Lab 4: AC circuits (II)

REVIEW

AC analysis of circuits using complex numbers

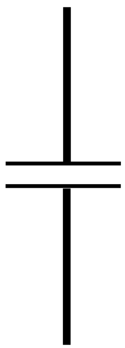
Assumptions:

i) Steady-state

ii) Sinusoidal waveforms: $V_o \sin \omega t$

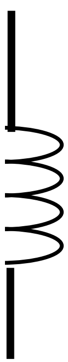
Ohm's Law for L and C:

Impedance (*Z*) **Measured in ohms**



C

$$Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C}$$

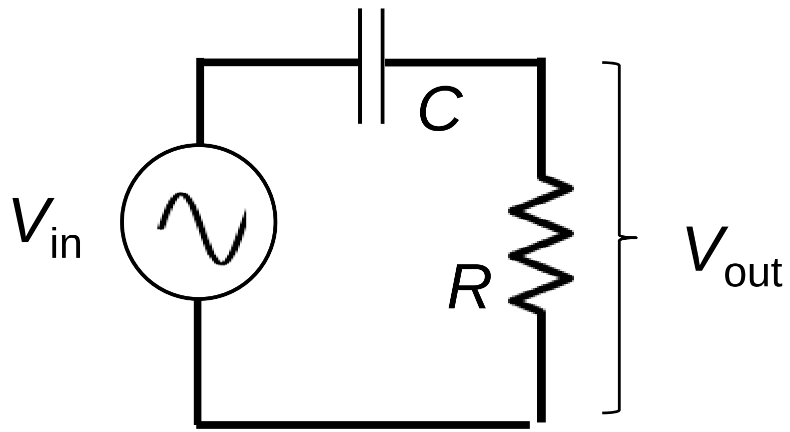


L

$$Z_L = \frac{V_L}{I_L} = \frac{\omega L}{-j} = j\omega L$$

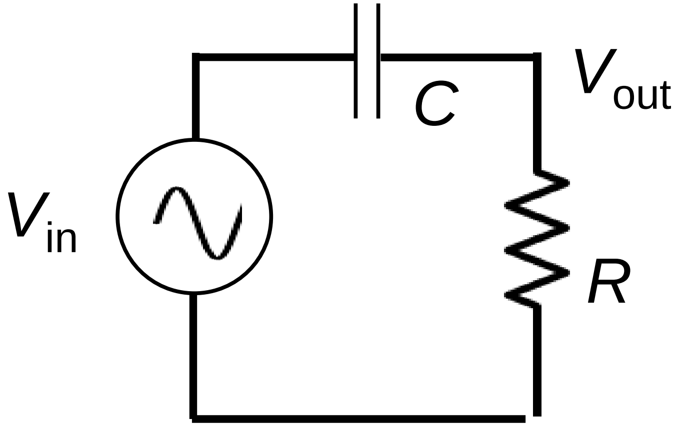
90o phase-shift in polar form: $e^{j\frac{\pi}{2}} = \cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}) = j$

AC circuit: High-pass

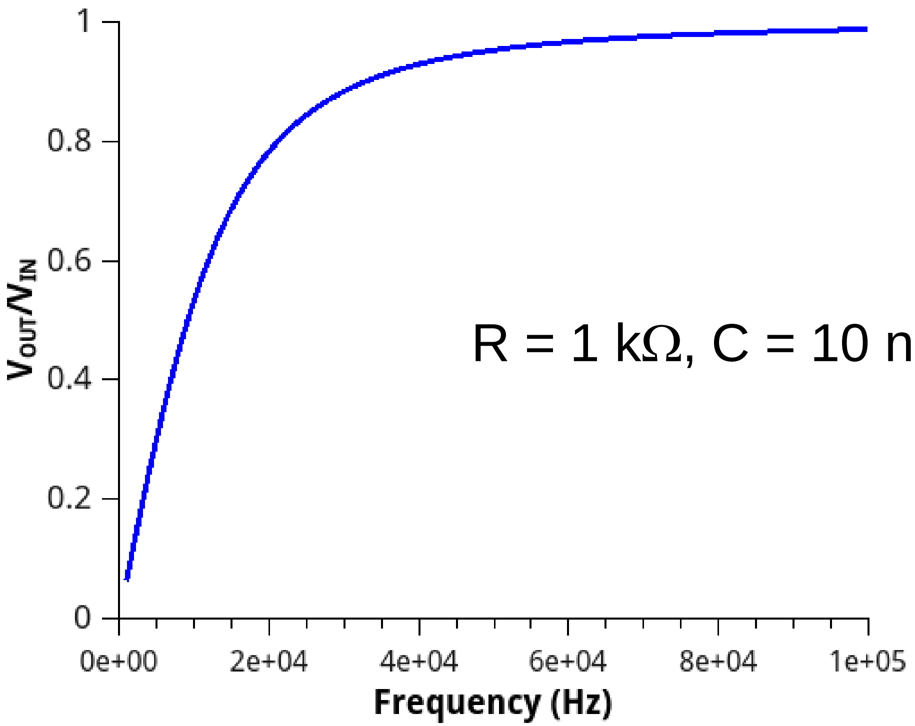


$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = \frac{R}{R + 1/j\omega C}$$

