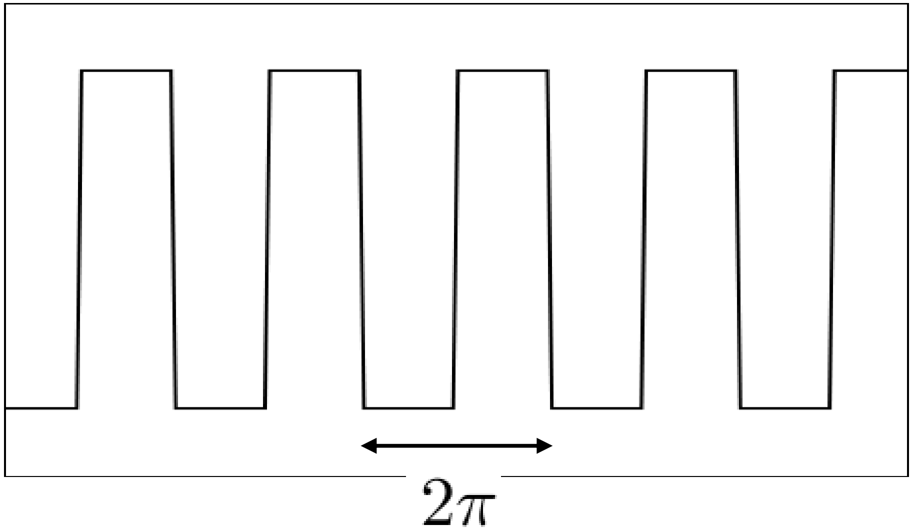
Fourier Analysis

Joseph Fourier
1768-1830

Fourier Series

Periodic function $f(x)$

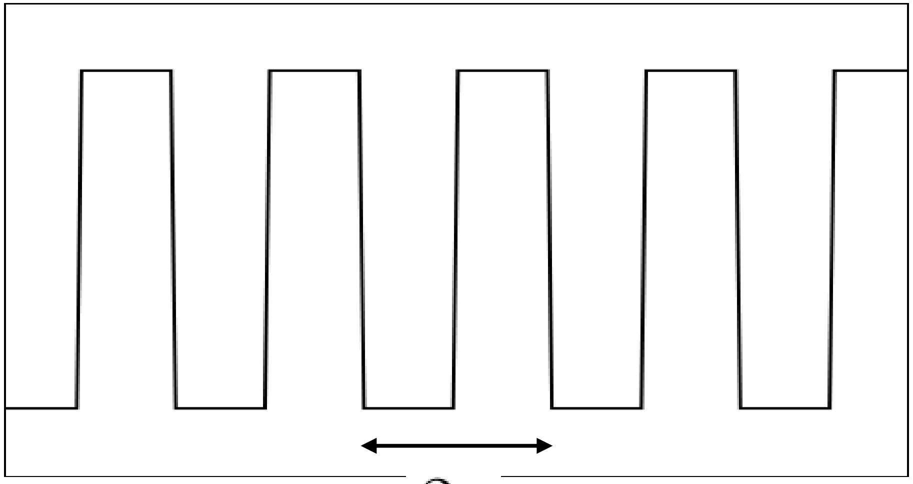


$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx \qquad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx$$

Fourier Series

Periodic function $f(x)$

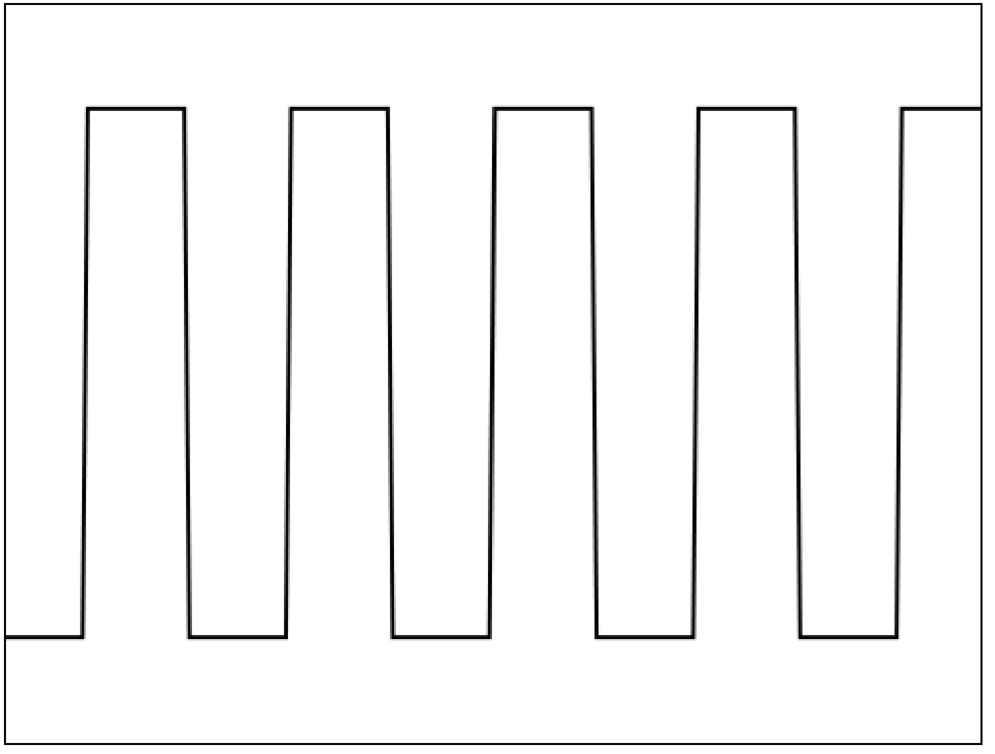


2π

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{jnx}$$

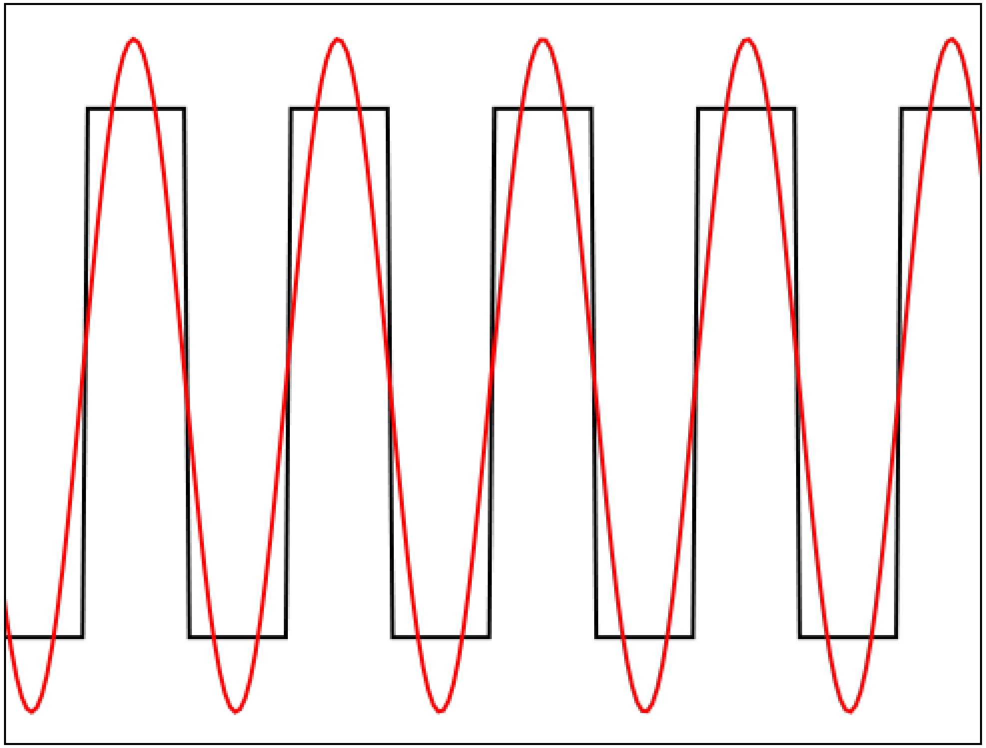
$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-jnx} dx$$

