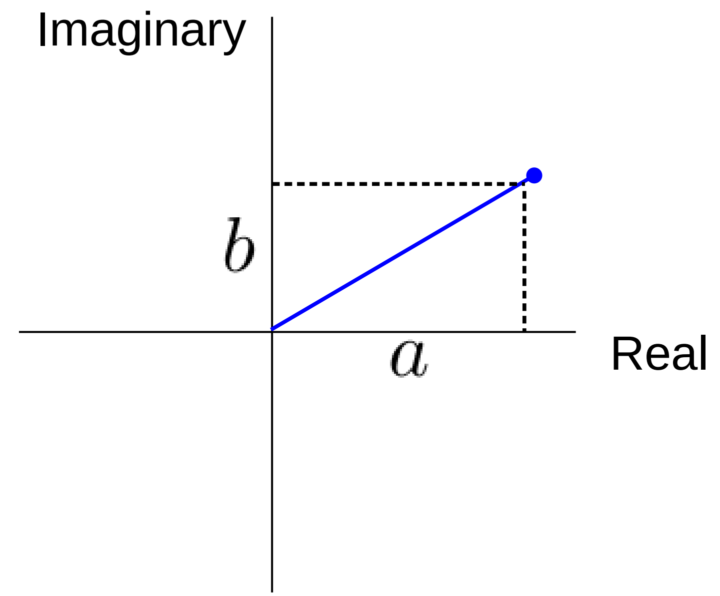
Lab 3: Capacitors and Inductors in AC circuits

Review of Complex Numbers

$$j = \sqrt{-1}$$

$$z = a + jb$$



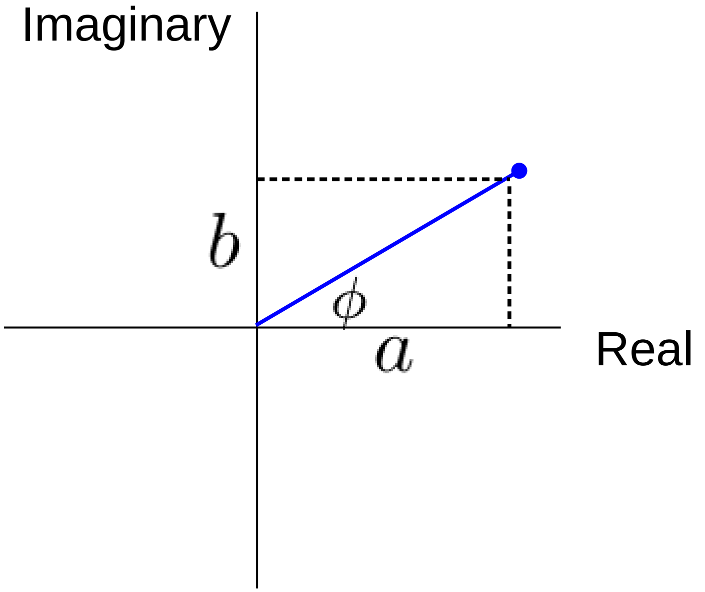
Review of Complex Numbers

$$j = \sqrt{-1}$$

$$z = a + jb = Ae^{j\phi}$$

$$A = \sqrt{a^2 + b^2}$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$



Review of Complex Numbers

$$j = \sqrt{-1}$$

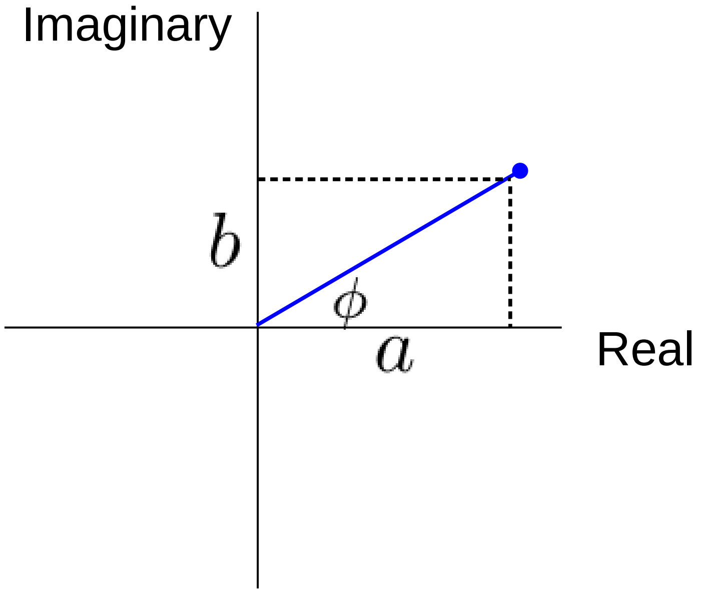
$$z = a + jb = Ae^{j\phi}$$

$$A = \sqrt{a^2 + b^2}$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right)$$