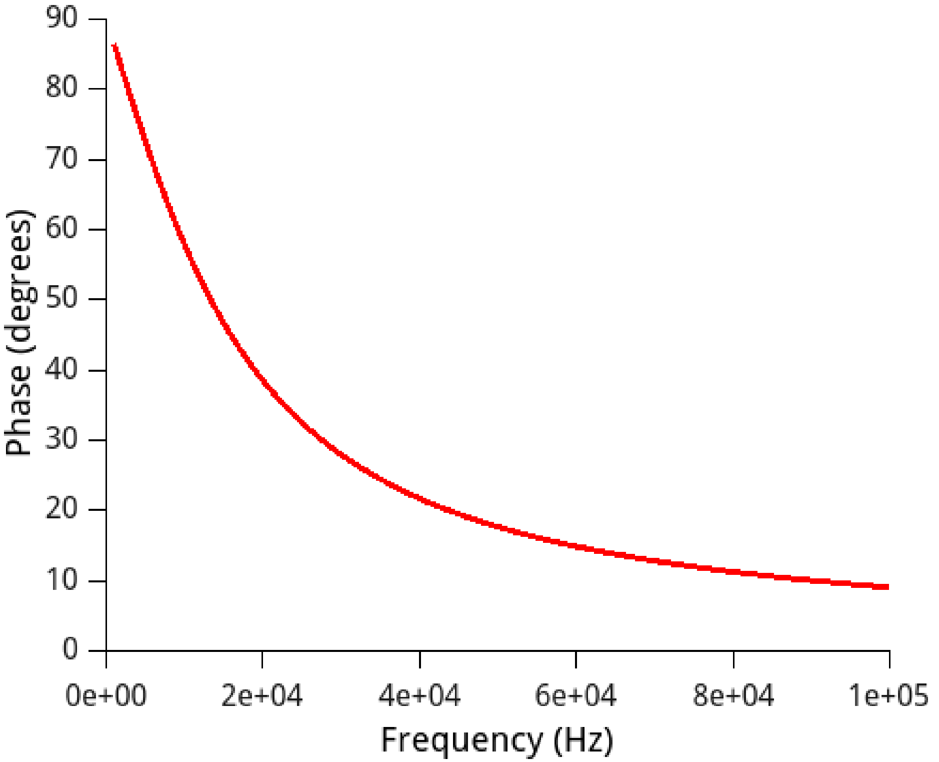
AC circuit: High-pass



$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = \frac{R}{R + 1/j\omega C}$$

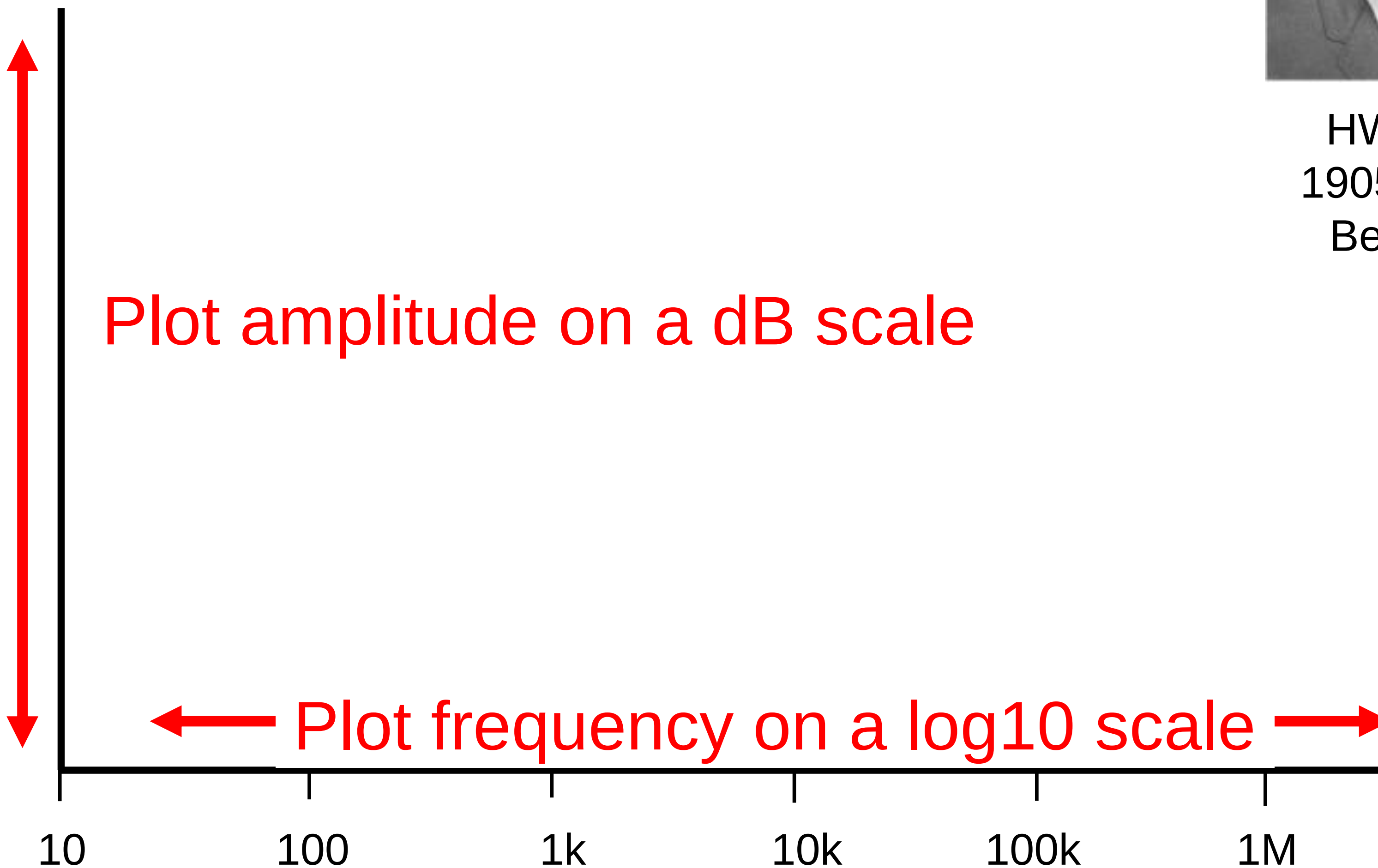


R = 1 kΩ, C = 10 nF



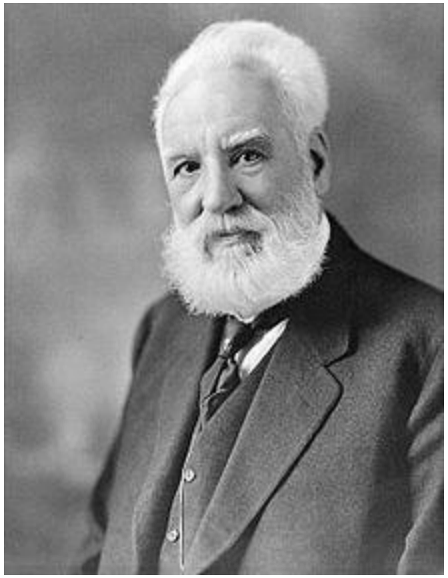
The Bode Plot for $|V_{out}/V_{in}|$

HW Bode
1905—1982
Bell Labs



The Decibel: A Ratio

RATIO	POWER $10 \log_{10}\left\{\frac{P_{\text{signal}}}{P_{\text{ref}}}\right\}$	FIELD $20 \log_{10}\left\{\frac{A_{\text{signal}}}{A_{\text{ref}}}\right\}$
1	0 dB	0 dB
10	10 dB	20 dB
100	20 dB	40 dB
2	3 dB	6 dB
0.01	–20 dB	–40 dB

