### Zeeman writeup:

### Study of the Zeeman effect in rubidium using saturated absorption spectroscopy.

The electromagnetic interaction between light and atoms can be used to investigate the rich structure of the energy levels of alkali atoms. This structure is the result of the coupling between the electron and the nuclear angular momenta. The coupling of the electron spin and the nuclear angular momentum give rise to hyperfine interactions resulting in the atomic energy level structure. In this experiment we will investigate the Zeeman effect in the magnetic energy sublevels within the hyperfine structure in rubidium. By applying a constant magnetic field, the degeneracy of the hyperfine levels is broken and allows us to resolve the manifold of these energy levels. By using saturated absorption spectroscopy, which is used to resolve quantum energy levels within the natural linewidths at room temperatures, it is possible to study the Zeeman shift of magnetic levels and infer the Landé g-factor of the quantum interaction.

**Goal**: In this experiment you will investigate the Zeeman effect in the magnetic sublevels of hyperfine manifold in rubidium vapor using saturated absorption spectroscopy to resolve quantum energy levels within the natural linewidths. You will investigate the effect of a magnetic field in the frequency shift of the magnetic sublevels and determine the specific transitions are involved in the interaction. From here, you can estimate the most consistent value of the Landé factor for your observation.

### Background:

### Saturated Absorption Spectroscopy: 2-level atoms (from Satspec manual)

Saturated absorption spectroscopy is a technique that allows for measuring narrow-line atomic spectral features, limited only by the natural linewidth  $\Gamma$  of the transition (for the rubidium D lines  $\Gamma \approx 6$  MHz), from an atomic vapor with large Doppler broadening of  $\Delta v_{Dopp} \sim 1$  GHz. Consider the experimental setup shown in Figure **1a**. Two lasers are sent through an atomic vapor cell from opposite directions; one, the "probe" beam, is very weak, while the other, the "pump" beam, is strong. Both beams are derived from the same laser, and therefore have the same frequency. As the laser frequency is scanned, the probe beam intensity is measured by a photodetector. If the probe frequency v coincides with the atomic resonance v0, atoms will absorb light going from the ground state  $|g\rangle$  to the excited state  $|e\rangle$  (see Fig. 2(a)) resulting in probe absorption around the atomic resonance.



If one had 2-level atoms in the vapor cell, one might record spectra like those shown in Figure **1b**. Fig. **2(c)** gives the probe beam absorption without the pump beam. Here one sees simple Doppler broadened absorption**1**; in our case the Doppler width is much larger than the natural linewidth,  $\Delta v$ Dopp  $\gg \Gamma$ , and the optical depth of the vapor is fairly small  $\tau$  (v)  $\leq$ 1 (the transmitted fraction of the probe is  $e^{-\tau(v)}$ , which defines the optical depth;  $\tau$  is proportional to the atomic vapor density and the path length), so the probe spectrum is essentially a simple Gaussian profile. Fig. **2(d)** shows the spectrum with

the pump beam, showing an additional spike right at the atomic resonance frequency. The reason this spike appears is as follows: If the laser frequency is  $v0 - \Delta v$ , with v0 the atomic resonant frequency, then the probe beam is absorbed only by atoms moving with longitudinal velocity  $v \approx c\Delta v/v0$ , moving toward the probe beam. These atoms see the probe beam blue-shifted into resonance; other atoms are not in resonance with the probe beam, and so they do not contribute to the probe absorption. These same atoms see the pump beam red-shifted further from resonance (since the pump beam is in the opposite direction) so they are unaffected by the pump beam. Thus, for laser frequencies  $v \neq v0$ , the probe absorption is the same with or without the pump beam. However, if v = v0, then atoms with v = 0 contribute to the probe absorption. These v = 0 atoms also see an on-resonance pump beam, which is strong enough to keep a significant fraction of the atoms in the excited state, where they do not absorb the probe beam (in fact they increase the probe beam intensity via stimulated emission). Thus, at v = v0 the probe absorption is less than it was without the pump beam. This is the idea of saturated absorption spectroscopy, allowing to measure sharp Doppler-free features in a Doppler-broadened vapor.<sup>1</sup>

### Saturated Absorption Spectroscopy: Multi-level atoms (from Satspec manual)

If the atoms in the absorption cell had a single ground state and two excited states (typically an electronic level split by the hyperfine interaction), and the separation of the excited states was less than the Doppler width, then one would see a spectrum like that shown in Figure **2 (left).** 



Figure<sup>(2)</sup> Saturated absorption spectrum for atoms with (left) a single ground state and two closely spaced excited states (right) schematic of situations giving rise to two saturated absorption peaks and one crossover peak.

