# Senior Laboratory PHYS 493L, Spring 2024 

Lab Time: Tuesdays \& Thursdays, 8am-10:50am
Lectures: most Tuesdays (PAIS 1405)
Lab Location: PAIS 1417

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Office Hours: arrange meeting with instructor or TA via email

## Error in Measurement

| Research Group | Reported value |
| :--- | :--- |
| Yale | 15 |
| Waterloo | 15 |
| UNM | 12 |
| UCSB | 15 |

## Error in Measurement

| Research Group | Reported value |
| :--- | :--- |
| Yale | $15 \pm 7$ |
| Waterloo | $15 \pm 8$ |
| UNM | $12 \pm 2$ |
| UCSB | $15 \pm 4$ |

## Error in Measurement

| Research Group | Reported value |
| :--- | :--- |
| Yale | $15 \pm 7 \mathrm{~g} / \mathrm{cm}^{3}$ |
| Waterloo | $15 \pm 8 \mathrm{~g} / \mathrm{cm}^{3}$ |
| UNM | $12 \pm 2 \mathrm{~g} / \mathrm{cm}^{3}$ |
| UCSB | $15 \pm 4 \mathrm{~g} / \mathrm{cm}^{3}$ |



Density of gold $=19.3 \mathrm{~g} / \mathrm{cm}^{3}$ Density of lead $=11.4 \mathrm{~g} / \mathrm{cm}^{3}$

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$$
\begin{aligned}
& \text { Density of gold }=19.3 \mathrm{~g} / \mathrm{cm}^{3} \\
& \text { Density of lead }=11.4 \mathrm{~g} / \mathrm{cm}^{3}
\end{aligned}
$$

Conclusion: It is very important to understand the reported errors on the UNM measurement.

## Uncertainty is Necessary

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- "Error" does not mean mistake
- Important related ideas:
- Error and Uncertainty
- Accuracy vs. Precision
- Statistical vs. Systematic Uncertainty
- Significant Figures
- Resolution
- Uncertainty in fitted data: errors on parameters vs. goodness of fit


## Uncertainty is Necessary

## (In physics) "Error" does not mean mistake!

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## A word on Precision and Accuracy



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Large statistical uncertainty Small systematic uncertainty


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Small statistical uncertainty Small systematic uncertainty


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## A word on Precision and Accuracy



Large statistical uncertainty systematic uncertainty = ??

Small statistical uncertainty systematic uncertainty = ??

Large statistical uncertainty systematic uncertainty = ??

Small statistical uncertainty systematic uncertainty = ??

It is always necessary to evaluate experimental/systematic sources of error/uncertainty, no matter how "good" your data.

Precision scientists often use "blinds" to prevent researchers from biasing data while it is being taken.
"We have learned a lot from experience about how to handle some of the ways we fool ourselves. One example: Millikan measured the charge on an electron by an experiment with falling oil drops, and got an answer which we now know not to be quite right. It's a little bit off because he had the incorrect value for the viscosity of air. It's interesting to look at the history of measurements of the charge of an electron, after Millikan. If you plot them as a function of time, you find that one is a little bit bigger than Millikan's, and the next one's a little bit bigger than that, and the next one's a little bit bigger than that, until finally they settle down to a number which is higher.
"Why didn't they discover the new number was higher right away? It's a thing that scientists are ashamed of—this history—because it's apparent that people did things like this: When they got a number that was too high above Millikan's, they thought something must be wrong-and they would look for and find a reason why something might be wrong. When they got a number close to Millikan's value they didn't look so hard. And so they eliminated the numbers that were too far off, and did other things like that ..."

## Significant Figures

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- The least significant digit tells you to what precision you've measured that value.


## Examples:

- 200.006 meters
$\rightarrow$ How many sig. figs.?
$\rightarrow$ What is the measurement precision? (order of magnitude)


## Significant Figures

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Examples: (How many sig figs? LSD?)

- 0.0032
- 32.00
- 320


## Significant Figures

- The least significant digit tells you to what precision you've measured that value.

Examples: (How many sig figs? LSD?)

- 320 (ambiguous!)


## Breaking ambiguity:

320. 

3.2 e 2 or $3.2 \times 10^{2}$

320 +/- 10
$\rightarrow$ Why does the last case break ambiguity?