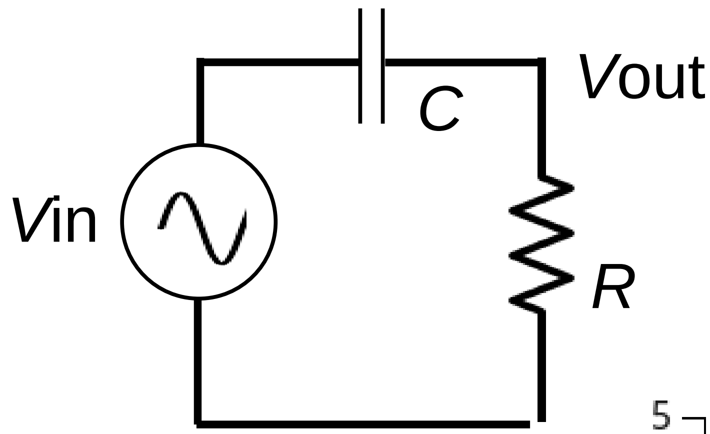


In honor of
Alexander Graham Bell

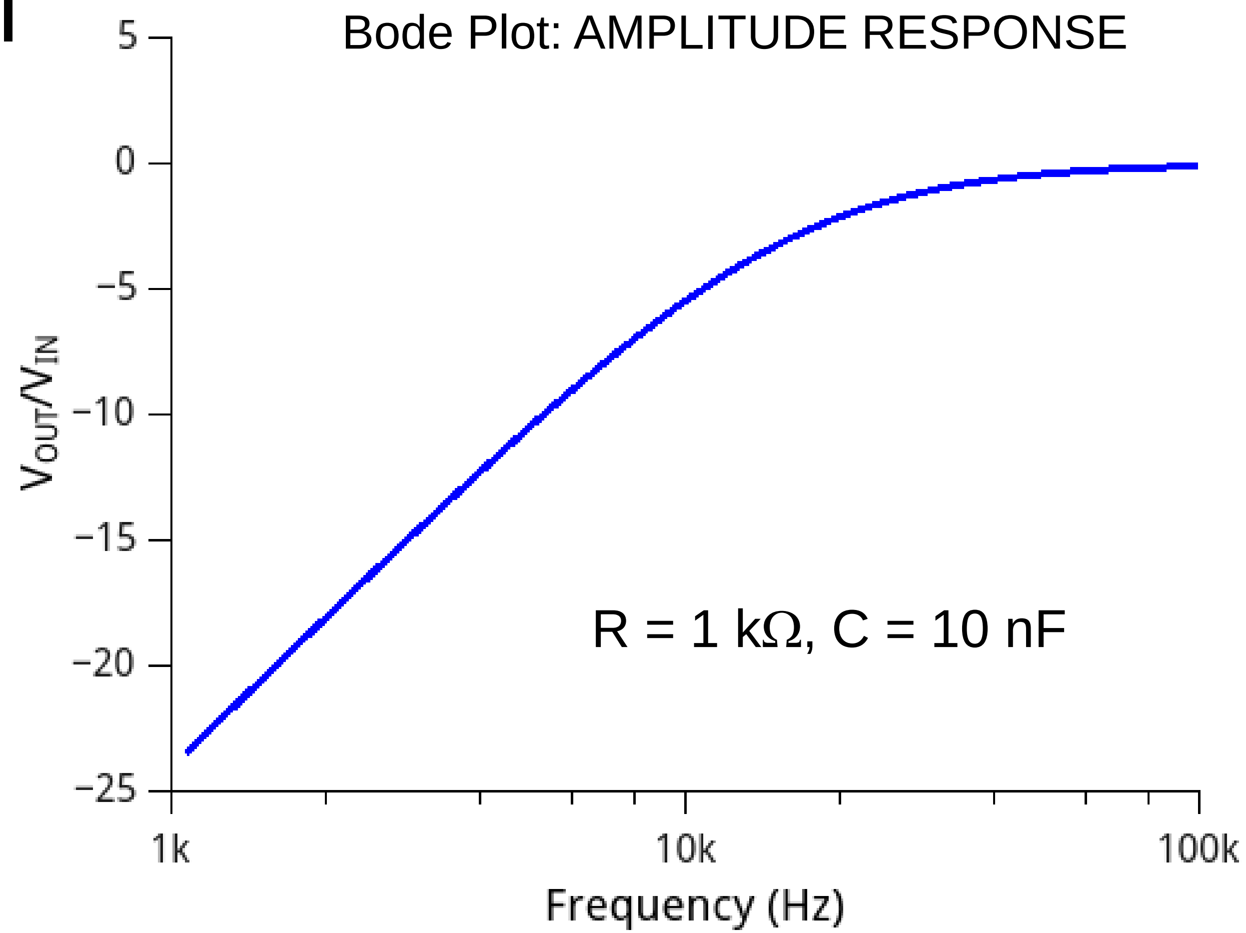
$$\text{dBm:}\ \ 10 \log_{10}\left\{\frac{P_{\text{signal}}}{1\ \text{mW}}\right\}$$