Fourier Series: Rectangular wave



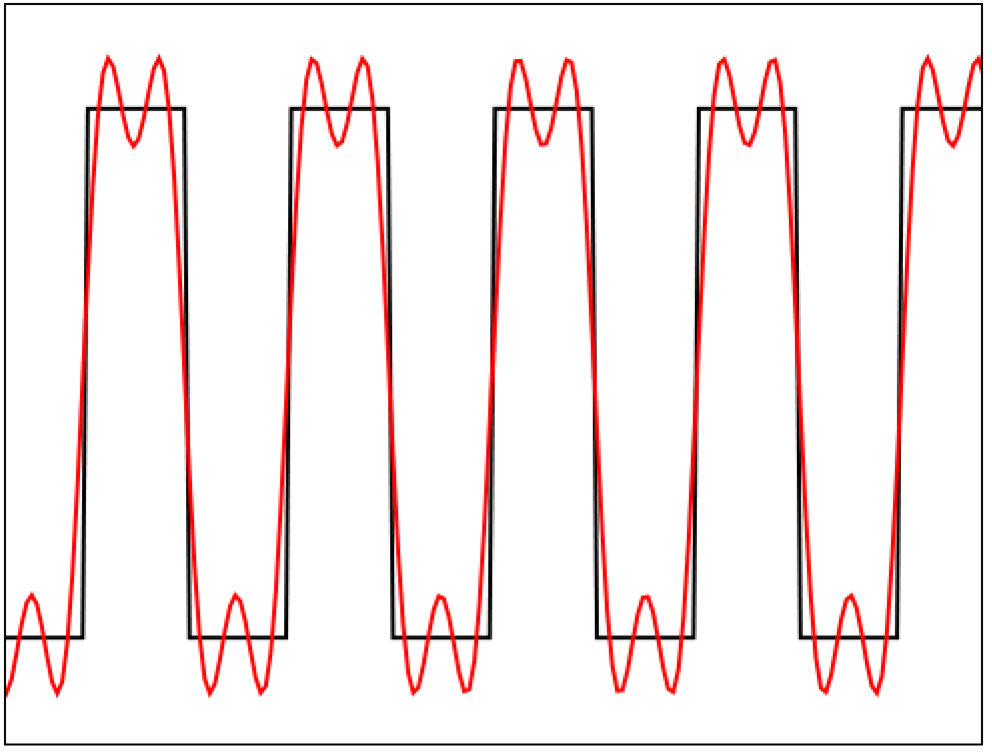
Fourier Series: Rectangular wave

1st order component



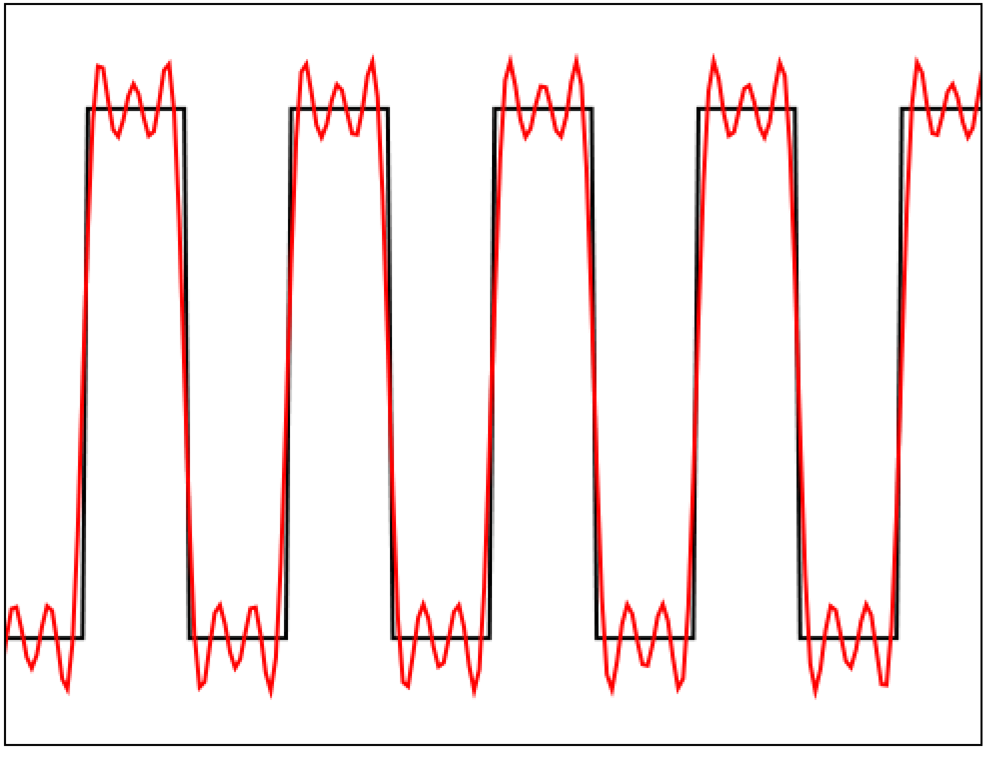
Fourier Series: Rectangular wave

1st + 2nd order components



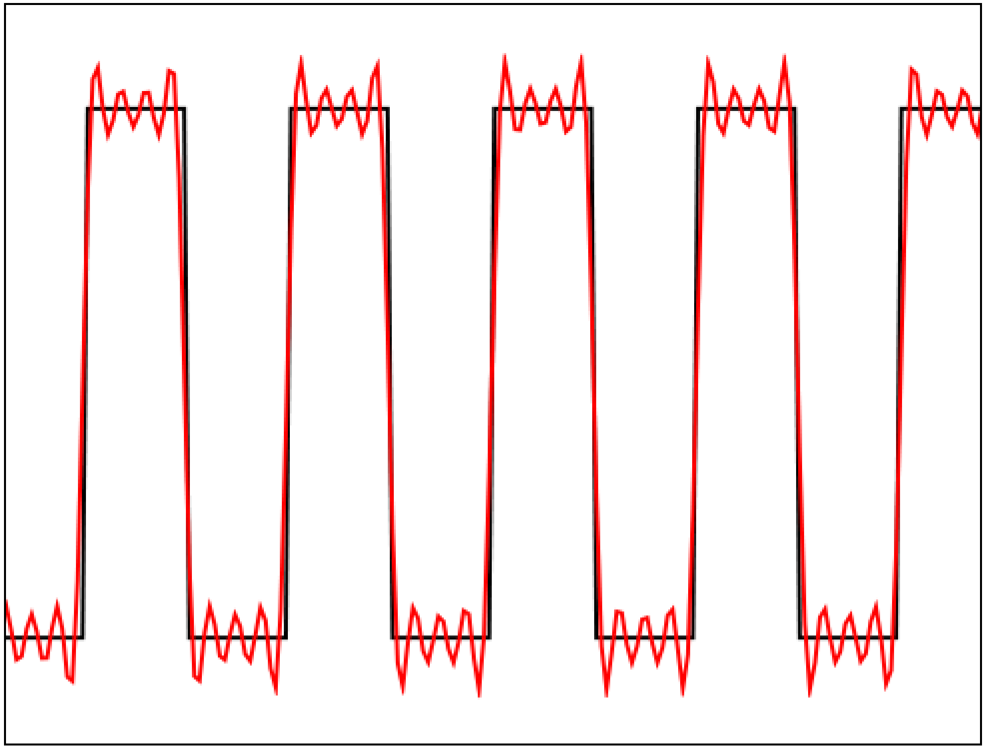
Fourier Series: Rectangular wave

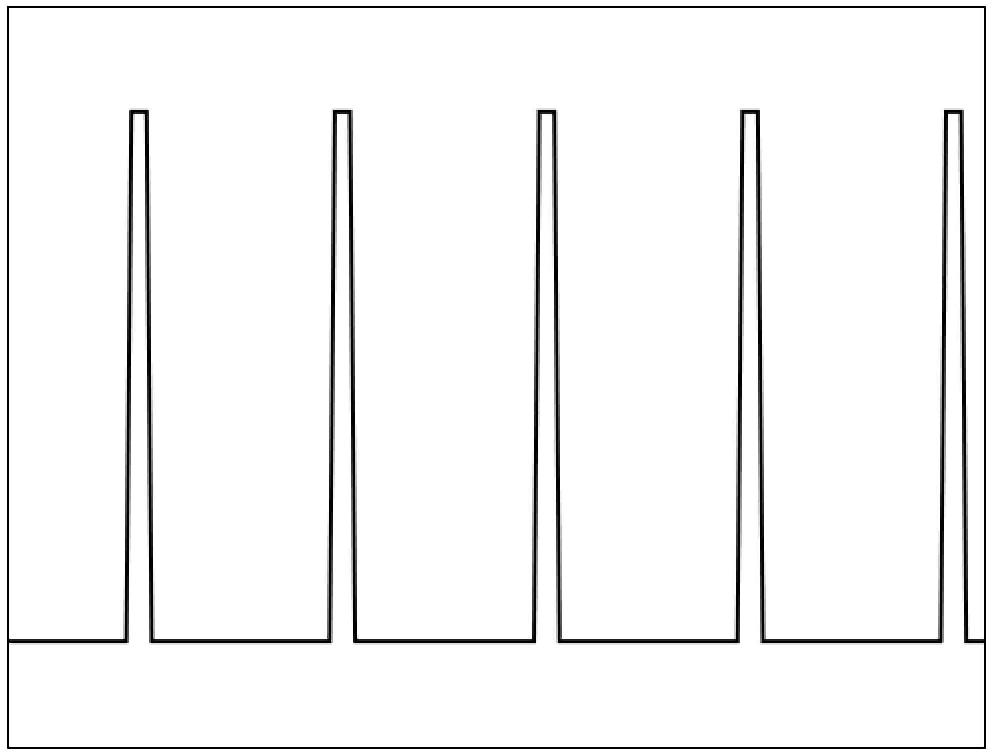
1st + 2nd + 3rd order components



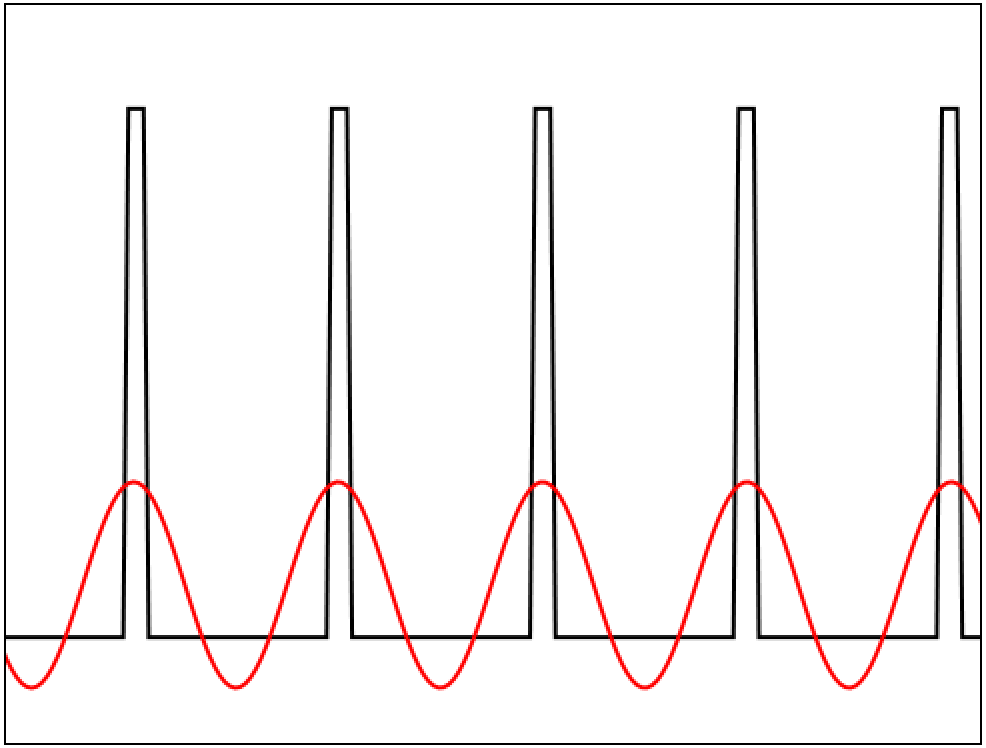
Fourier Series: Rectangular wave

1st + 2nd + 3rd + 4th order components

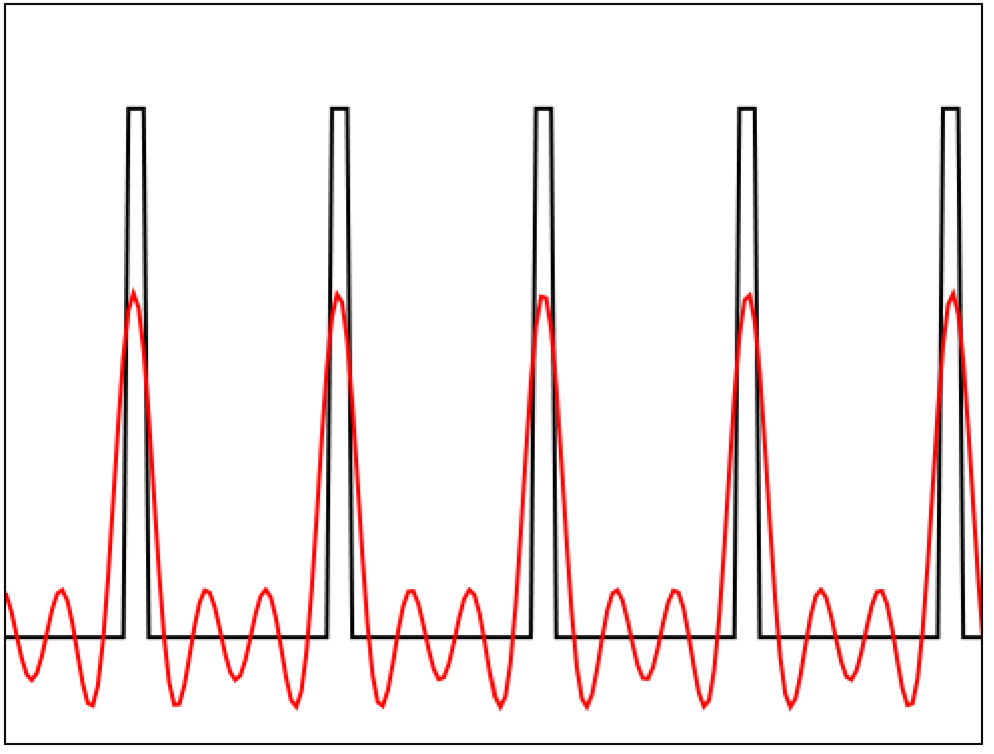




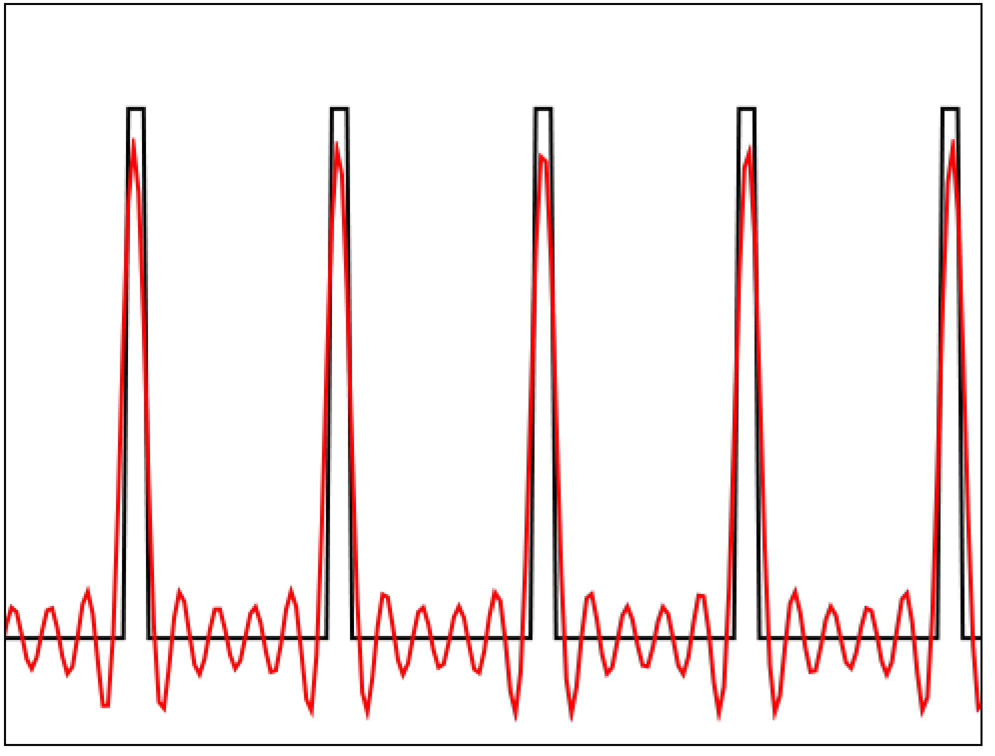
1st order



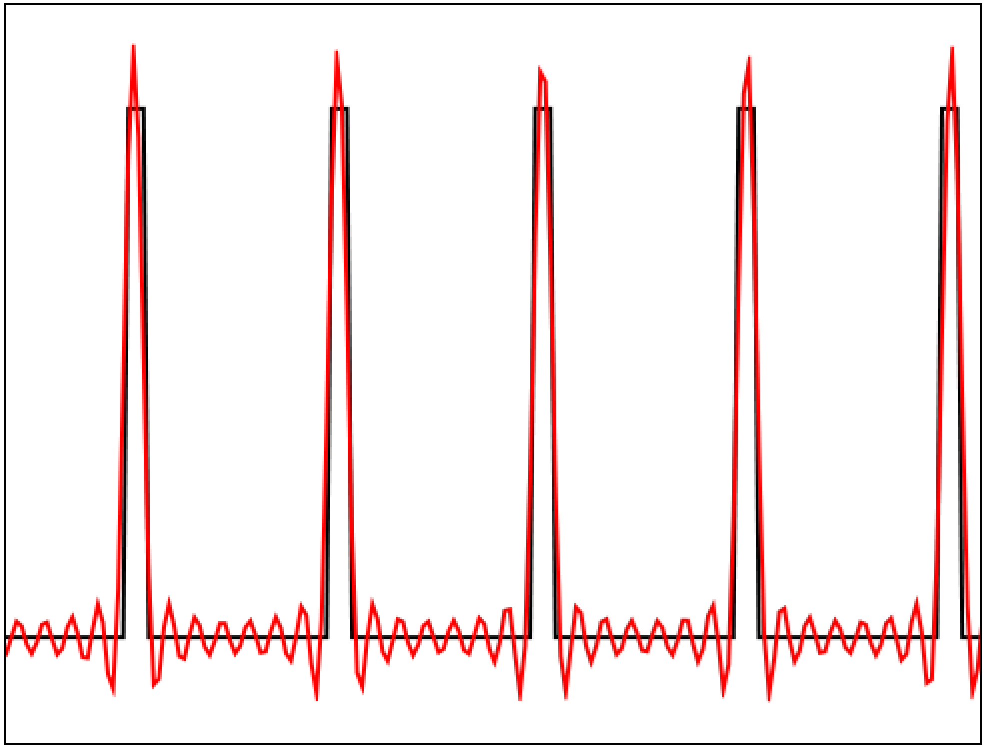
1st + 2nd order



1st + 2nd + 3rd order

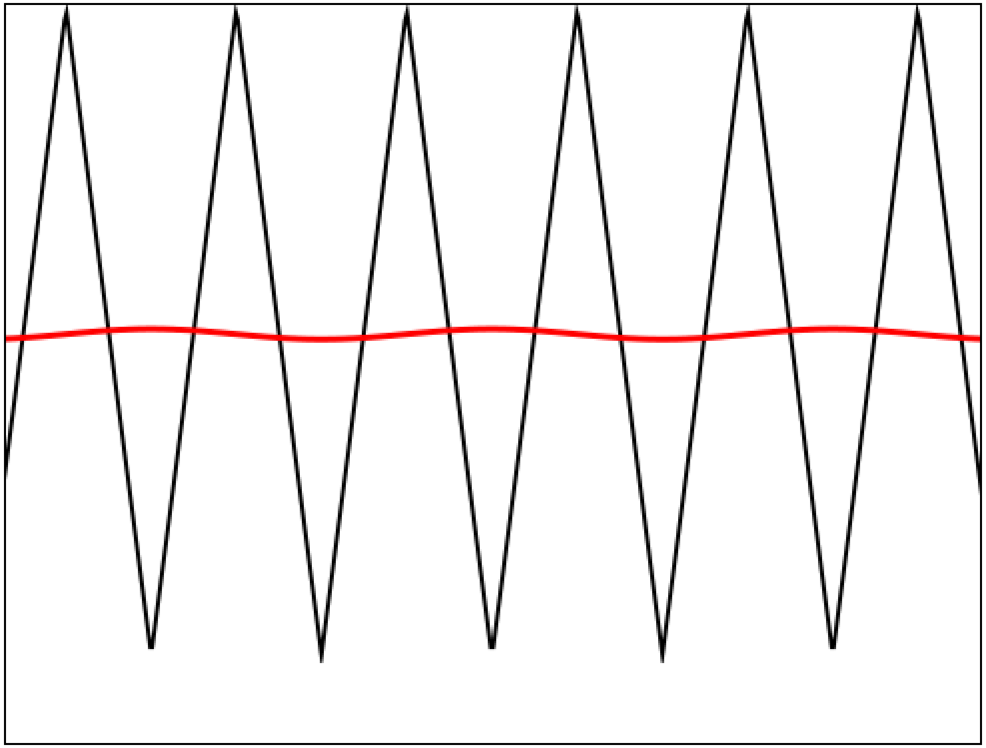


1st + 2nd + 3rd + 4th order components



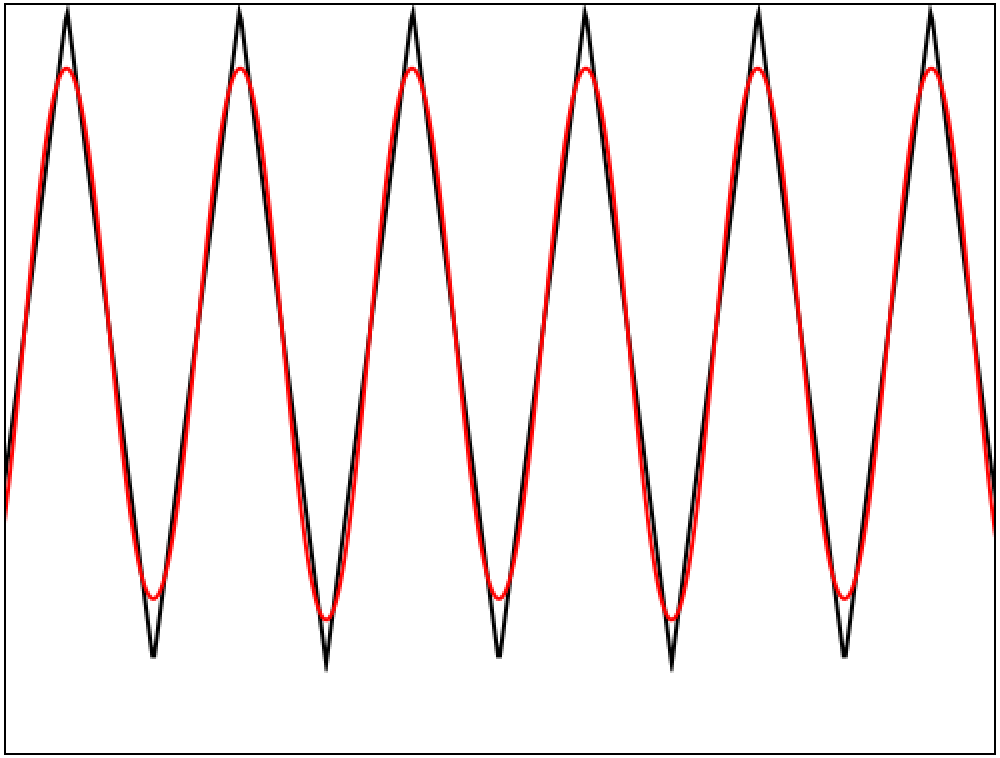
Triangle (Sawtooth) Wave

1st order



Triangle (Sawtooth) Wave

1st + 2nd order