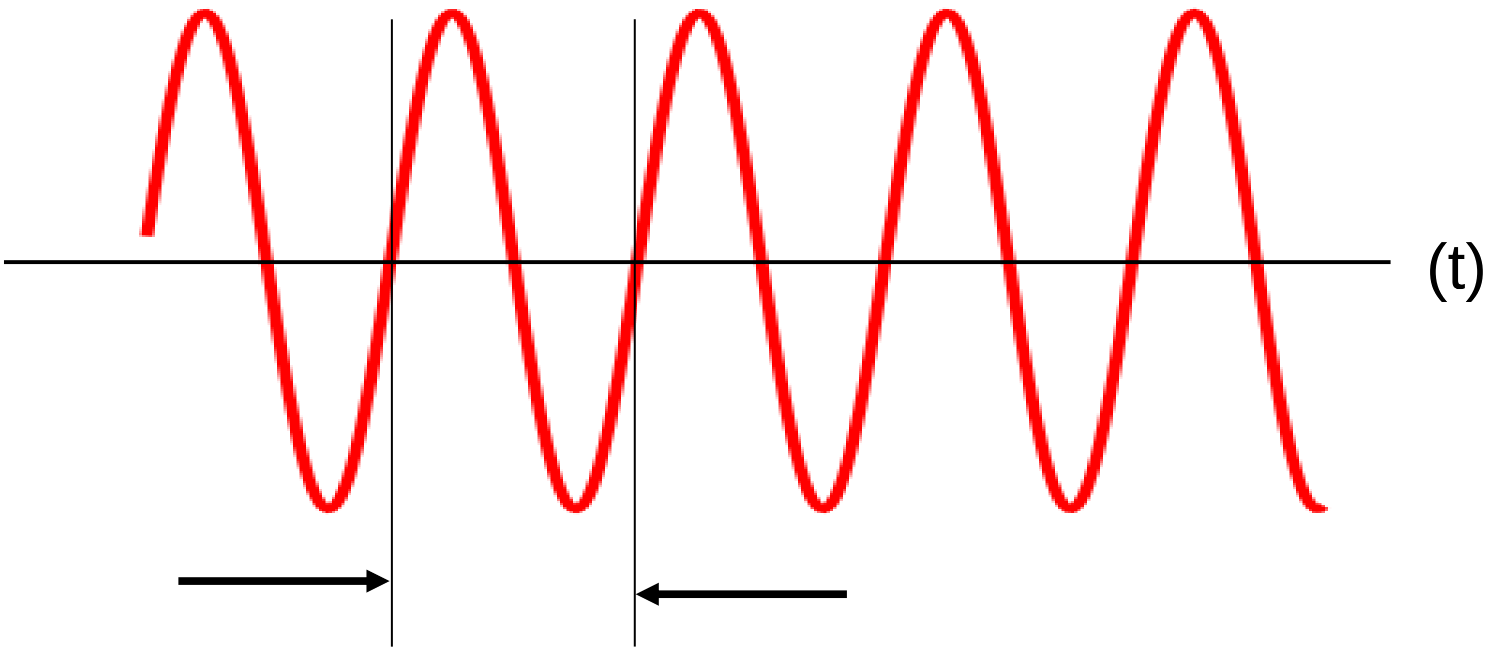
$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$e^{j\frac{\pi}{2}} = \cos(\tfrac{\pi}{2}) + j\sin(\tfrac{\pi}{2}) = j$$



OUR ESSENTIAL WORKING ASSUMPTION:

AC ≡ **SINUSOIDAL FUNCTION**



T: *Period*

Frequency: **f** = 1/**T**

Capacitor in AC circuit

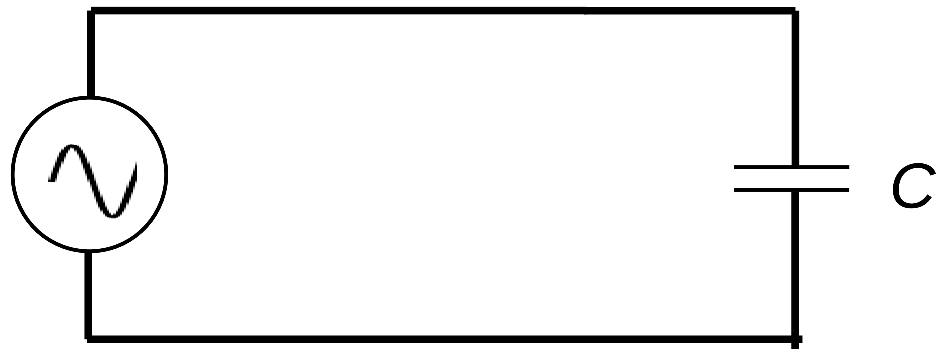
$$V_{ac} = V_o \sin \omega t$$

$$\omega = 2\pi f$$

ω: radians/sec
f: Herz (1/sec)

Capacitor in AC circuit

$$V_{ac} = V_o \sin \omega t$$

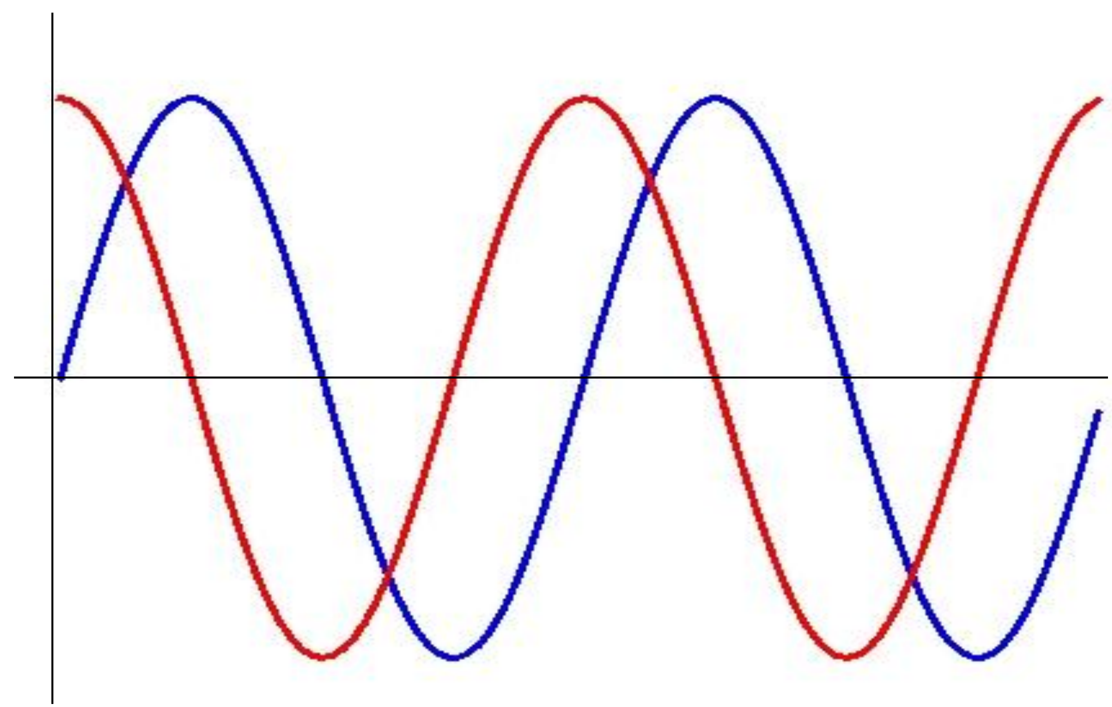


$$\omega = 2\pi f$$

ω : radians/sec
f: Herz (1/sec)