$$0\ \text{dBm} = 1\ \text{mW}$$

AC circuit: High-pass

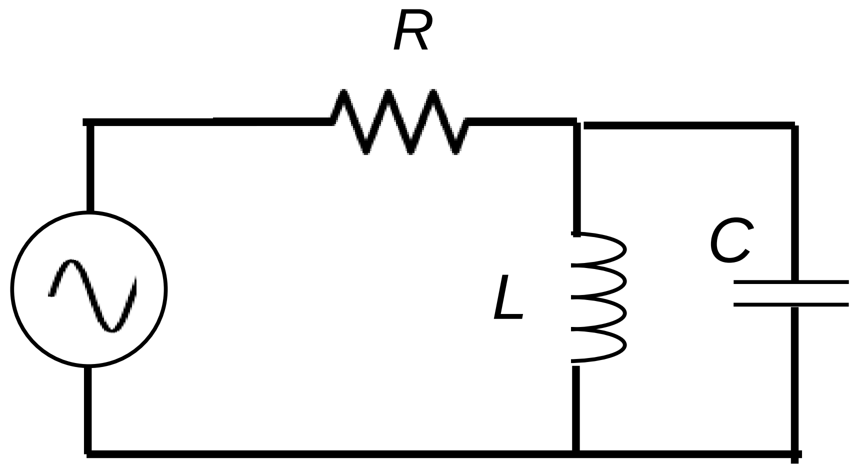


$$\frac{V_{OUT}(\omega)}{V_{IN}(\omega)} = \frac{R}{R + 1/j\omega C}$$



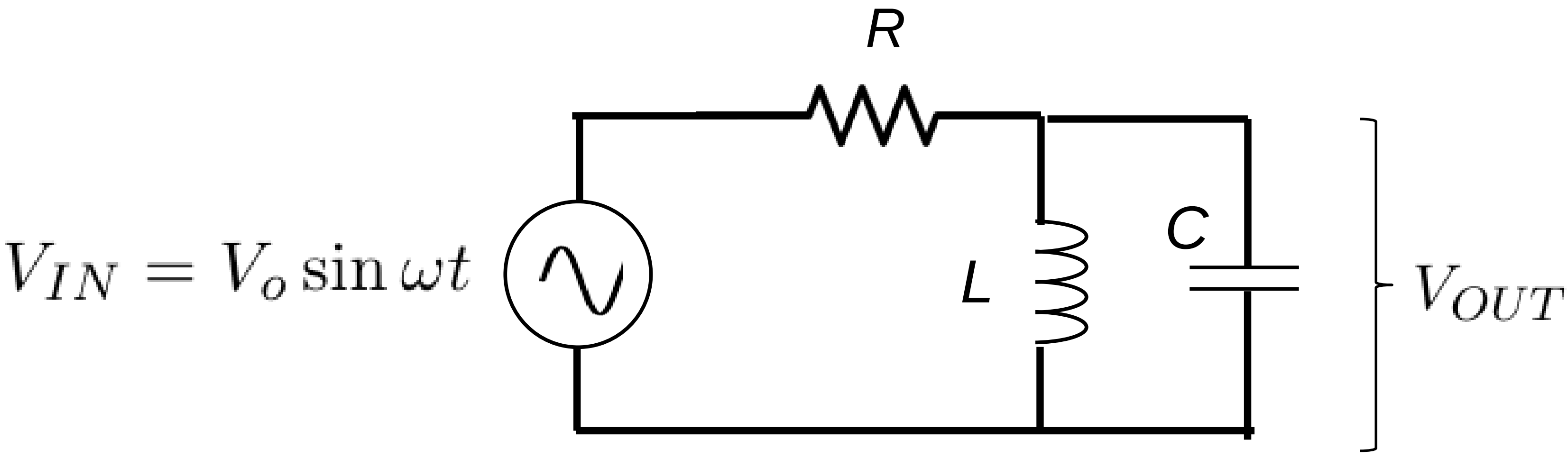
Inductor-capacitor in AC circuit: Resonance