Fourier Series $\Longrightarrow$ Fourier-Transform

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n x \frac{1}{T}} \qquad c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(\hat{x}) e^{-j2\pi n \hat{x}\frac{1}{T}} d\hat{x}$$

$$f(x) = \frac{1}{T}\sum_{n=-\infty}^{\infty} e^{j2\pi n x \frac{1}{T}} \left[\int_{-T/2}^{T/2} f(\hat{x}) e^{-j2\pi n \hat{x}\frac{1}{T}} d\hat{x}\right]$$

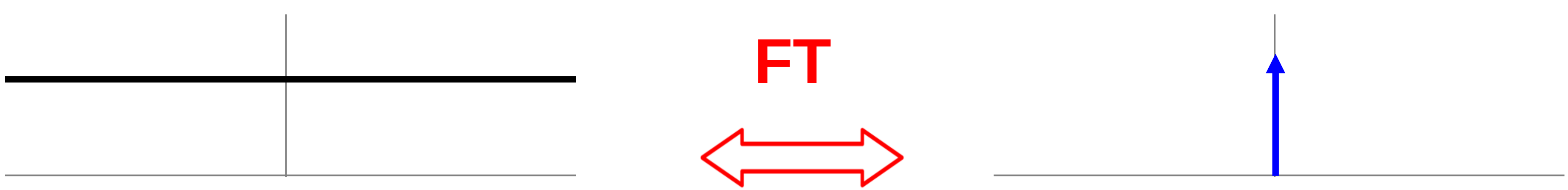
$$\nu = \frac{n}{T} \qquad f(x) = \int_{-\infty}^{\infty} d\nu\; e^{j2\pi \nu x} \left[\int_{-\infty}^{\infty} f(\hat{x}) e^{-j2\pi \nu \hat{x}} d\hat{x}\right]$$

Fourier Transform: $f(x) = \int_{-\infty}^{\infty} d\nu\; e^{j2\pi \nu x} F(\nu)$

Fourier-Transform

$$f(x) = \int_{-\infty}^{\infty} d\nu \; e^{j2\pi\nu x} F(\nu)$$

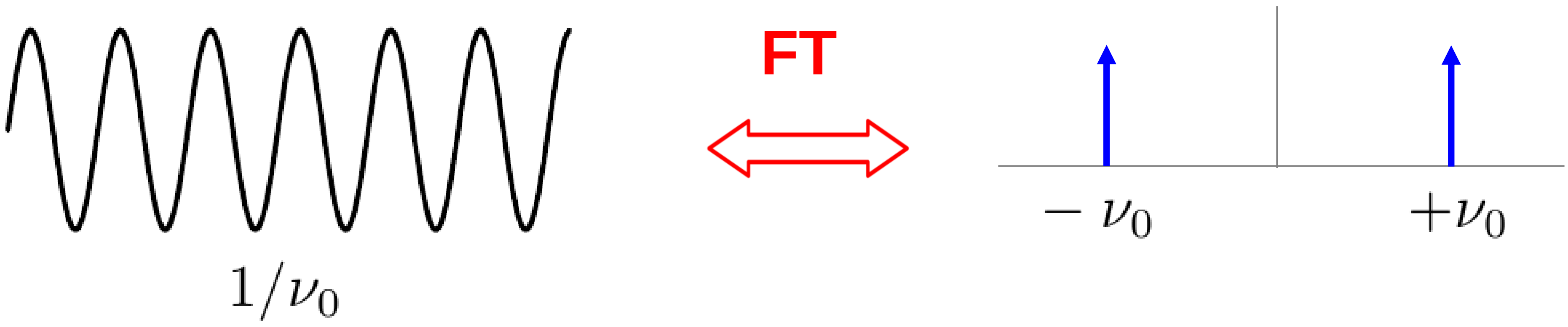
$$F(\nu) = \int_{-\infty}^{\infty} f(\hat{x}) e^{-j2\pi\nu\hat{x}} d\hat{x}$$