The peaks on the left and right are ordinary saturated absorption peaks at v1 and v2, the two resonance frequencies. The middle peak at (v2+v2)/2 is called a "cross-over resonance." If you think about it for a while you can see where the extra peak comes from. It arises from atoms moving at velocities such that the pump is in resonance with one transition, and the probe is in resonance with the other transition, as shown in Fig.2 (iii) right. If you think about it a bit more you will see there are two velocity classes of atoms for which this is true atoms moving toward the pump laser, and away from it.

#### Atomic Structure of Rubidium (from Satspec manual)

The structure of the atomic energy levels in rubidium is due to the interaction between its nuclear spin (*I*) and the outer-most electron's total angular momentum J = L + S, and is called hyperfine structure. The ground-state electronic configuration of rubidium consists of closed shells plus a single 5S valence

<sup>&</sup>lt;sup>1</sup> The Doppler effect states that atoms moving towards (away from) a light source see the light frequency blue (red) shifted, so that atoms with different velocities absorb light at different frequencies. In a thermal vapor atoms move with velocities following the Maxwell-Boltzmann distribution, which produces broadening in the absorption profile.

electron. This gives a spectrum which is similar to hydrogen. For the first excited state the 5S electron is moved up to 5P. Rubidium has two stable isotopes: **85Rb** (72% abundance), with nuclear spin quantum number I = 5/2, and **87Rb** (28% abundance), with I = 3/2.

The different energy levels are labeled by "term states", with the notation  ${}^{25+1}L'_J$ , where *S* is the spin quantum number, *L* is the spectroscopic notation for the angular momentum quantum number (i. e. S, P, D, ..., for orbital angular momentum quantum number *L* = 0, 1, 2, ...), and *J*=*L*+*S* is the total angular momentum quantum number. For the ground state of rubidium *S* = 1/2 (since only a single electron contributes), and *L* = 0, giving *J* = 0 + 1/2 = 1/2 and the ground state  ${}^{2}S_{1/2}$ . For the first excited state we have S = 1/2, and *L* = 1, giving *J* = 1/2 or J = 3/2, so there are two excited states  ${}^{2}P_{1/2}$  and  ${}^{2}P_{3/2}$ . Spin-orbit (S-L) coupling lifts splits the otherwise degenerate  $P_{1/2}$  and  $P_{3/2}$  levels, as shown in Fig. **3**.

The interaction between the nuclear spin I and the total electron angular momentum J gives rise to the hyperfine interactions [2]. This interaction leads to a hyperfine energy shift of:

$$\Delta Ehfs = \frac{1}{2}A_{hfs}K + B_{hfs}\frac{\frac{3}{2}K(K+1) - 2I(I+1)J(J+1)}{2I(2I-1)2J(2J-1)}$$
$$K = F(F+1) - I(I+1) - I(I+1)$$

where F = I + J is the total angular momentum quantum number including nuclear spin,  $A_{hfs}$  is the magnetic dipole constant, and  $B_{hfs}$  is the electric quadrupole constant. Figures **3** and **4** shows the lower S and P energy levels for 85Rb and 87Rb, including the hyperfine splitting.



Fig. 3. (Left) Energy diagrams for the D2 lines in the two stable rubidium isotopes in the ground and excited states. (right) Typical absorption spectrum for a rubidium vapor cell, with different lines.



Fig. 4. Detailed hyperfine splittings of the  $5S_{1/2}$  and  $5P_{1/2}$  levels in rubidium.

#### Zeeman effect: (from Steck modified)

Each of the hyperfine (F) energy levels contains 2F + 1 magnetic sublevels labeled  $m_F$  ( $-F < m_F < F$ ) that determine the angular distribution of the electron wave function. In the absence of external magnetic fields, these sublevels are degenerate. However, when an external magnetic field is applied, their degeneracy is broken producing shifts in the magnetic sublevels depending on the value of. If the energy shift due to the magnetic field **B** is small compared to the fine-structure splitting, then **J** is a good quantum number and the Hamiltonian describing the atomic interaction with the weak magnetic field  $H_B$  can be written as [2,STECK]

$$H_B = \frac{\mu_B}{\hbar} \big( g_J J_z + g_I I_z \big) B_z$$

with

$$g_J \approx 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

Here, we have taken the magnetic field to be along the z-direction (i.e., along the atomic quantization axis). The quantities  $g_I$  and  $g_I$  are respectively the total electron and nuclear "g-factors" that account for various modifications to the corresponding magnetic dipole moments.<sup>2</sup>