$$V_{IN} = V_o \sin \omega t$$



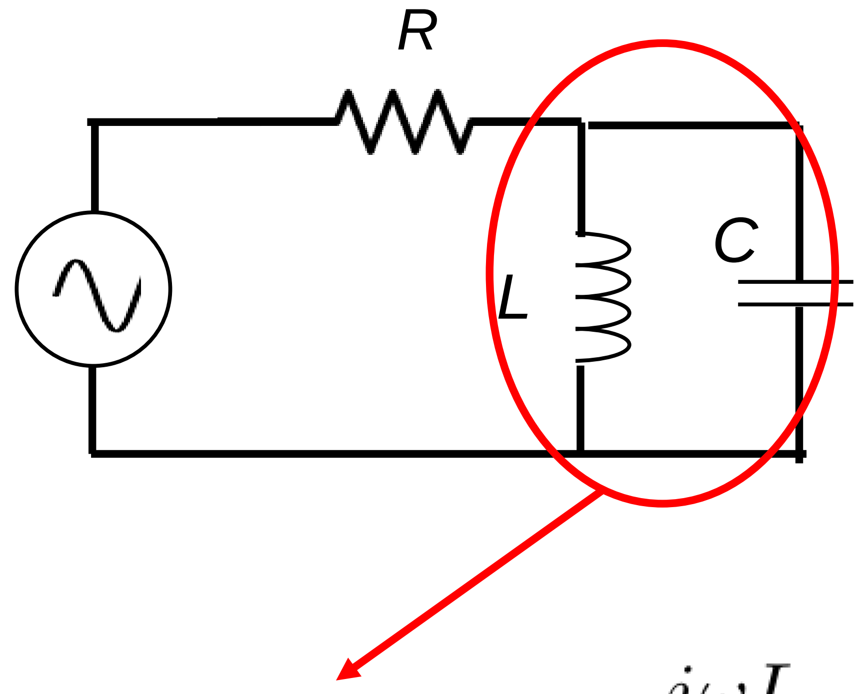
Parallel LC circuit

Inductor-capacitor in AC circuit: Resonance



Parallel LC circuit

Inductor-capacitor in AC circuit: Resonance

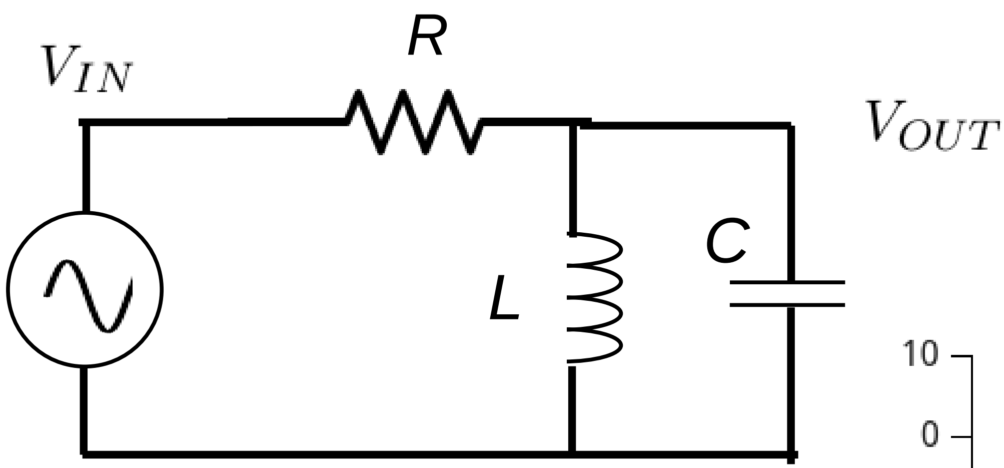


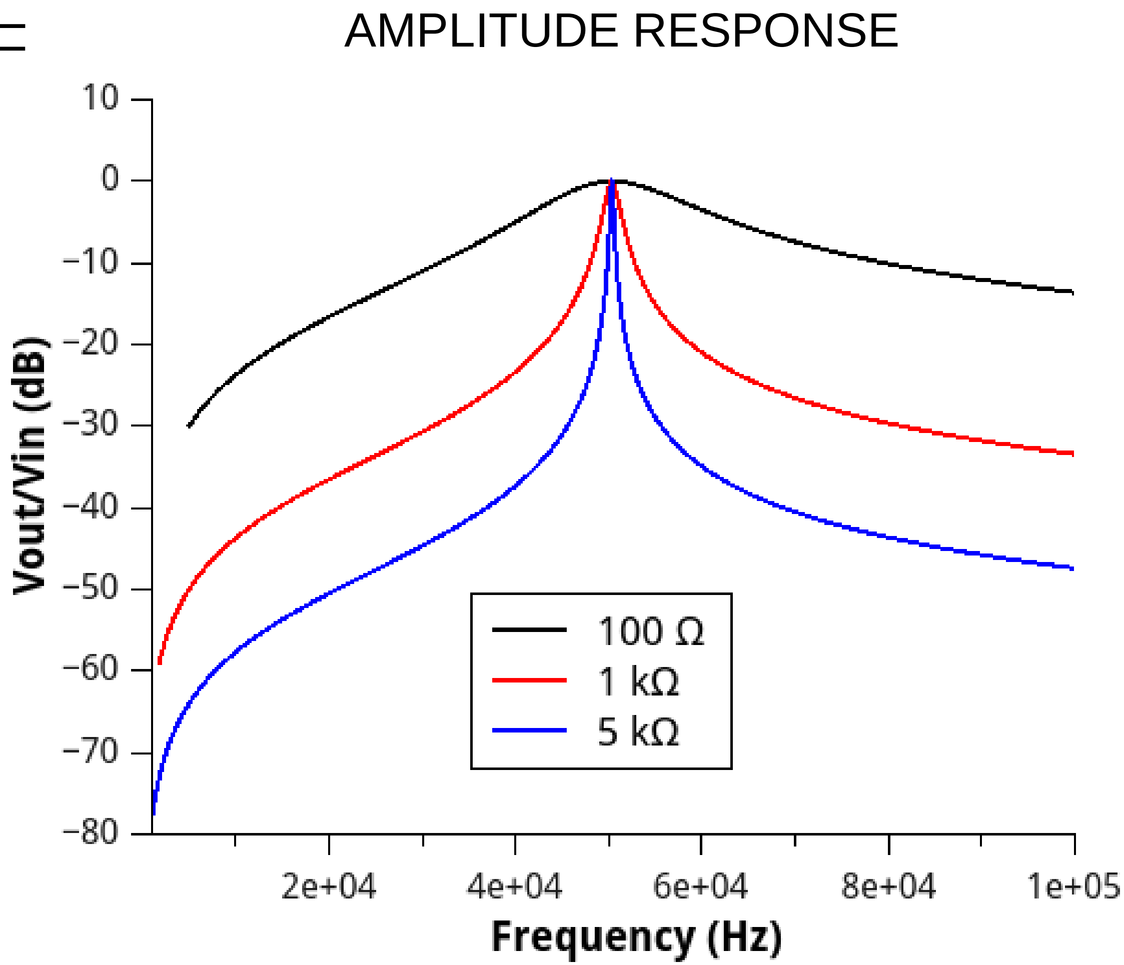
$$V_{IN} = V_o \sin \omega t$$