VOLTAGE

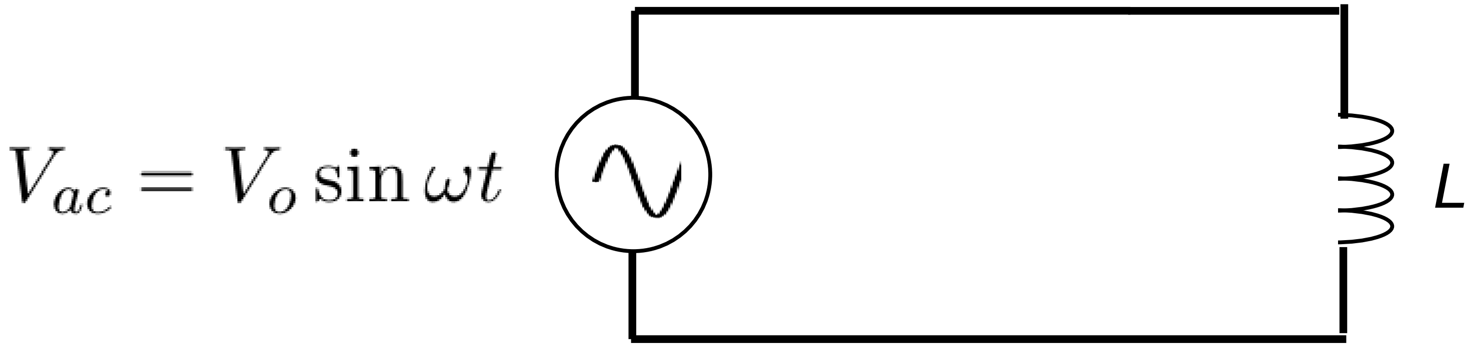
CURRENT

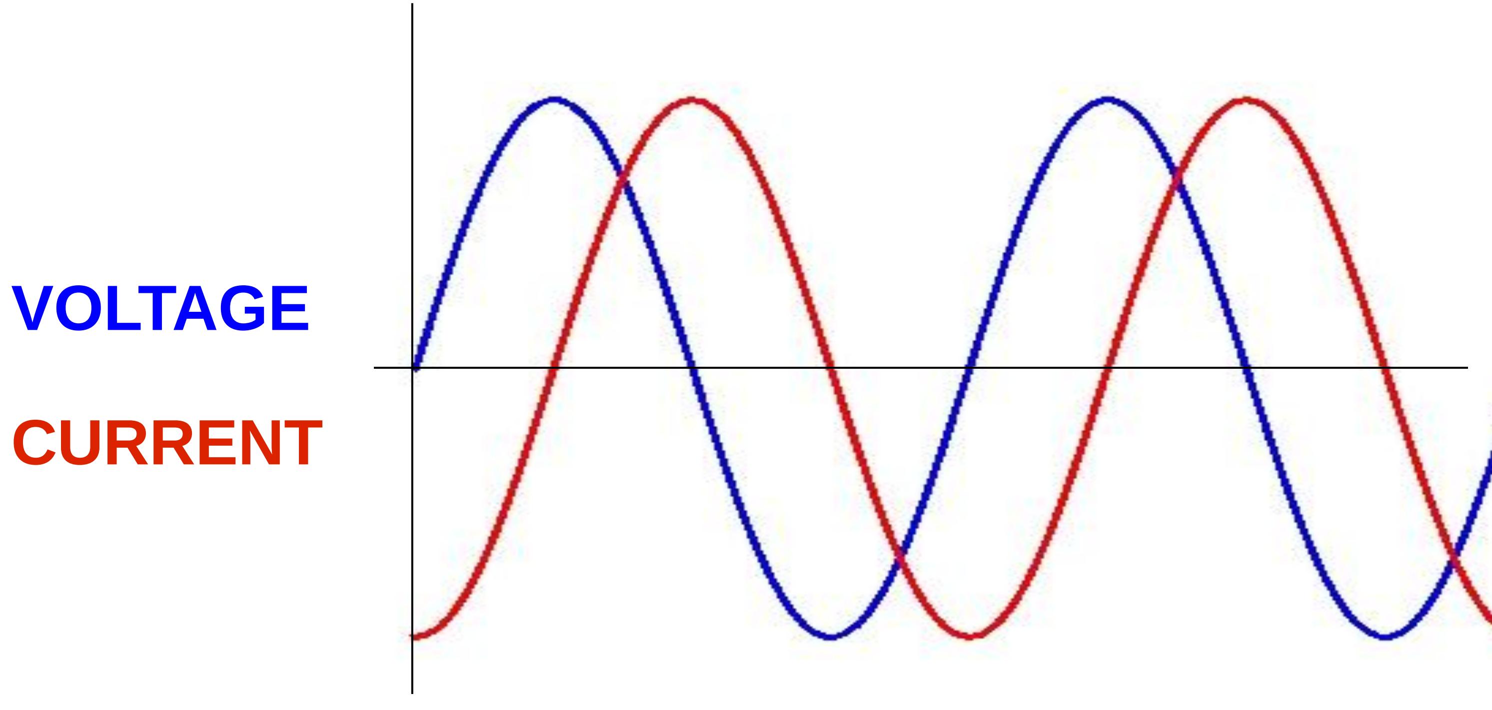


Inductor in AC circuit

$$V_{ac} = V_o \sin \omega t$$

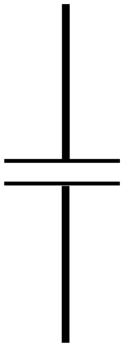
Inductor in AC circuit





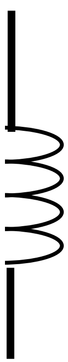
Ohm's Law for L and C:

Impedance (*Z*)



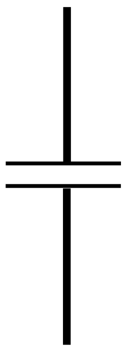
C

$$Z_C = \frac{V_C}{I_C} = \frac{V_o \sin \omega t}{\omega C V_o \cos \omega t} = \frac{V_o \sin \omega t}{\omega C V_o \sin\left(\omega t + \frac{\pi}{2}\right)}$$



L

Ohm's Law for L and C: Impedance (*Z*)



C

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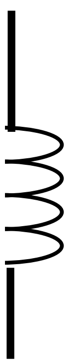
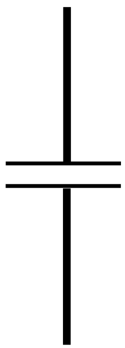


L

$$Z_L = \frac{V_L}{I_L} = \frac{V_o \sin \omega t}{-\left(V_o/\omega L\right) \cos \omega t} = \frac{V_o \sin \omega t}{-\left(V_o/\omega L\right) \sin\left(\omega t + \frac{\pi}{2}\right)}$$