Fourier-Transform

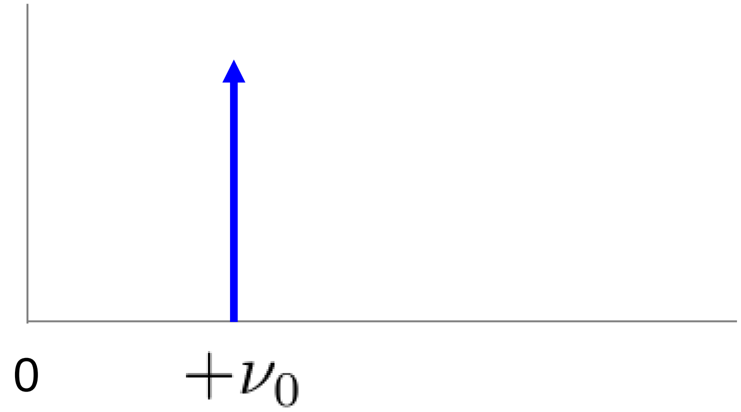
$$f(x) = \int_{-\infty}^{\infty} d\nu \; e^{j2\pi\nu x} F(\nu)$$

$$F(\nu) = \int_{-\infty}^{\infty} f(\hat{x}) e^{-j2\pi\nu\hat{x}} d\hat{x}$$



Power Spectrum

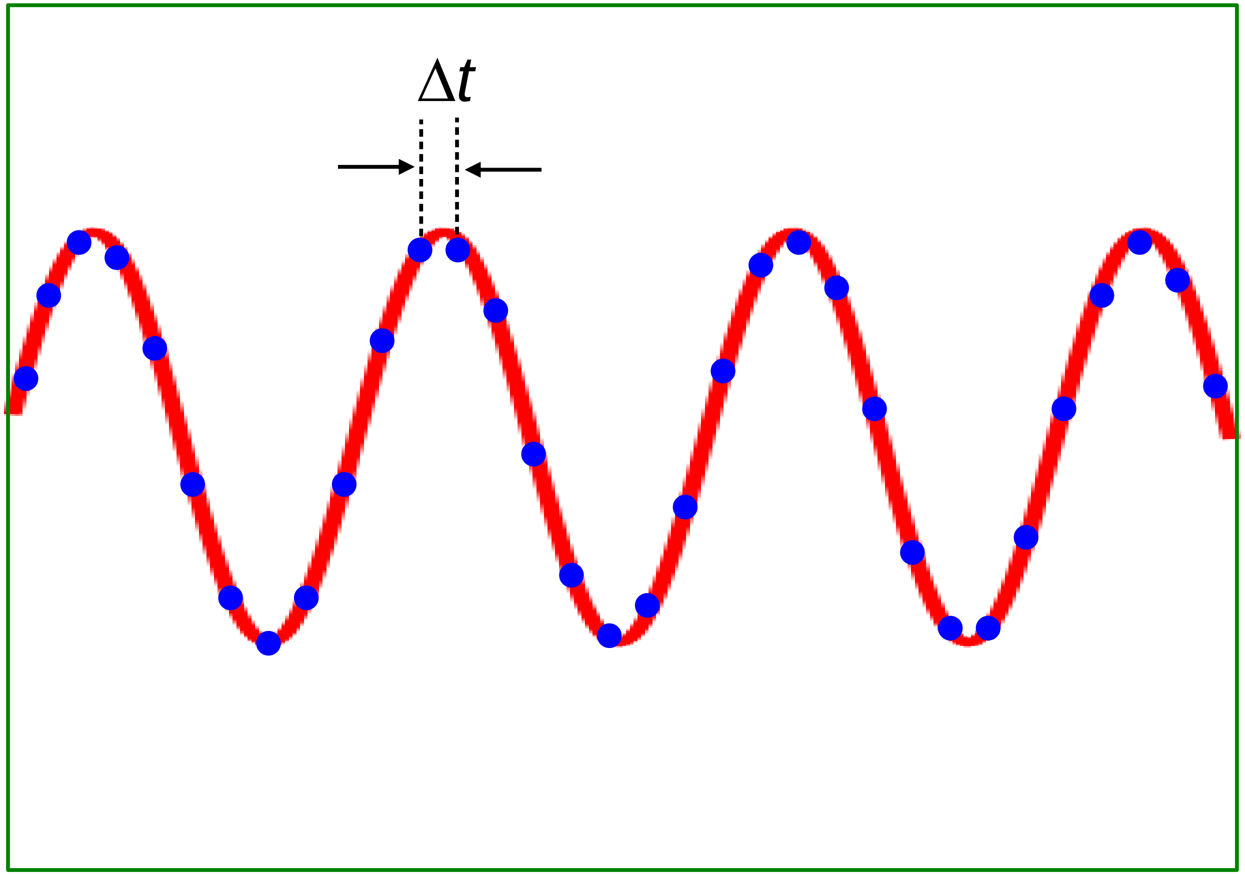
$$|F(\nu)|^2 = F(\nu)F^*(\nu)$$



Nyquist theorem
Sampling theorem

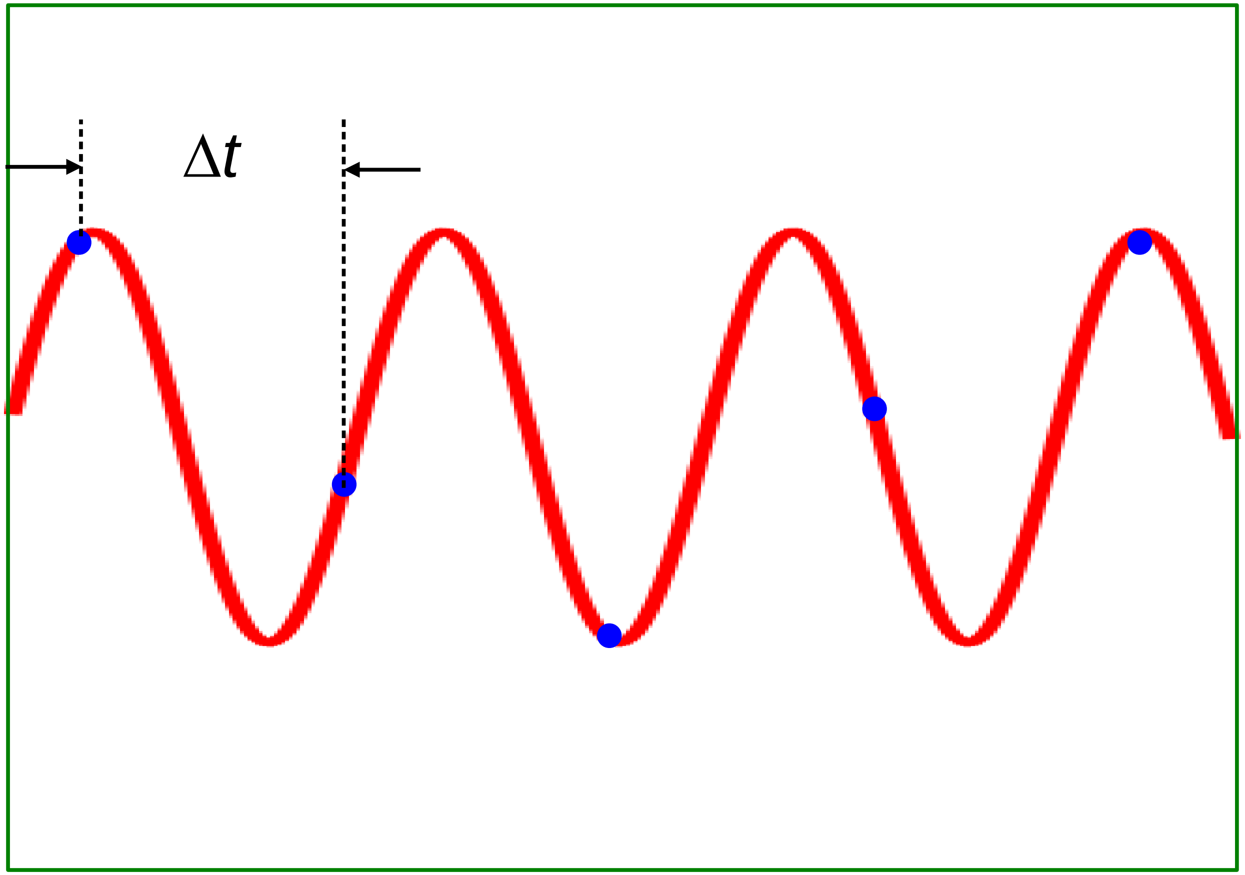
Temporal spacing of signal sampling

$$\Delta t \leq \frac{1}{2\nu}$$



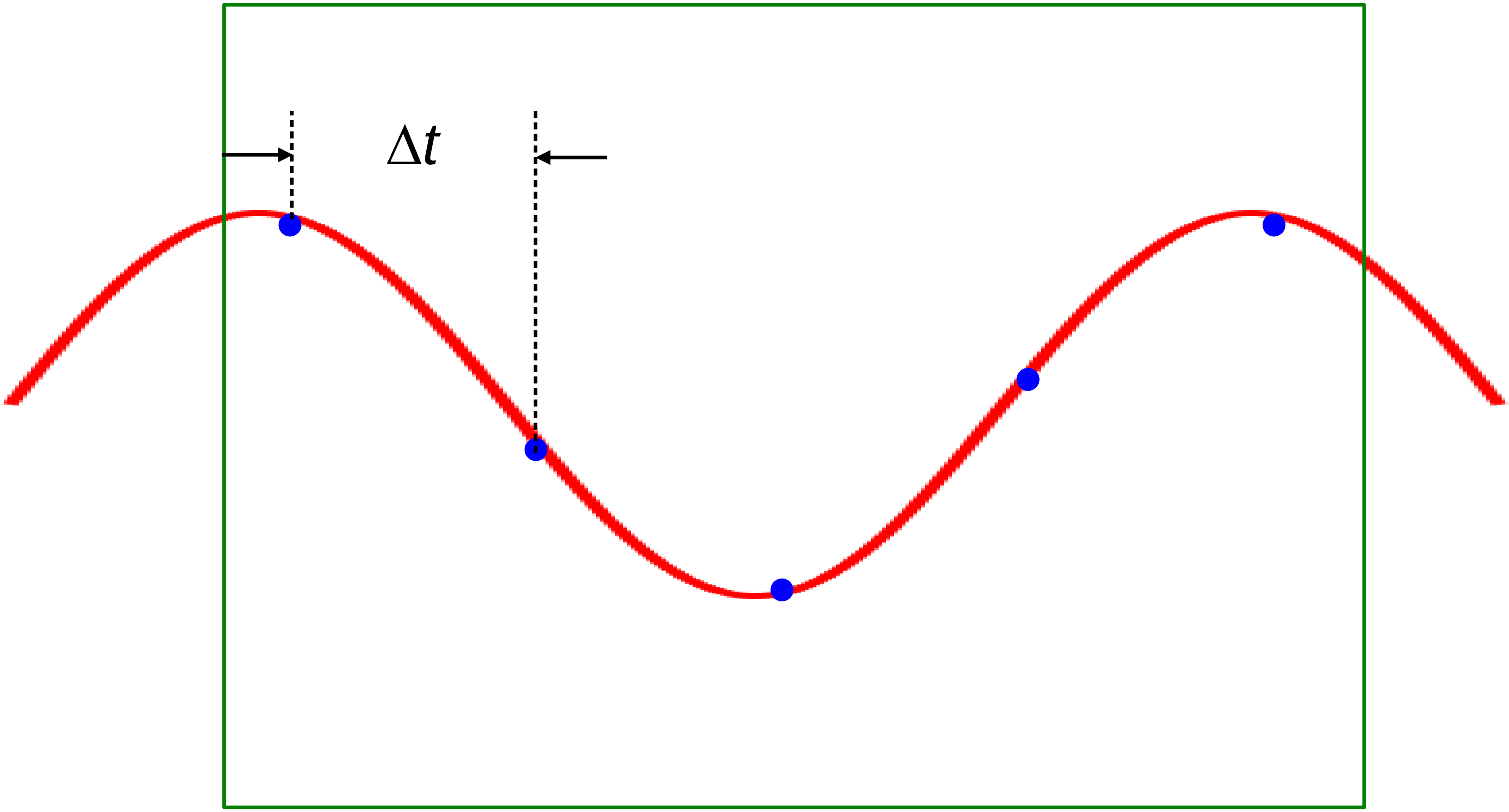
Nyquist theorem
Sampling theorem

Temporal spacing of signal sampling $\Delta t > \frac{1}{2\nu}$



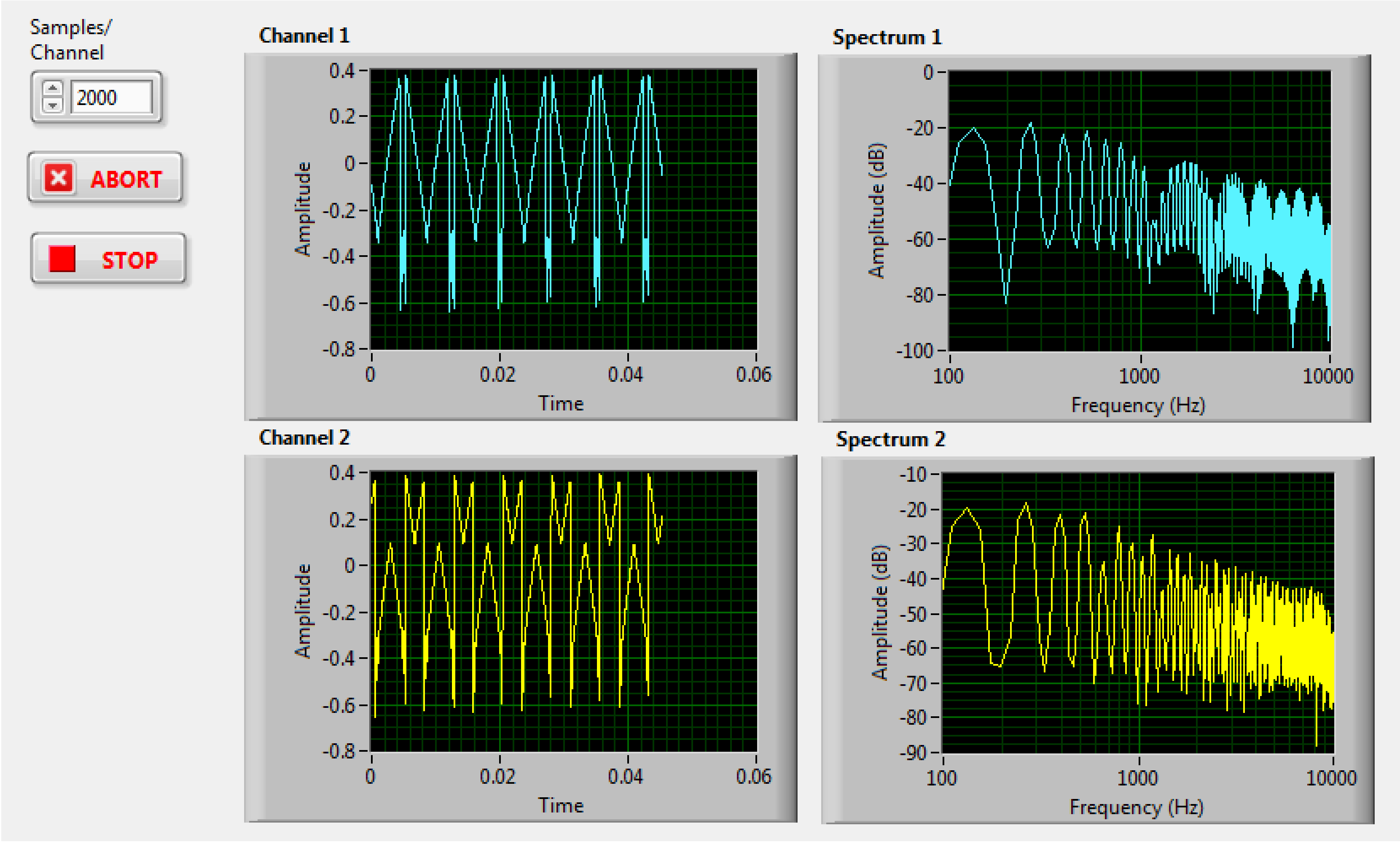
Nyquist theorem
Sampling theorem

Temporal spacing of signal sampling $\Delta t > \dfrac{1}{2\nu}$

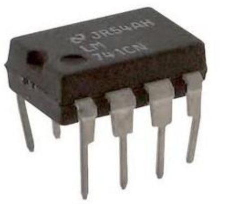


ALIASING