$$Z_{parallel} = \frac{j\omega L}{1 - \omega^2 LC}$$

Resonance at: $f = \frac{1}{2\pi\sqrt{LC}}$

Inductor-capacitor in AC circuit: Resonance

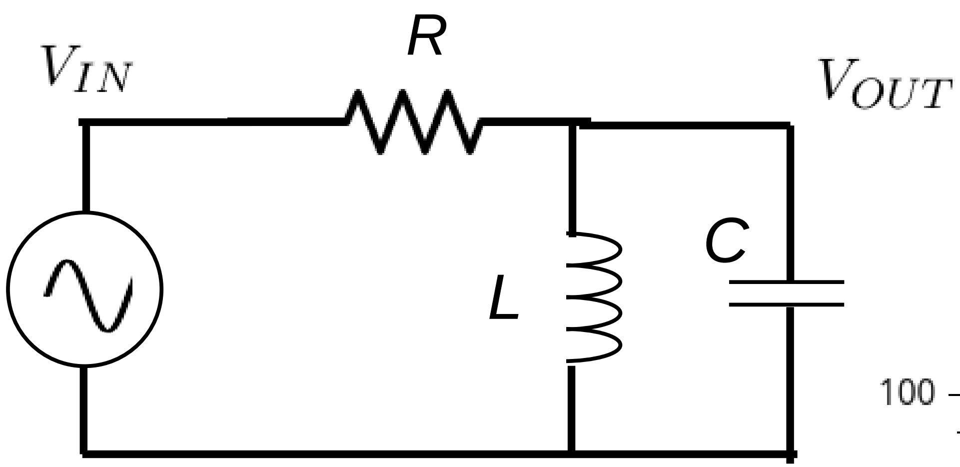




C = 100 nF
L = 100 uH
*f*res = 50.3 kHz

R = 100Ω, 1 kΩ, 5kΩ

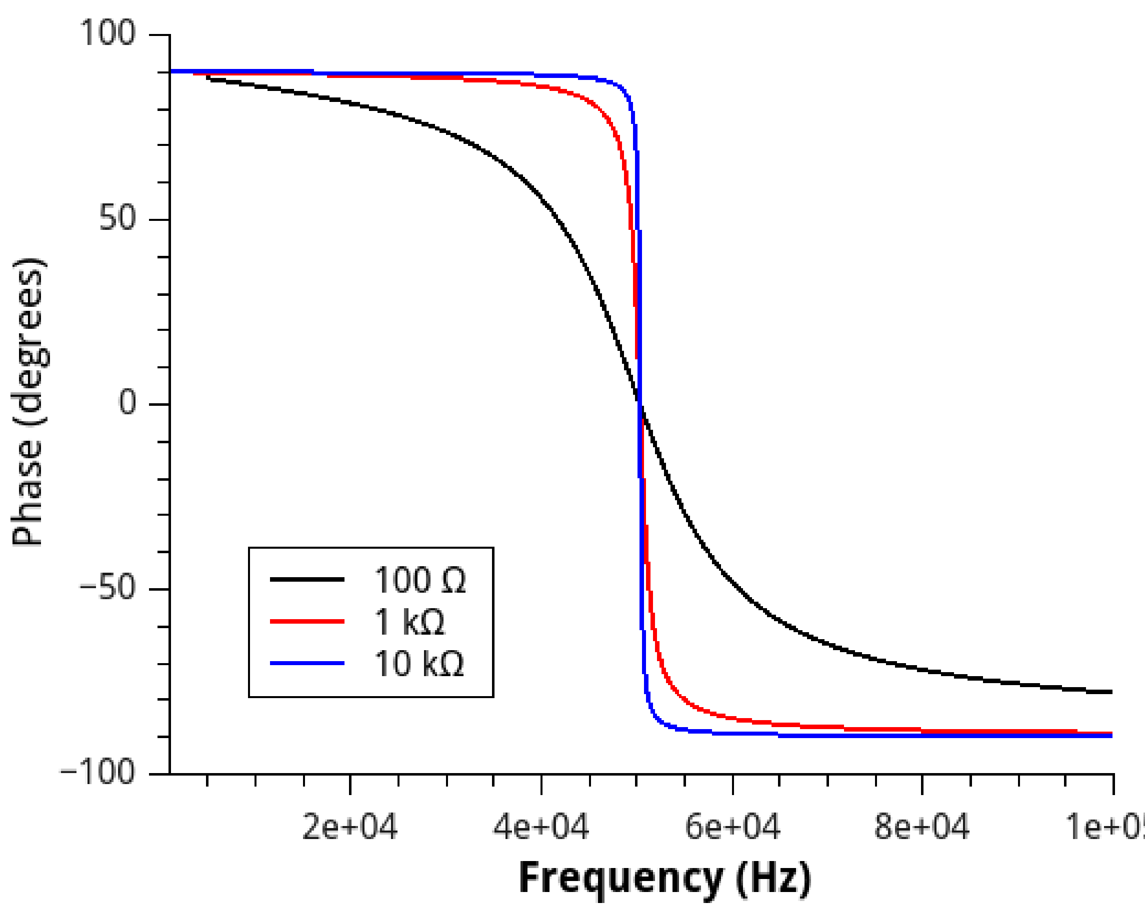
Inductor-capacitor in AC circuit: Resonance