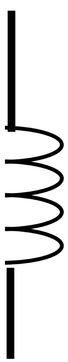
Ohm's Law for L and C:
Impedance (*Z*)



C

$$Z_C = \frac{V_C}{I_C} = \frac{V_o \sin \omega t}{\omega C V_o \cos \omega t} = \frac{V_o \sin \omega t}{\omega C V_o \sin\left(\omega t + \frac{\pi}{2}\right)}$$

90º phase shift

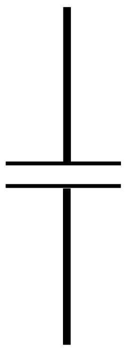


L

$$Z_L = \frac{V_L}{I_L} = \frac{V_o \sin \omega t}{-\left(V_o/\omega L\right) \cos \omega t} = \frac{V_o \sin \omega t}{-\left(V_o/\omega L\right) \sin\left(\omega t + \frac{\pi}{2}\right)}$$

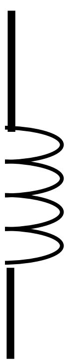
90º phase shift

Ohm's Law for L and C:
Impedance (*Z*)



C

$$Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C}$$



L

$$Z_L = \frac{V_L}{I_L} = \frac{\omega L}{-j} = j\omega L$$

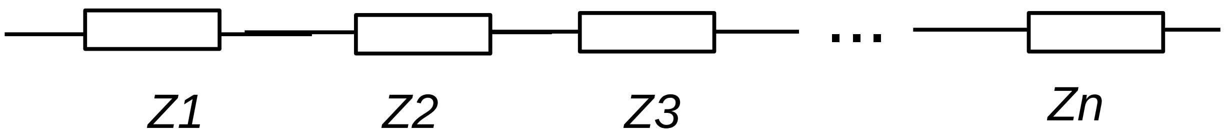
90º phase-shift in polar form: $e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = j$

Impedance of a Resistor:

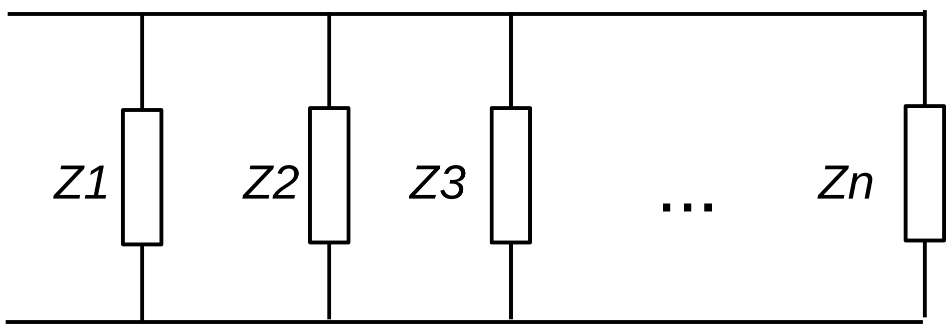
R

$$Z_R = \frac{V_R}{I_R} = R$$

Impedance arithmetic: same as a resistor

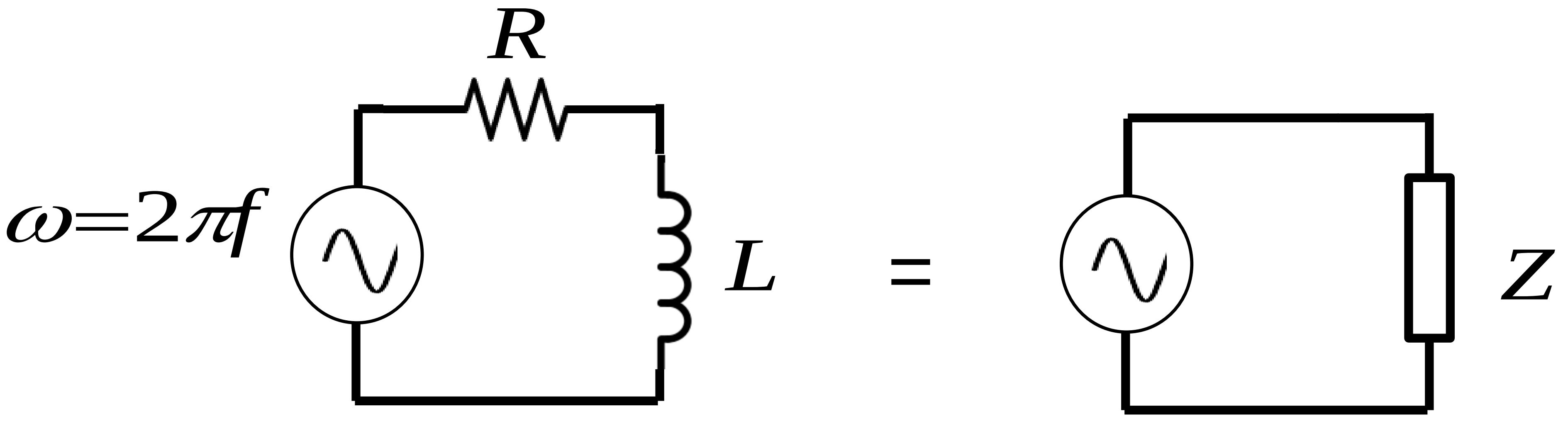


$$Z_{total} = \sum_{i}^{n} Z_i$$



$$\frac{1}{Z_{total}} = \sum_{i}^{n} \frac{1}{Z_i}$$

EXAMPLE 1



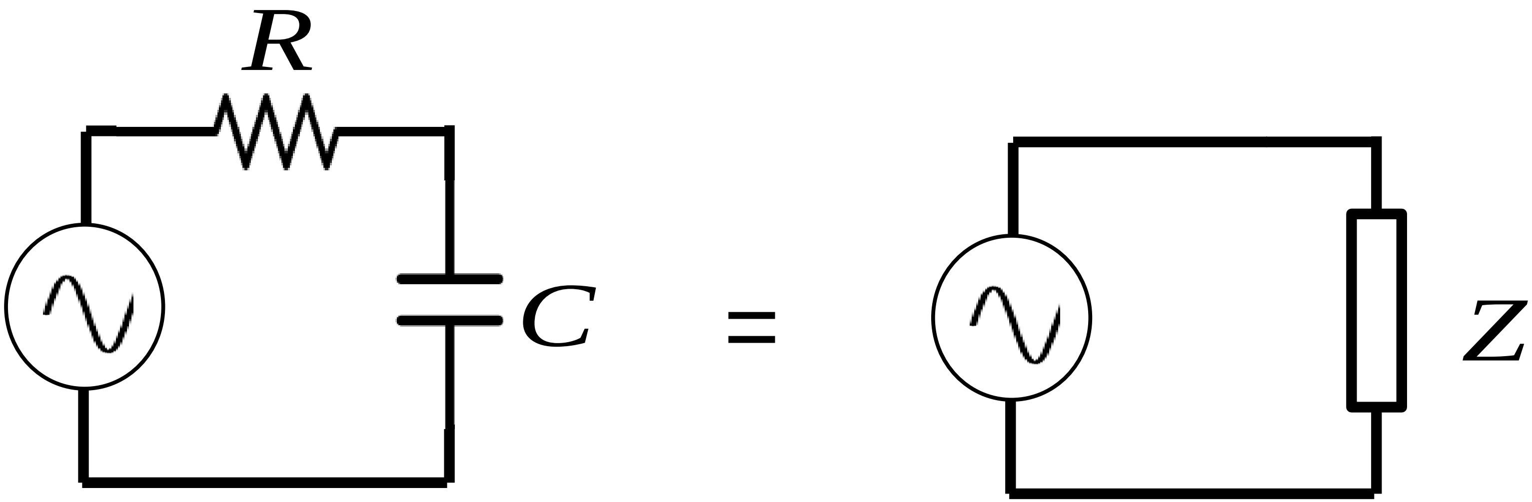
