistance in parallel with 20 pF of stray (unintentional but unavoidable) capacitance. This means the scope may act as a low-pass filter with a cutoff frequency of around 8 kHz . The cutoff can be increased to nearly 160 MHz by using a $50\ \Omega$ BNC terminator, which will be in parallel with the internal $1\text{ M}\Omega$ resistance. This value is chosen because it is comparable with the characteristic impedance of coaxial cable. When a $50\ \Omega$ terminator is used, however, the measured voltage levels will drop accordingly.

Low-pass filter with inductor

The low-pass filter can also be implemented with an inductor and resistor arranged as follows:



Build this circuit with the second resistor having a value of around $1000\ \Omega$ and the inductor measured previously (in the range $3\text{--}5\text{ mH}$) to produce a cutoff frequency near 40 kHz (not critical) according to the formula $f_c = \omega_c/2\pi = R/2\pi L$. Perform the same set of measurements as with the R-C filter, but adjust your data range to account for the higher cutoff frequency. The goal is to show the frequency response rolloff with data points

above and below the cutoff frequency. Get data for both amplitude and phase.

Note 4: The L-R circuit is rarely a concern in electronic design except at radio frequencies (rf) and higher. This is because stray inductance is usually negligible. If the inductor and resistor are exchanged, the circuit becomes a high-pass filter.

Analysis and writeup

The voltage measured at the output of the R-C circuit is given by:

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

and for the L-R circuit:

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega L/R}$$

To model the experimental response, you will need to put the right-hand side of the above equations in the form $a + jb$, where a and b are the real and imaginary parts of the complex number. The frequency-dependent amplitude response is then:

$$\frac{V_{out}}{V_{in}} = \sqrt{a^2 + b^2}$$

and the phase response is:

$$\phi(\omega) = \tan^{-1} \left(\frac{b}{a} \right)$$

Don't forget that the function generator frequency display (Hz) is related to the angular frequency (radians/sec) by the following equation: $f = \omega / 2\pi$.

The lab writeup requires the following:

1) Plot V_{out}/V_{in} for the R-C circuit. Because this is a complex response, you will need to plot the amplitude and phase data on separate graphs. The amplitude response is a voltage ratio and is dimensionless on the y-axis. Display the phase data in degrees (not radians). The x-axis of both plots should be frequency (f) in units of Hz or kHz.

2) Repeat the analysis described in 1) for the R-L circuit.

3) The four data sets generated in 1) and 2) have corresponding theoretical curves. These continuous curves are obtained with the equations described above and using your measured values for R, L, and C. Add the theory curves to the four plots from 1) and 2) (i.e. you should produce two amplitude curves and two phase curves). List the measured component values. Include your equation formulas on each plot.

There is no additional writing or explanation required. Your assignment is to generate four graphs (2 amplitude graphs, 2 phase graphs with your data compared to theory) corresponding to the two circuits. The report is due before the next class. E-mail your 4 graphs as a single electronic file (.pdf preferred) to the instructor.