C = 100 nF
L = 100 uH
*f*res = 50.3 kHz

R = 100Ω, 1 kΩ, 5kΩ

PHASE RESPONSE



Q-factor: Sharpness of Resonance

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f}$$

Sharper resonance $\longrightarrow$ Higher Q

Δf = frequency range between the – 3 dB points

– 3 db ≈ 0.707 of the peak

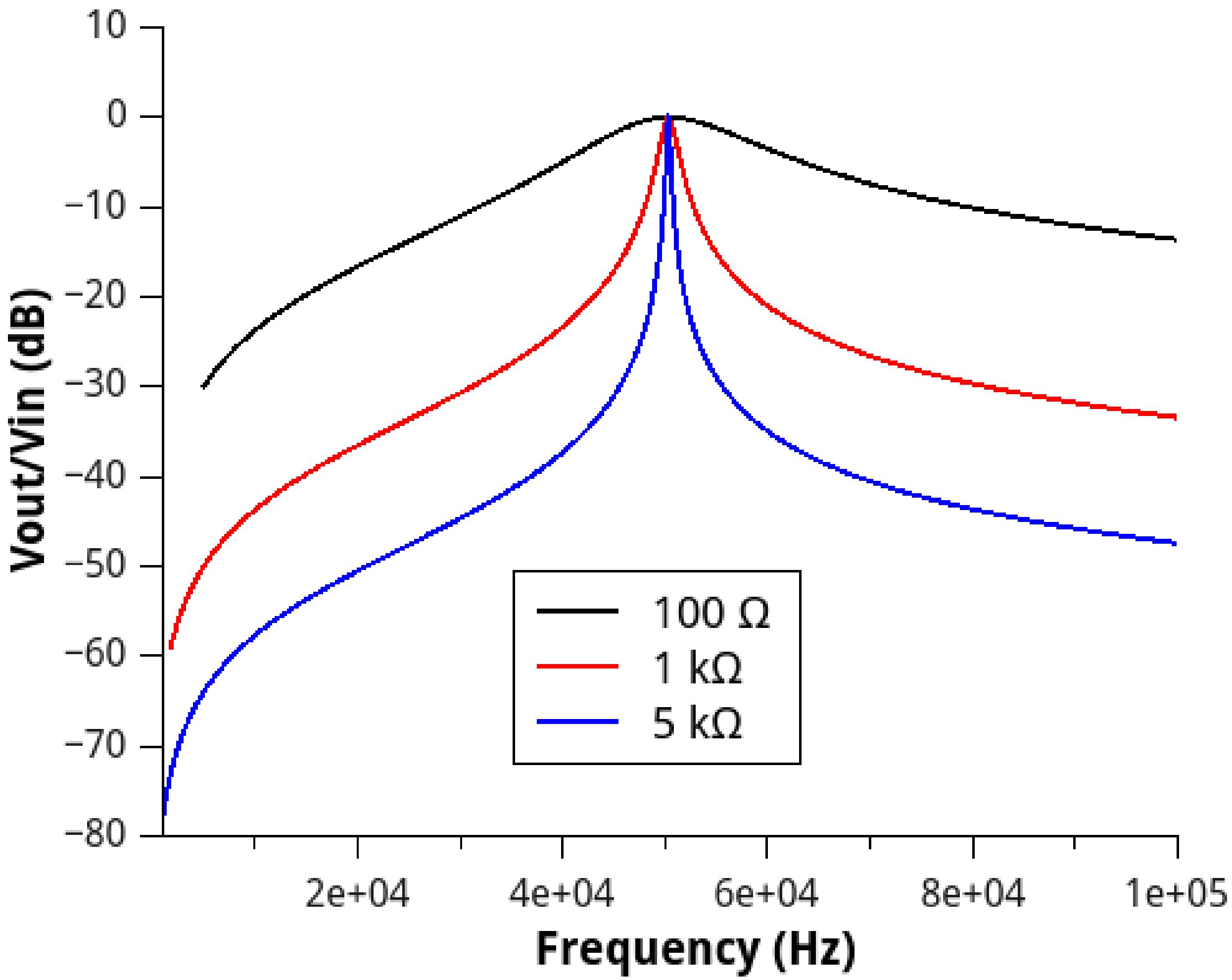
Q-factor: Sharpness of Resonance

Example RLC circuit:

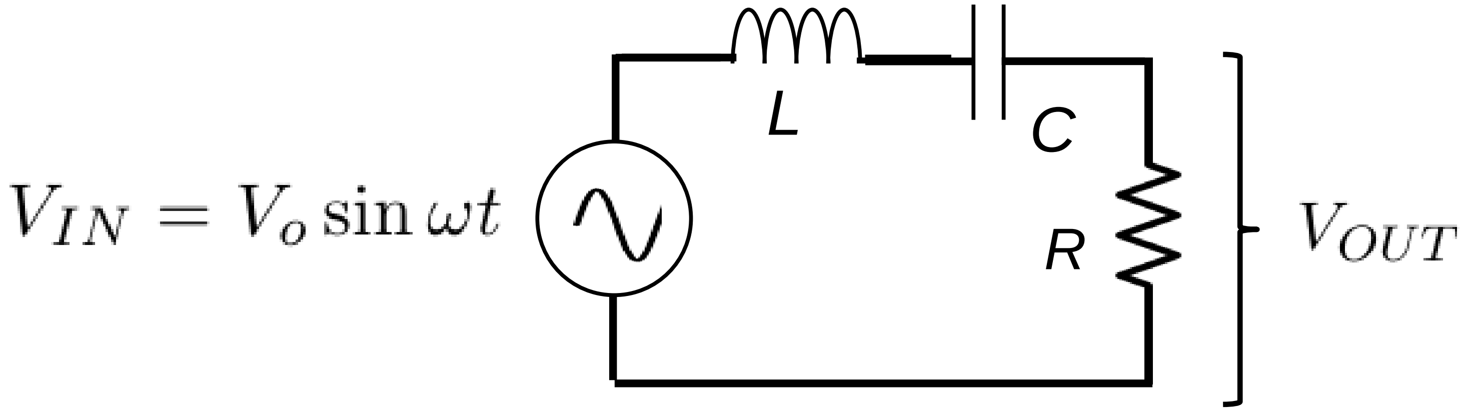
R = 100 Ω, 1 kΩ, 5 kΩ

Q = 3.1, 31, 158

$$Q = R\sqrt{\frac{C}{L}}$$



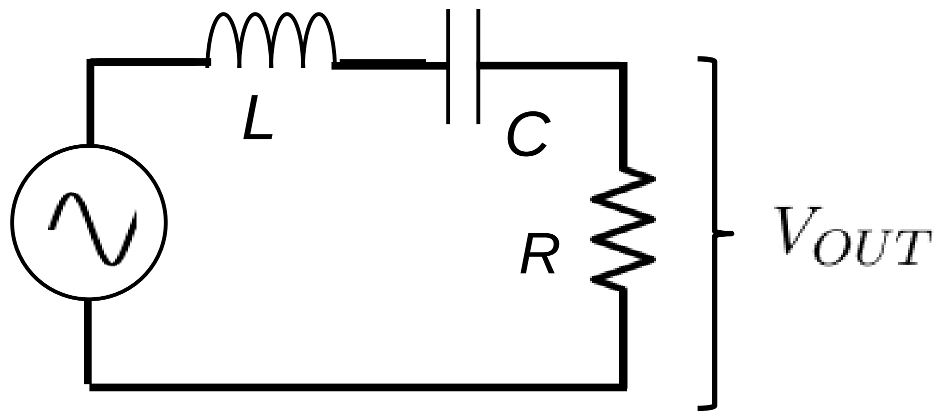
Resonance in series LC circuit



Series LC circuit

Resonance in series LC circuit

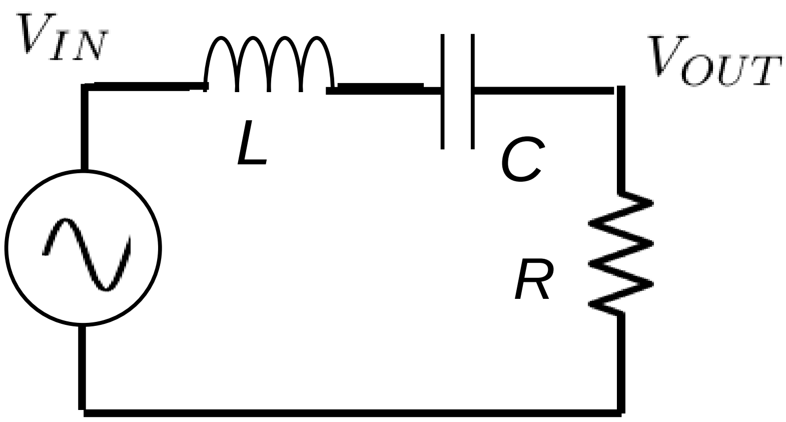
$$V_{IN} = V_o \sin \omega t$$



Series LC circuit

$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{R + j\omega L + 1/j\omega C}$$