LabVIEW Assignment 8: Audio Spectrum Analyzer



Lab 8: Operational Amplifiers



Often better alternative to simple transistor amplifiers

Stability – circuits nearly immune to temperature drift

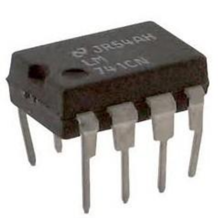
Versatile especially with use of feedback

Electrical implementation of **mathematical operations**

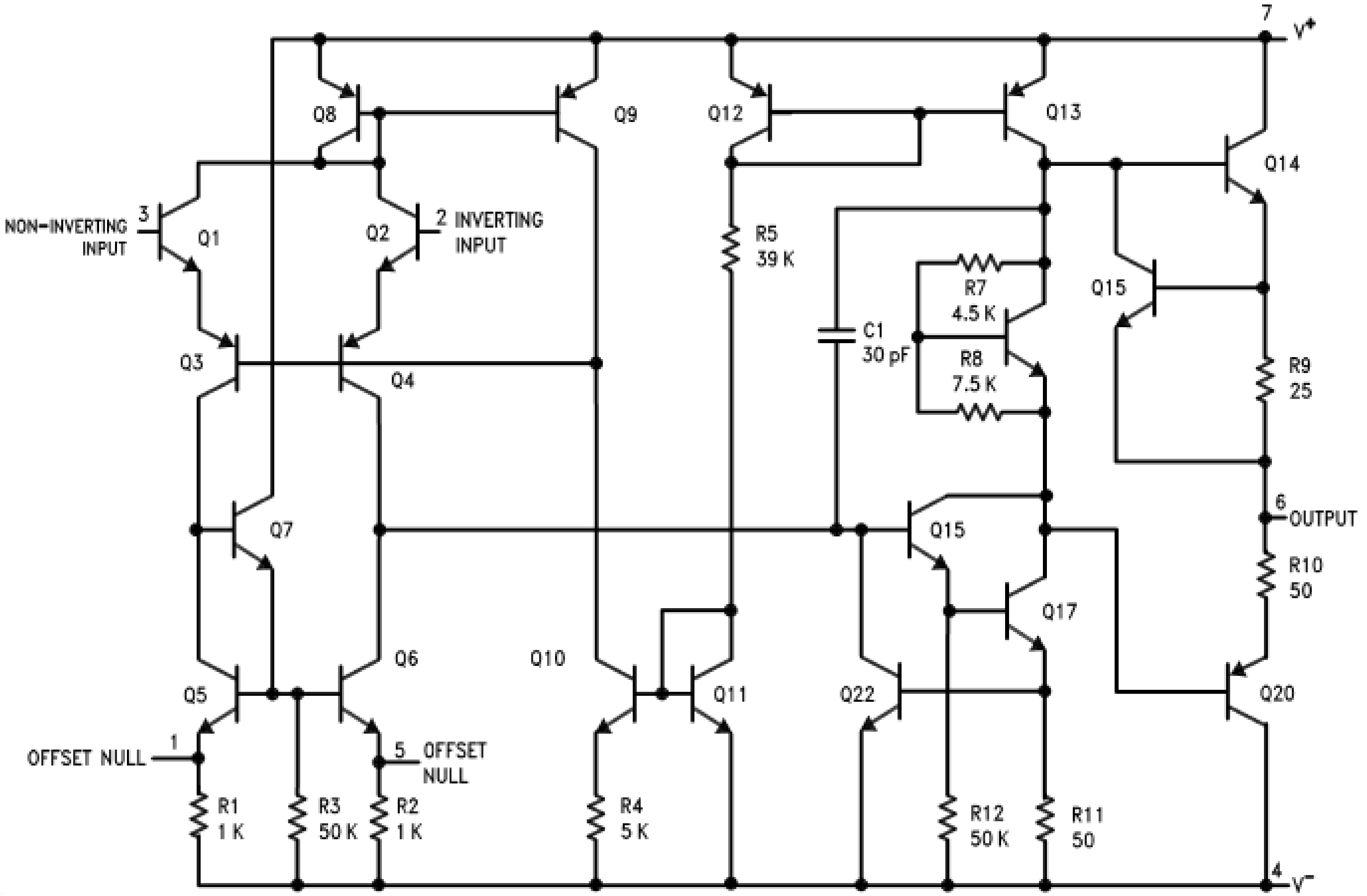
Individual transistors: highest frequency operation, high power

Op amps packaged as an integrated circuit (IC)

LM741 Operational amplifier

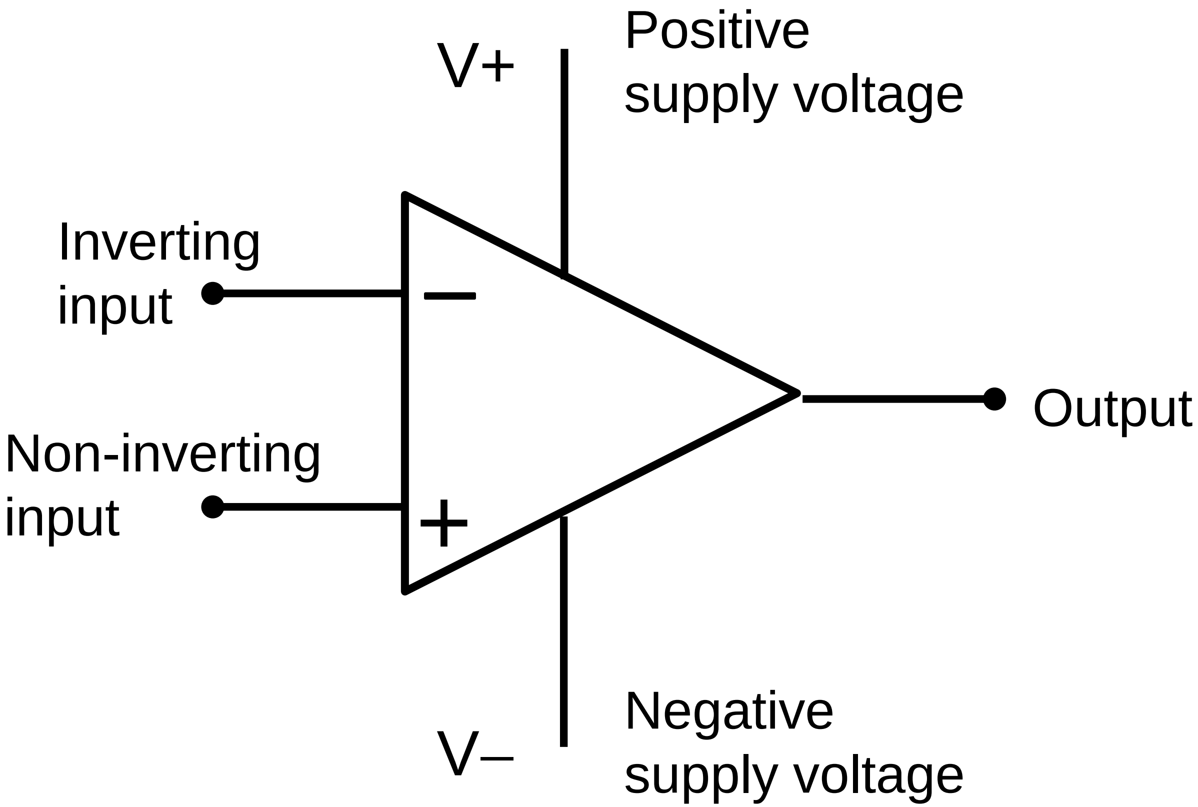


SCHEMATIC DIAGRAM



IDEAL OP-AMP

* Infinite gain
* Infinite input impedance/resistance
* Output current can go to infinity if needed



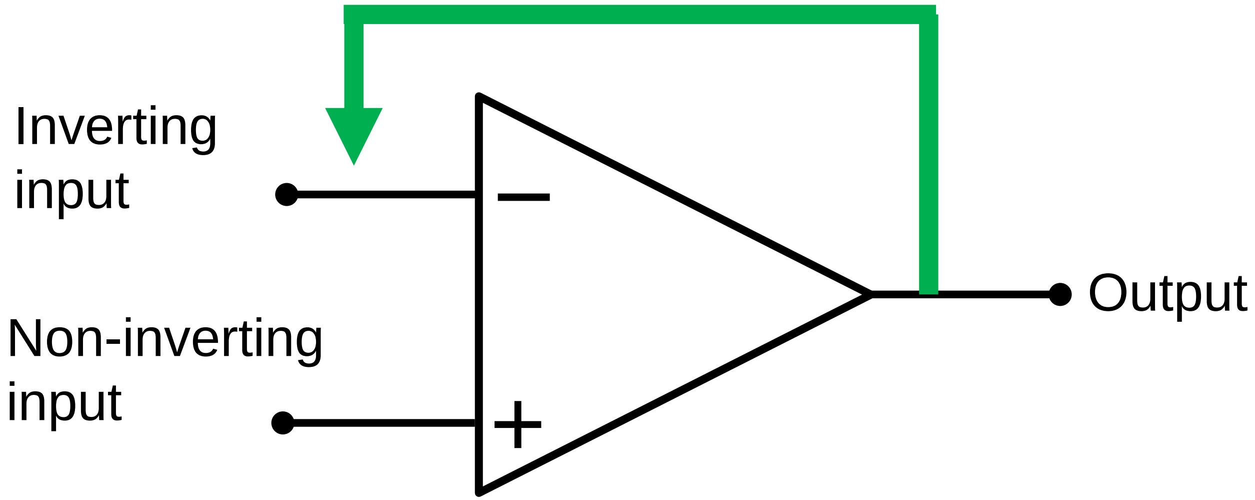
<u>"Golden Rules"</u>

I. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero.