$$Z = Z_R + Z_L = R + j\omega L$$

Resistance

Reactance

EXAMPLE 2

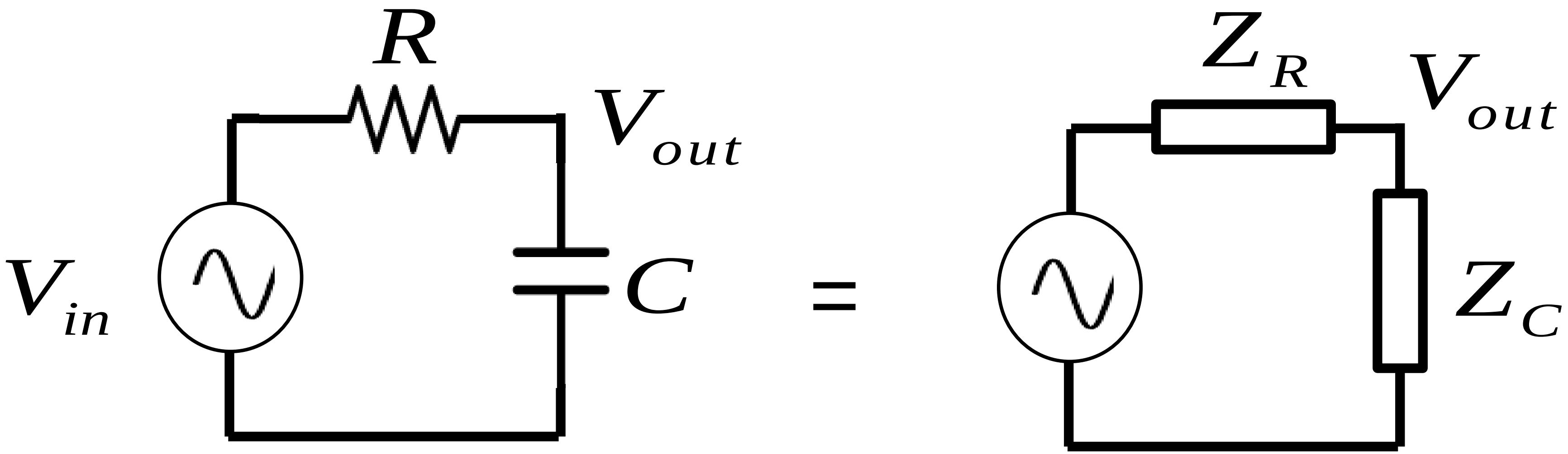
$$\omega = 2\pi f$$



$$Z = Z_R + Z_L = R - \frac{j}{\omega C}$$

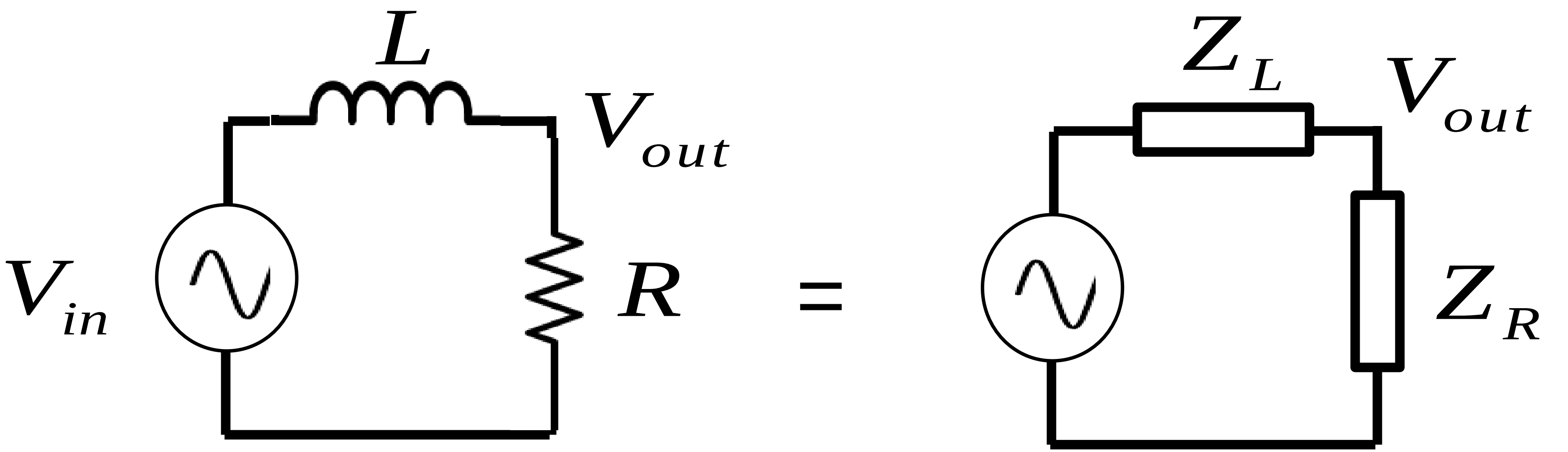
Resistance **Reactance**

EXAMPLE 3: Voltage Divider