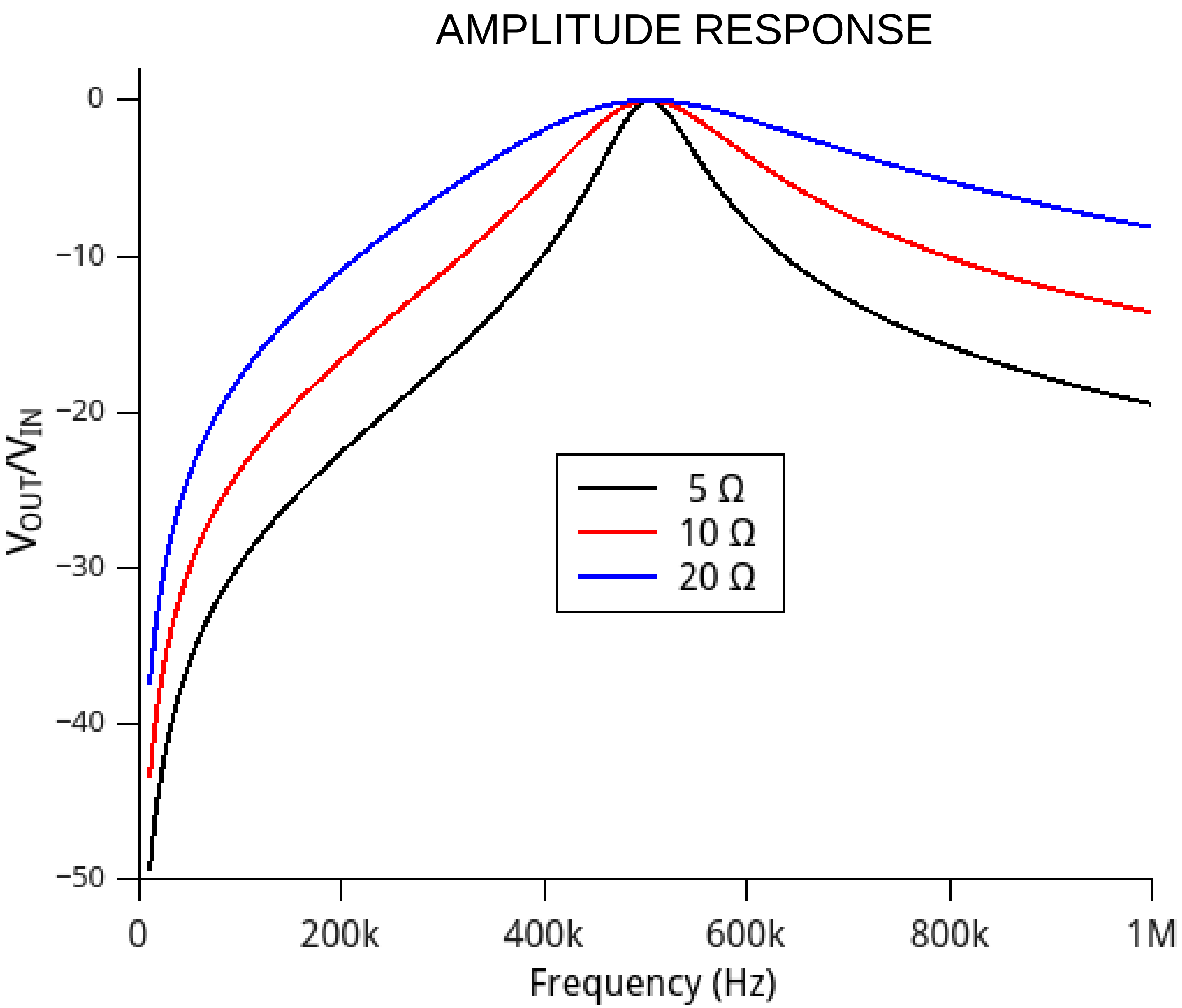
Resonance in series LC circuit



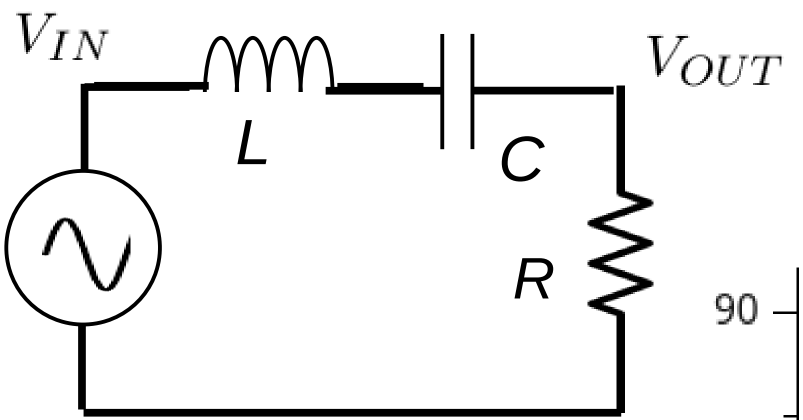
C = 10 nF
L = 10 uH
*f*res = 503 kHz

R = 5 Ω, 10 Ω, 20 Ω
Q = 6.3, 3.1, 1.6

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Resonance in series LC circuit



C = 10 nF
L = 10 uH
$f$res = 503 kHz