If the energy shift due to the magnetic field is small compared to the hyperfine splittings (See Fig. 5), then similarly F is a good quantum number, so the interaction Hamiltonian becomes [2,STECK]

$$H_B = \mu_B g_F F_z B_z$$

where the hyperfine Landé g-factor is given by

$$g_F \approx g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)}$$

For weak magnetic fields, the interaction Hamiltonian  $H_B$  perturbs the hyperfine levels so that (to lowest order) the hyperfine levels split linearly according to

$$\Delta E_{|F,m_F\rangle} = \mu_B g_F m_F B_z$$

This interaction changes the resonance frequency from the initial state  $|F, m_F\rangle$  in the ground state level  $5S_{1/2}$  to the final state  $|F', m_{F'}\rangle$  in the exited state level  $5P_{3/2}$ , so that

$$\Delta_1 = \Delta_0 + \mu_B \left( g_F \, m_F - g_{F'} \, m_{F'} \right) B_z$$

In this experiment you will study the Zeeman effect using saturated absorption spectroscopy. The aim is to determine the Hyperfine Landé g-factors  $g_F$  and  $g_F'$  for the ground  $5S_{1/2}$  and excited  $sP_{3/2}$  states in rubidium. To this end, you will excite the atoms with circularly polarized light to selectively excite transitions with a change in quantum number of  $|m_F - m_{F'}| = 1$ . Quantum selection rules state that circular polarized light with helicity  $\sigma^+$  will make a transition from the ground state  $m_F$  to  $m_{F'} = m_F + 1$ . And circular polarized light with helicity  $\sigma^-$  will make a transition from the ground state  $m_F$  to  $m_{F'} = m_F + 1$ .

<sup>&</sup>lt;sup>2</sup> This expression for  $g_J$  assumes the Landé factor for the spin  $g_S \approx 1$ , and the orbital angular momentum  $g_L \approx 2$ .

 $m_F - 1$ . This circularly polarized light can be generated from laser light, which is linearly polarized, using a quarter waveplate QWP [Hetch].

### **Experimental Procedure**

The goal of this section is first to observe saturated absorption spectra for the rubidium lines. Then by using the saturated absorption spectrum peaks in the P3/2 hyperfine splitting of 87Rb, investigate the Zeeman effect in the shifts of the magnetic energy levels. You will use the information of the Hyperfine splitting in the saturated absorption spectra for calibration of the frequency.

**Eye safety.** The laser operates at 780 nm, which is very close to being, with power ~20 mW, which is concentrated in a narrow beam. It is possible to cause permanent eye damage using the Ph76 laser if you are not careful. Therefore be careful. **ALWAYS WEAR LASER GOGGLES WHEN THE LASER IS ON!** As long as you keep the goggles on, your eyes will be protected.

# Resonant absorption and laser tuning. (Detailed instructions are in Satspec exp.)

The first step for this experiment is to tune the laser on resonance the rubidium lines. Figure 4 shows the absorption profile in Rb vapor over a frequency span of 8 GHz. To observe resonant absorption, build the set-up shown in Figure 5.



Fig. 5. (Left) Energy diagrams for the D2 lines in the two stable rubidium isotopes in the ground and excited states. (right) Typical absorption spectrum for a rubidium vapor cell, with different lines.

The ND filter (attenuator) can be removed when aligning the laser beam. Once you have the beam going about where you want it, sweep the high-voltage going to the grating PZT with a triangle wave, so that the voltage varies from about 0 to 100 volts, but keep the scan frequency of the RAMP generator  $\leq$ 10 Hz. Use the HV/100 "MONITOR" output to monitor the high voltage on the oscilloscope. Sweeping this voltage sweeps the grating position of the lasers using a small piezoelectric actuator (PZT), which changes the frequency of the laser. A good starting value of the knob of the coarse DC offset of the PZT controller can be about 4.5. While the high voltage is scanning you should then also change the laser injection current up and down by hand explore the full range. The current makes large changes in the laser frequency, while the PZT makes small changes (see the laser primer for details). By sweeping the laser frequency and exploring the full range, you will sweep over the rubidium lines and you will see some fluorescence inside the vapor cell. You can use the camera to look at the florescence in the cell through the aperture in the cell enclosing. This will appear as a bright line inside the cell. Once you see fluorescence, compare the photodiode output to the rubidium spectrum shown in Figure 4. Adjust the laser current and ramp amplitude to get the spectrum shown in Fig. 4, and record it using a digital oscilloscope. Sometimes you may only get the laser to scan over part of this spectrum without mode hopping (see the laser primer). At this point the laser is tuned to the rubidium lines. Before proceeding with the rest of the experiment, move the ND filter (attenuator) from its location in Figure 5 to a new

position right in front of the photodiode. You may see the absorption lines become much weaker. This is because atoms become saturated with the high power, which also reduces the absorption.