## Significant Figures

- Adding or subtracting:
- Answer has same LSD as the least precise measurement
$13.04 \mathrm{~s}+10.2 \mathrm{~s}$
103.42-0.42


## Significant Figures

- Multiplying or Dividing:
- Answer has same number of sig. figs. as the measurement with fewest sig. figs.
13.05*10.0

1105/5.0

## How to report uncertainties

$3.0 \pm 0.7 \mathrm{~cm}=3.0(7) \mathrm{cm}$

## How to report uncertainties

- Either:

$$
\begin{gathered}
3.0 \pm 0.7 \mathrm{~cm} \text {, or } \\
3.0(7) \mathrm{cm}
\end{gathered}
$$

Almost always rounded to one sig fig: $3.0052 \pm 0.0004 \mathrm{~cm}$

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- Last sig fig in answer should usually be same order of magnitude as uncertainty



## How to report uncertainties

$3.0 \pm 0.7 \mathrm{~cm}$ Not
$3.0 \pm \sqrt{0.5} \mathrm{~cm}$

# Mathematics of Error Propagation 

Ask yourself:
Should I be thinking about absolute uncertainty or fractional uncertainty?

## Addition of measurements

10.7(3) ft + 9.3(4) ft

## Addition of measurements

$$
10.7(3) \mathrm{ft}+9.3(4) \mathrm{ft}
$$

$$
=20.0(5) \mathrm{ft}
$$

*Error reported $=\sqrt{(\text { error } 1)^{2}+(\text { error } 2)^{2}}$
*for uncertainties which are independent and random

## Subtraction of measurements

10.7(3) ft-9.3(4) ft

## Subtraction of measurements

$$
10.7(3) \mathrm{ft}-9.3(4) \mathrm{ft}
$$

$$
=1.4(5) \mathrm{ft}
$$

Error reported $=\sqrt{(\text { error } 1)^{2}+(\text { error } 2)^{2}}$

Note: Subtraction of large and similarly valued measurements can lead to a big increase in fractional uncertainty

Multiplication or Division of measurements

$$
1.4(1) \mathrm{kg} * 3.5(5) \mathrm{m} / \mathrm{s}^{2}
$$

# Multiplication or Division of measurements 

$$
\begin{gathered}
\text { 1.4(1) } \mathrm{kg} * 3.5(5) \mathrm{m} / \mathrm{s}^{2} \\
=4.9(8) \mathrm{N} \\
z=x * y \\
\frac{d z}{z}=\sqrt{\left(\frac{d x}{x}\right)^{2}+\left(\frac{d y}{y}\right)^{2}}
\end{gathered}
$$

(for uncertainties which are independent and random)

## General formula for error propagation

$$
\begin{gathered}
y=\mathrm{f}(\mathrm{x}) \\
\delta y=\left|\frac{d y}{d x}\right| * \delta x \\
y=\mathrm{f}\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{N}\right) \\
\delta y=\sqrt{\left(\left|\frac{\partial y}{\partial x_{1}}\right| * \delta x_{1}\right)^{2}+\cdots+\left(\left|\frac{\partial y}{\partial x_{N}}\right| * \delta x_{N}\right)^{2}}
\end{gathered}
$$

(for uncertainties which are independent and random)

## Reporting uncertainty in your lab reports

- When reporting uncertainties, tell the reader where they come from. These could be:
- The error bar on a fit. (A fit (to the expected function) gives a rate of 4.0(1) liters/s.)
- The resolution of an instrument you used to measure. ("The analyzer had a resolution bandwidth of 100 kHz .")
- The expected number noise on random events. $(\sqrt{N})$
- The standard deviation on repeated measurements. (We measure 100(9) microorganisms per sample.)
- Most often, one source of uncertainty dominates the uncertainty in your results. Learn to identify this!

Reporting uncertainty in your lab reports

- When reporting uncertainties, tell the reader where they come from.
- Most often, one source of uncertainty dominates the uncertainty in your results. Learn to identify this!
- Do not try to combine statistical deviation and uncertainty of in measuring devices. Report these separately!


## Mean and standard deviation

## Mean:

$$
\mu=\frac{x_{1}+x_{2}+\cdots+x_{N}}{N}
$$

(Population) Standard Deviation:

$$
\sigma=\sqrt{\frac{\sum_{i=1}^{N}\left(x_{i}-\mu\right)^{2}}{N}}
$$

Sample Standard Deviation:

$$
\sigma=\sqrt{\frac{\sum_{i=1}^{N}\left(x_{i}-\mu\right)^{2}}{N-1}}
$$

## Standard error is different:

> Mean:
> $\mu=\frac{x_{1}+x_{2}+\cdots+x_{N}}{N}$

Standard Deviation:

$$
\sigma=\sqrt{\frac{\sum_{i=1}^{N}\left(x_{i}-\mu\right)^{2}}{N}}
$$

Standard Error ("Standard Deviation of the Mean"):

$$
\sigma_{\mu}=\frac{\sigma}{\sqrt{N}}
$$

## Error Analysis: an excellent reference



## Homework \#1

Let's go through it.