II. The inputs draw no current.

FEEDBACK: **Sending a portion of the output back to the input**

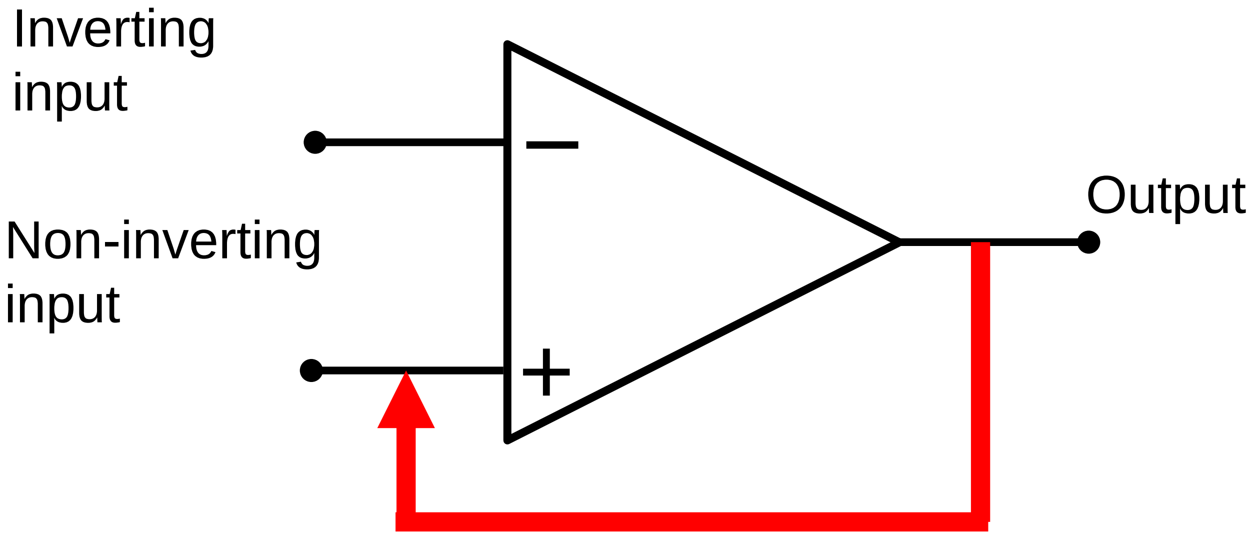
Negative feedback



Stabilizes the output of an amplifier

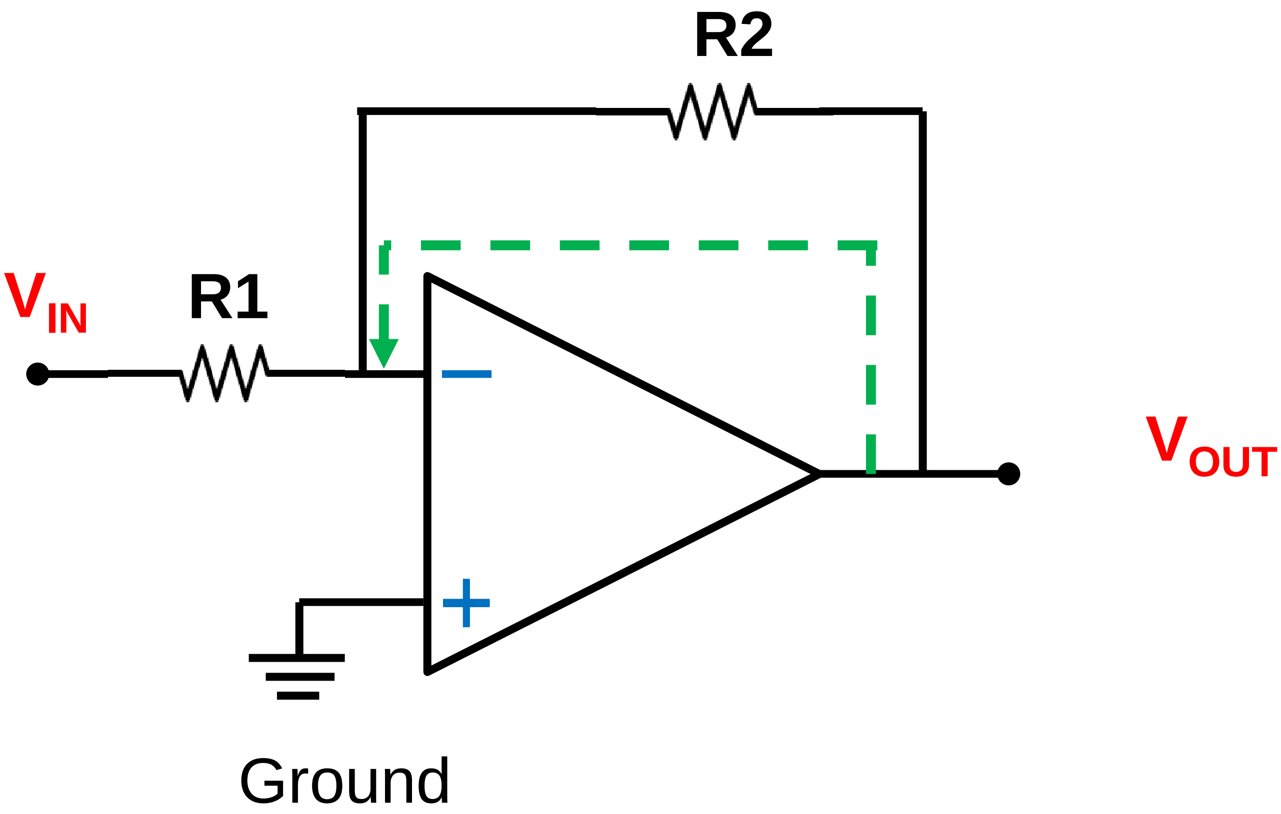
FEEDBACK: **Sending a portion of the output back to the input**

Positive feedback



Runaway amplification – Oscillation

Op Amps use negative feedback to produce stable amplification

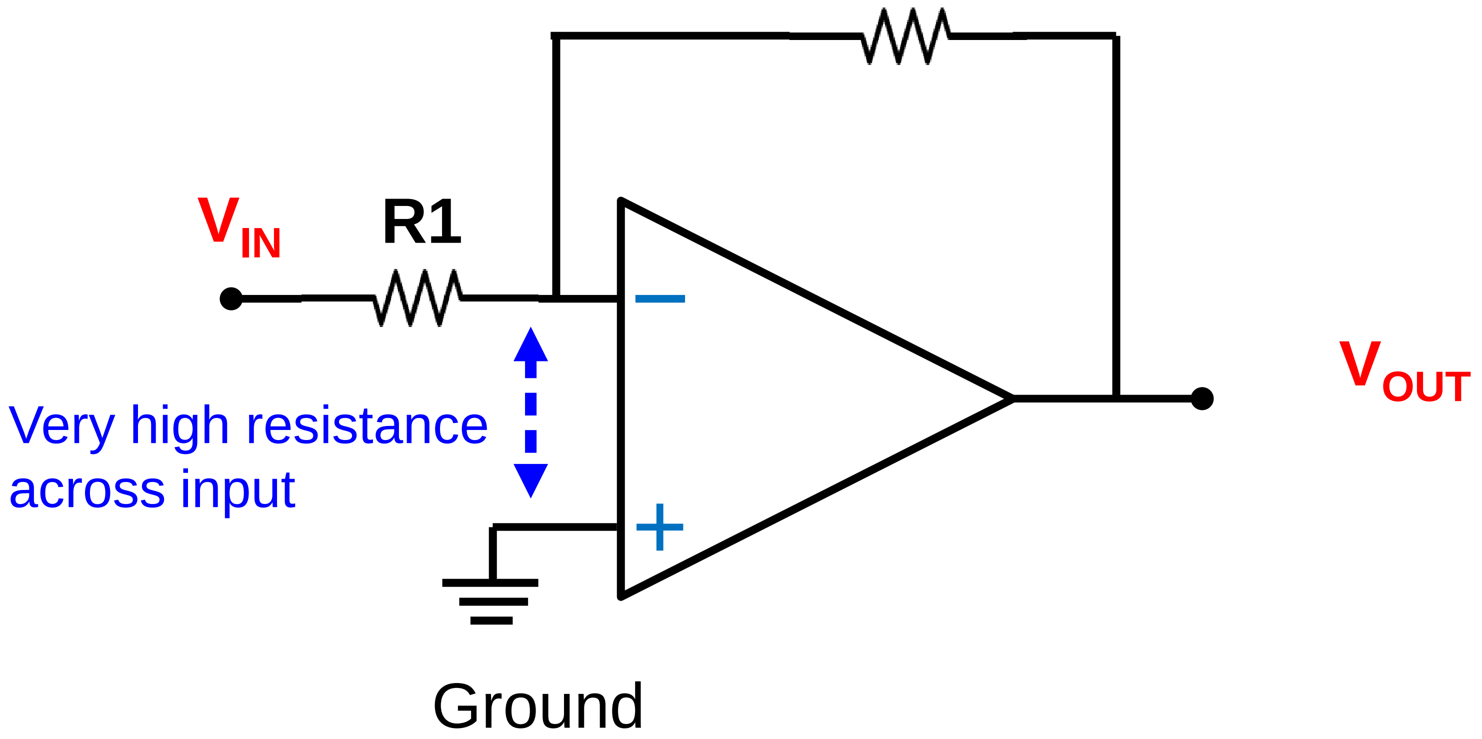


"Golden Rules"

I. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero.

II. The inputs draw no current.

Analysis



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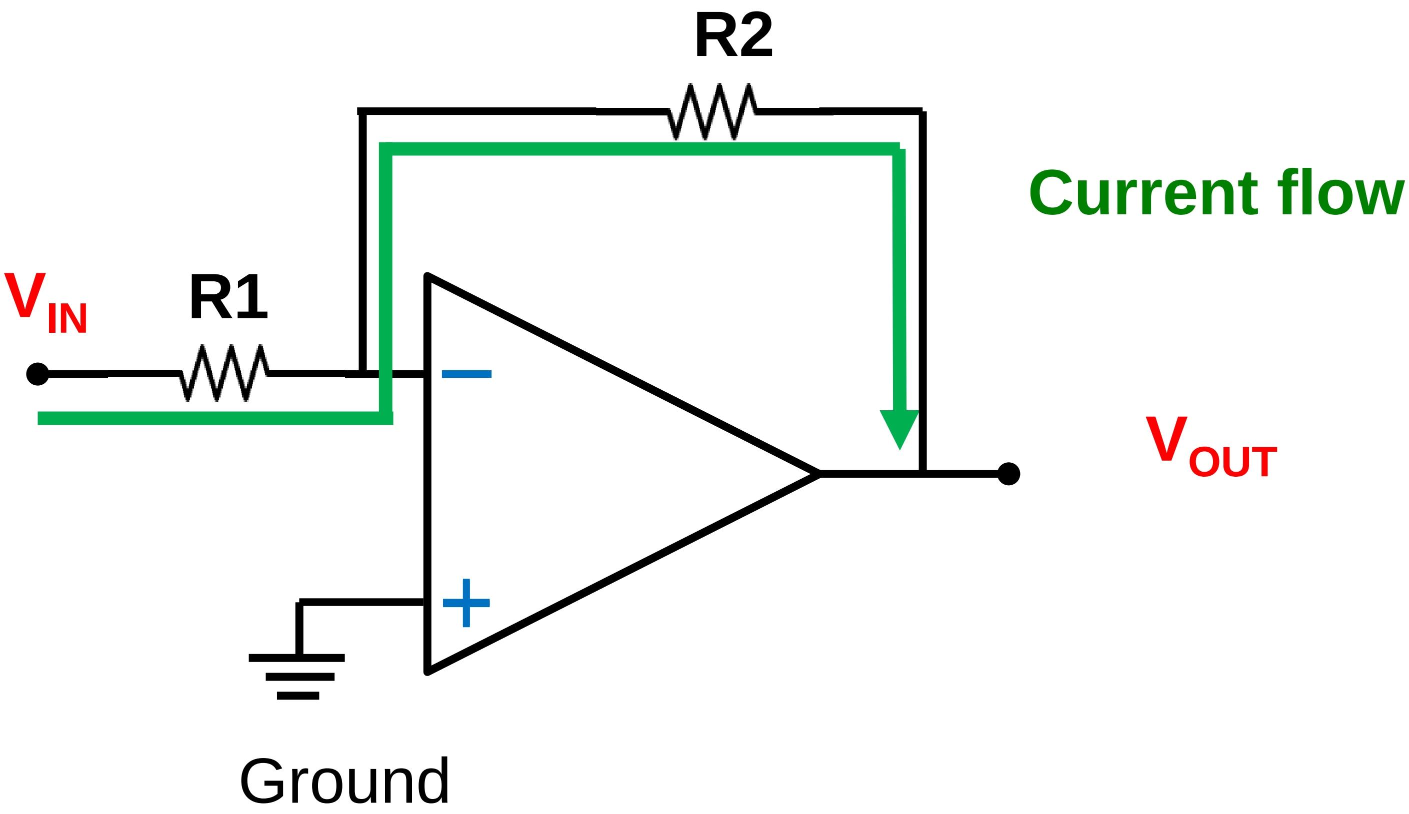