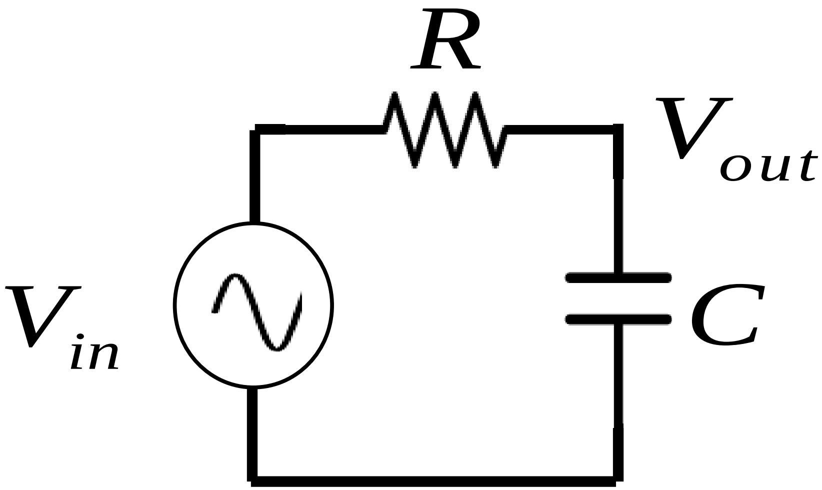


$$\frac{V_{out}}{V_{in}} = \frac{Z_C}{Z_R + Z_C} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

EXAMPLE 4: Voltage Divider



$$\frac{V_{out}}{V_{in}} = \frac{Z_R}{Z_R + Z_L} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L/R}$$



R = 217.8 Ω
C = 145.6 nF