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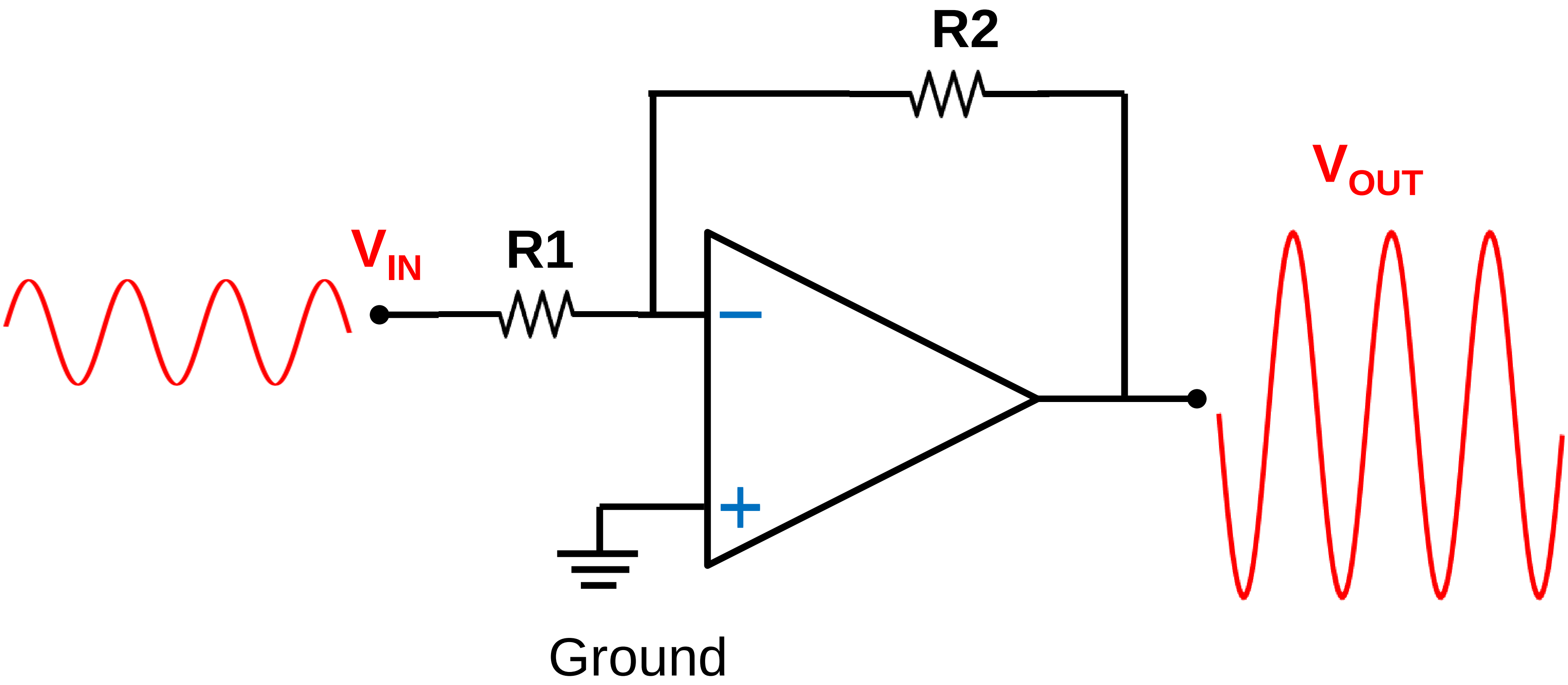
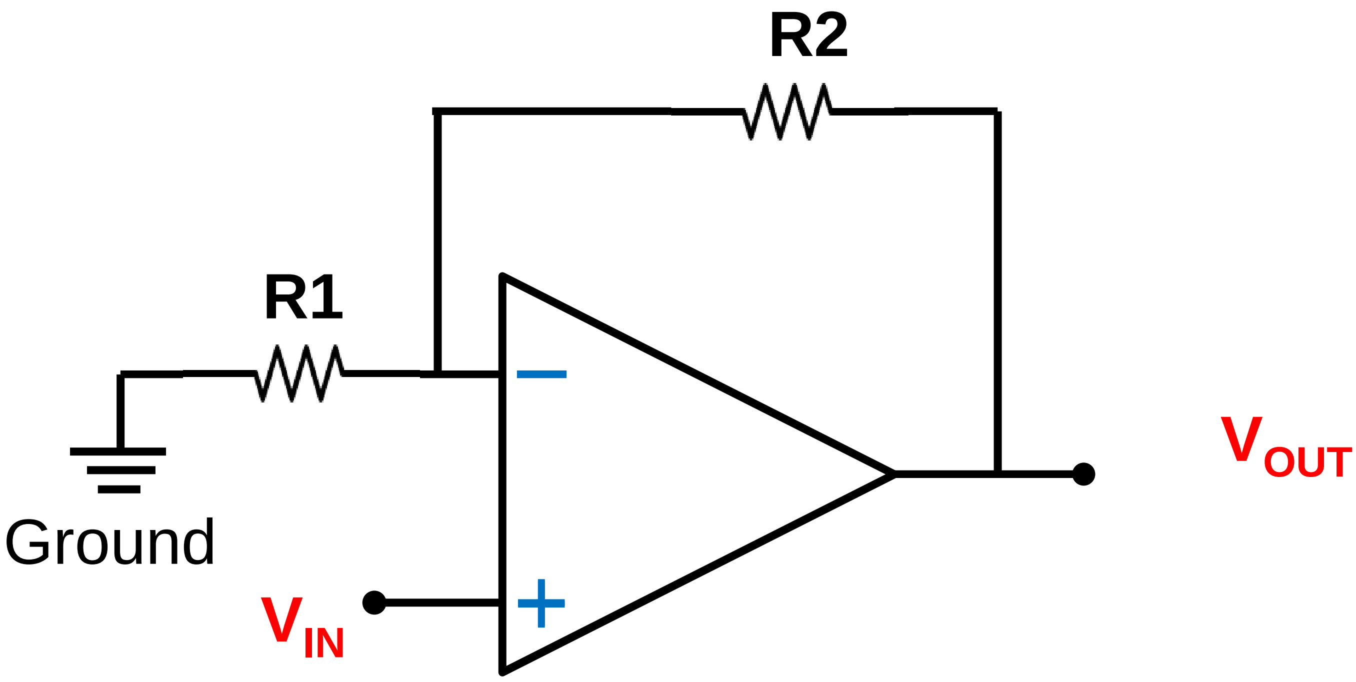
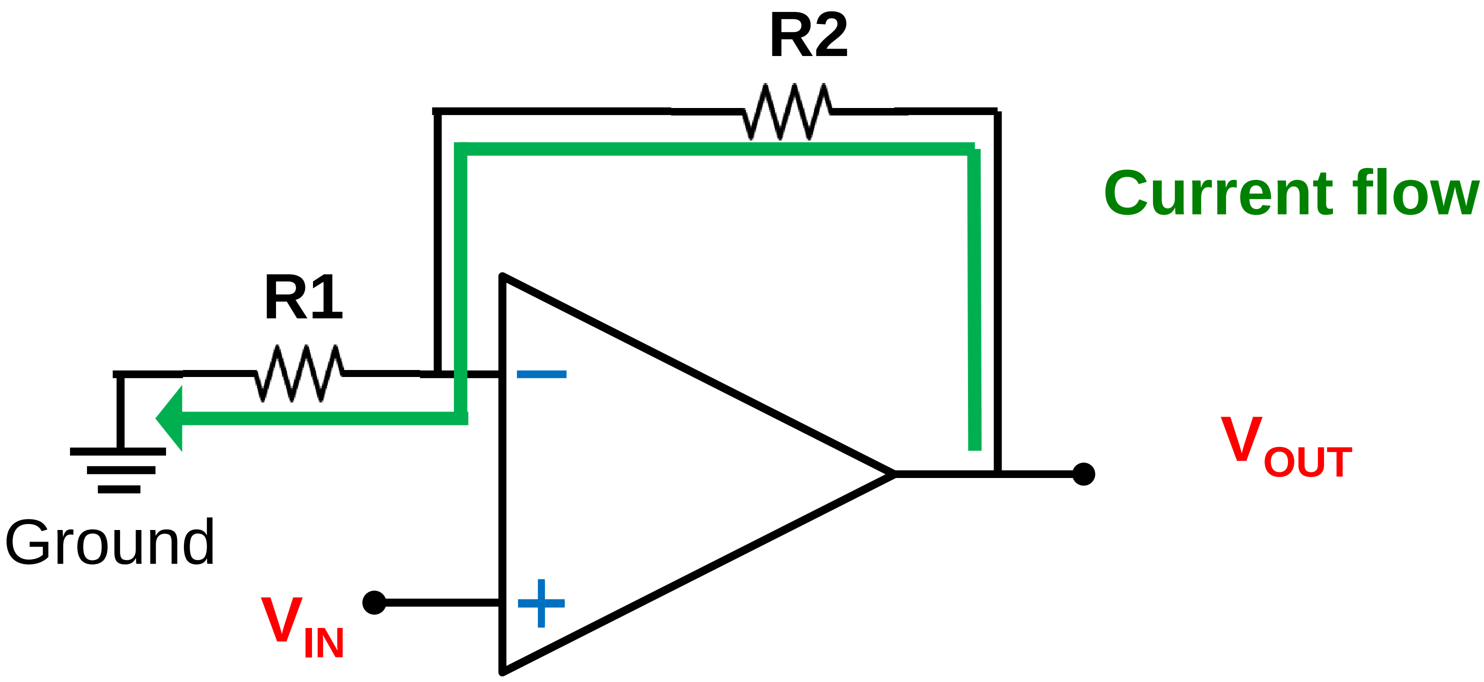


Inverting amplifier: **GAIN =** $-\dfrac{R2}{R1}$

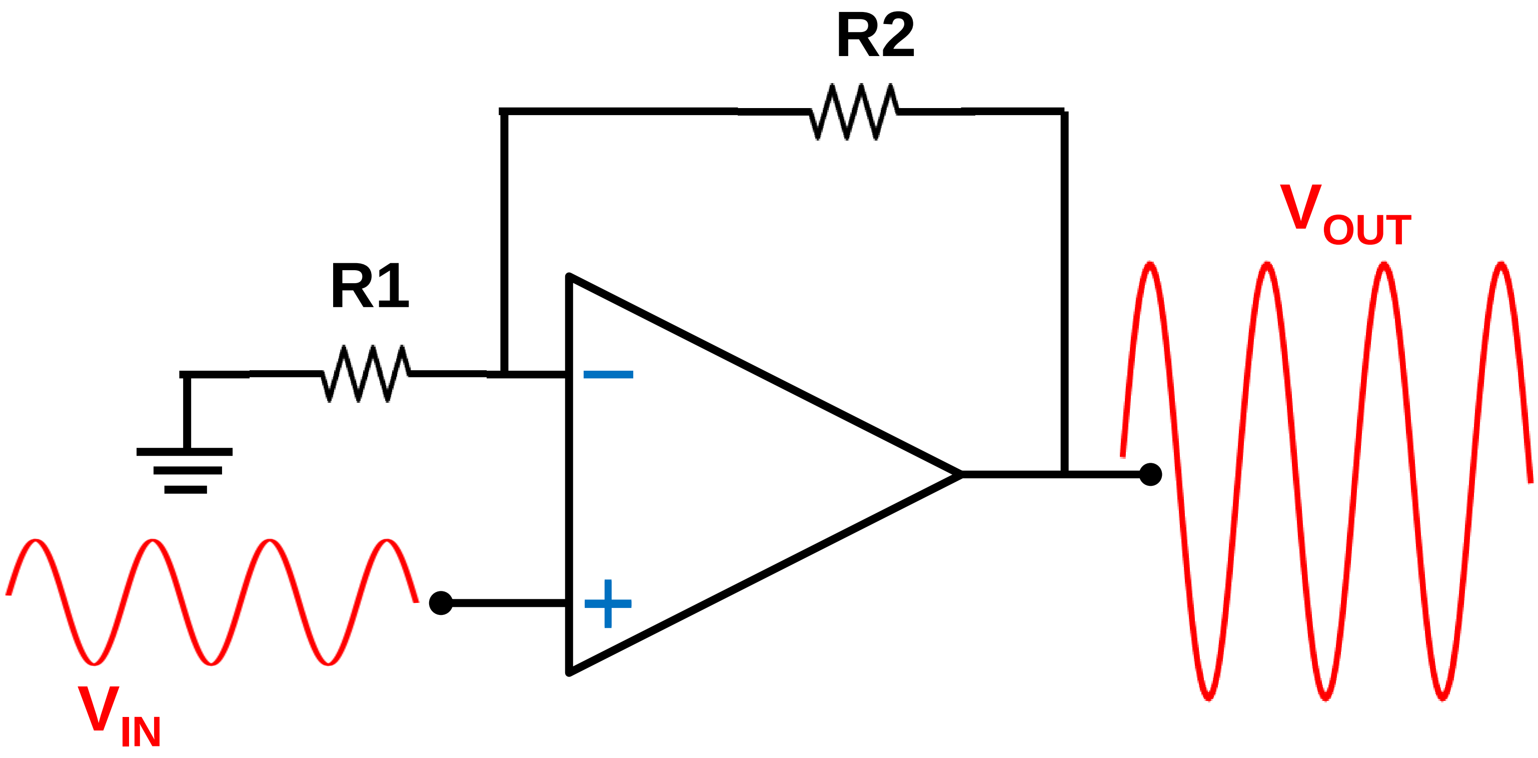
Op Amps use negative feedback to produce stable amplification



Op Amps use negative feedback to produce stable amplification

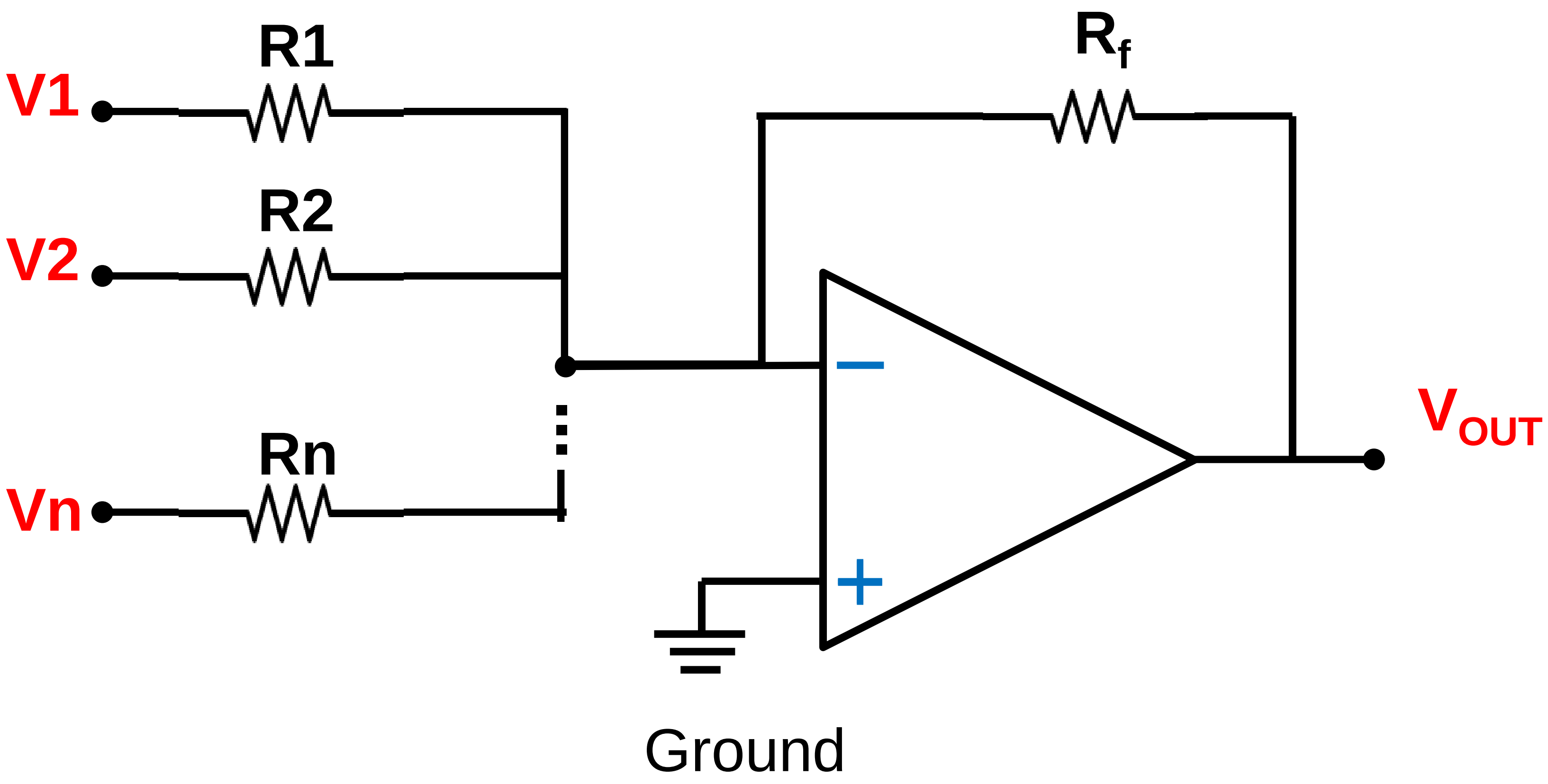


Op Amps use negative feedback to produce stable amplification



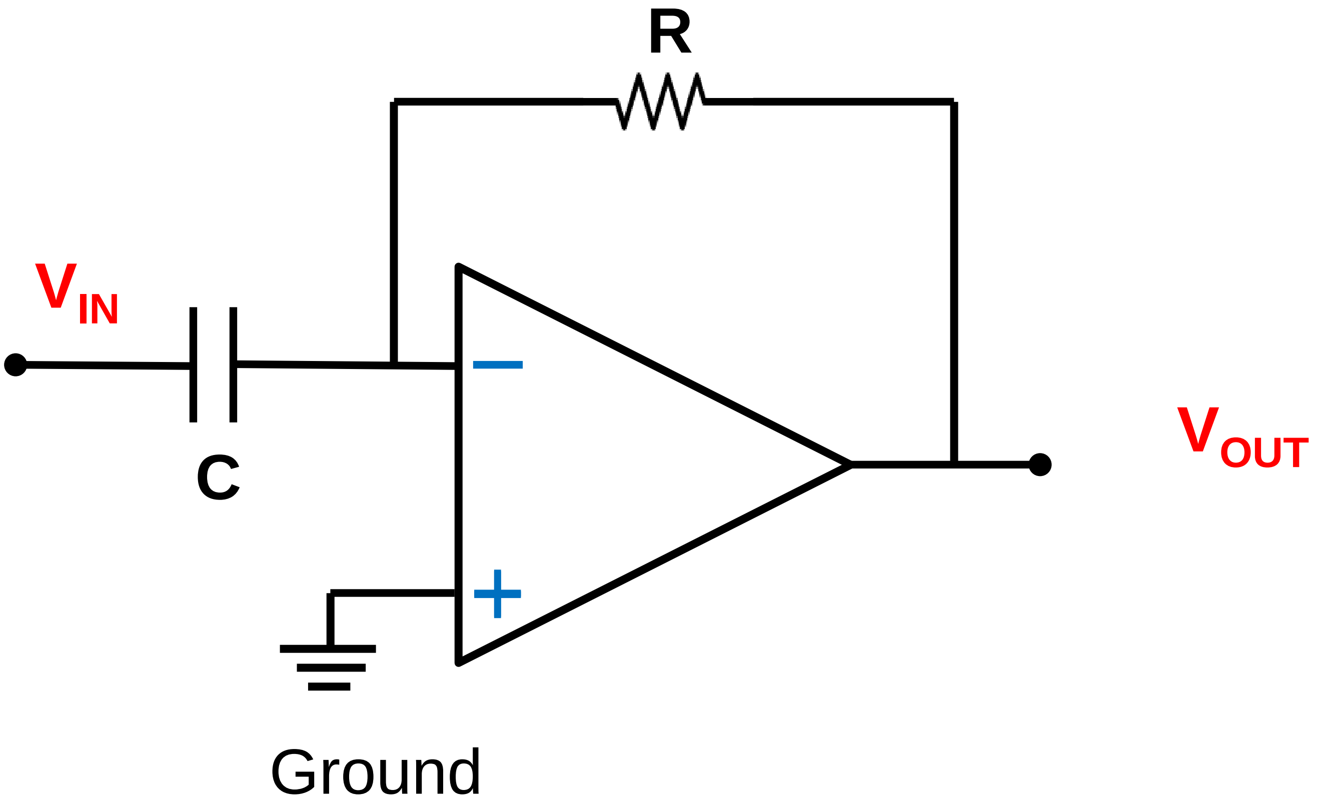
Non-inverting amplifier: GAIN = $1 + \frac{R2}{R1}$

Summing Amplifier



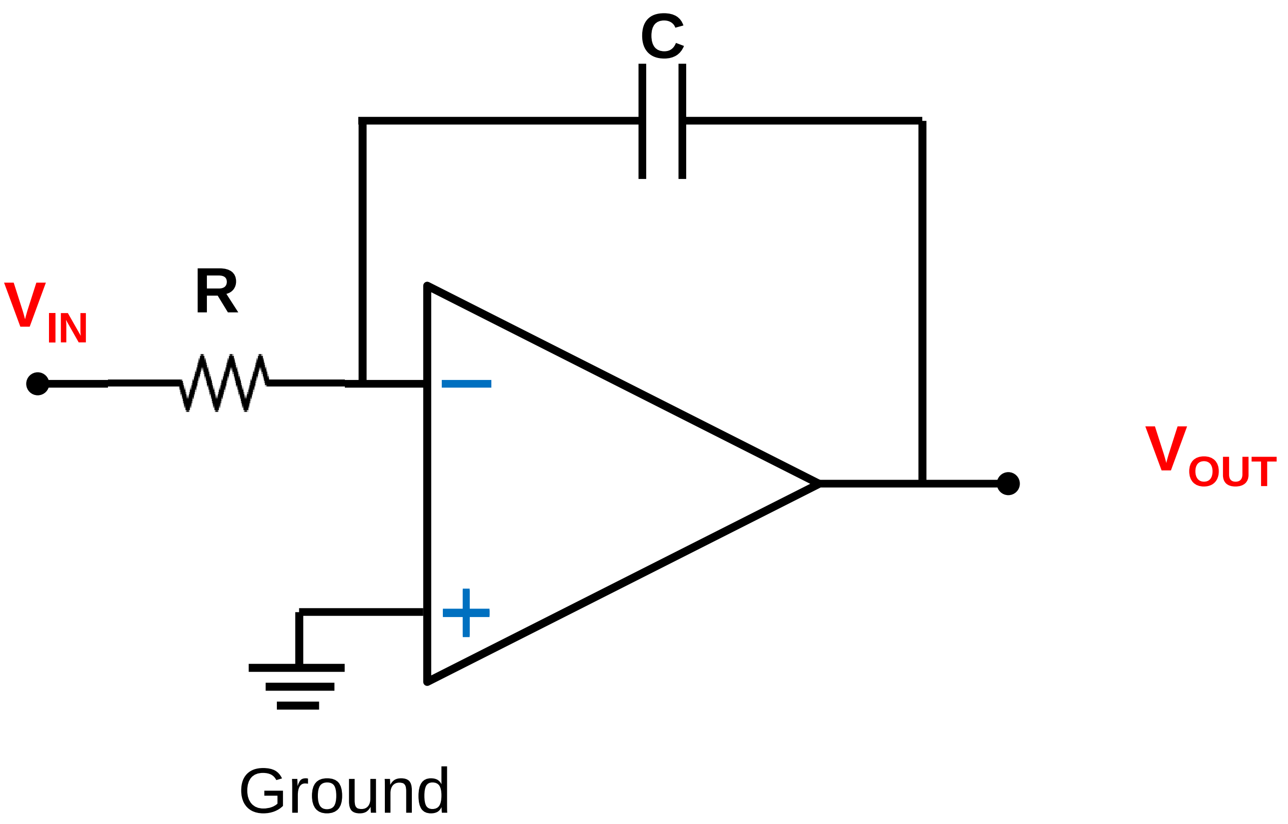
$$V_{OUT} = -V_1\frac{R_F}{R_1} - V_2\frac{R_F}{R_2} \ldots - V_n\frac{R_F}{R_n}$$

Differentiating Amplifier



$$V_{OUT} = -RC\frac{dV_{IN}}{dt}$$

Integrating Amplifier



$$V_{OUT} = -\frac{1}{RC}\int V_{IN}\,dt$$