# **Obtaining a Saturated Absorption Spectrum.**

The suggested set-up for observing saturated absorption spectra is shown in Figure 6. The laser should remain on resonance while you change the absorption set-up from Fig. 6 for saturated absorption. This setup uses "**Probe 1**" to observe the saturated absorption setup saturated by the "**Pump**". The beam labeled "**Reference**" is a reference beam that will allow you to subtract the signal of the Doppler profile to observe saturated absorption peaks more clearly for the study of the Zeeman. The glass slab allows you to obtain two parallel beams (Probe 1 and Reference) with roughly the same intensity from external and internal reflections. The irises are an alignment guide; if you have both the pump and probe beams going through small irises, then you can be assured that the beams overlap in the rubidium cell. If you block the pump beam you should get a spectrum that looks pretty much the same as you had in the previous section.



Fig. 6 (Left) Energy diagrams for the D2 lines in the two stable rubidium isotopes in the ground and excited states. (right) Typical absorption spectrum for a rubidium vapor cell, with different lines.



Fig. **7** (Left) Saturated absorption signal in rubidium vapor (87Rb). (right) Saturated absorption peaks with subtraction of the Doppler profile using the reference beam. Note the extra crossover peaks.

# Measuring the Zeeman shift in Rb atoms.

For this measurement, you will use the information of the hyperfine splittings of Fig. **4** as a frequency reference. Use hyperfine splitting spectra in 87 Rb, which have larger splittings and will allow you to resolve peaks easier. The Satspec spectrum provides the reference to calibrate the horizontal scale of the traces in the oscilloscope. Use the oscilloscope to observe and record the effect of the external magnetic field in the magnetic sublevels from the Satspec peaks, as you scan the laser frequency. Zoom in on the hyperfine features you want to use to study the Zeeman splitting and energy shift. You will need to know which features of the satspec spectrum belong to which lines, so identify the features by

comparing your spectra with the level diagrams in **Figures 4 and 5**. Note that some of the Satspec peaks will correspond to crossover peaks. Estimate your frequency calibration error.

Use your frequency calibration to measure the shifts of the various features in the spectra in the presence of the external magnetic field *B*. Focus on a single hyperfine peak and observe the Zeeman splitting when you apply *B*. Use the HWP to prepare circularly polarized light in the probe. As you increase B, there should be a shift of the magnetic sublevels of the Satspec resonances. Study the frequency shift as a function of *B*. You should observe a linear relation between the magnetic field B and the Zeeman shift that you observe, See **Eq. 7**. The slope of this line should correspond to  $\mu_B (g_F m_F - g_F, m_{F'})$ , for which  $m_{F'} = m_F \pm 1$  for circularly polarized light. Record the shifts observed as a function of B of as many features as you can (3 or more).

# Analysis

Plot the Zeeman shifts and fit a line to obtain the dependance of the shift with B. From here, determine which line corresponds to which magnetic levels  $(m_F, m_{F'})$  assuming the Landé factors are known. Can you determine which quantum energy magnetic sublevels are involved in the transitions you are observing?

What is the accuracy of your measurement?

What are the various uncertainties you encountered along the way?

Are you results good enough to determine such transmissions? If not, discuss potential reasons: why?

# References

[STECK] (Bowie, Boyce et al. 1995) (Lee, Park et al. 1994)

# [1] https://steck.us/alkalidata/cesiumnumbers.1.6.pdf

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Using an external cavity-tuned diode laser in Doppler-free saturation spectroscopy, we are able to resolve the weak-field Zeeman splittings in the hyperfine levels of the rubidium D1 line. The number of lines observed in the Zeeman structure of rubidium and their measured energy-level splittings are as expected from theory. The measured amplitudes of the saturated-absorption signals, crossover resonances, and velocity-selective optical pumping signals are used in the analysis of various theories concerning saturated-absorption spectroscopy.

[3] Lee, H. S., et al. (1994). "Zeeman effect in the saturation spectroscopy of the 87Rb D2 line." <u>Journal</u> of the Optical Society of America B **11**(4): 558-563.

The saturated absorption spectra of the 87Rb D2 line were observed in an almost-zero magnetic field, in the geomagnetic field, and in a magnetic field of 10 G. The relative magnitudes of the resonance signals were calculated and tabulated by application of Nakayama's four-level model to the 87Rb D2 line. The observed spectra were in good agreement with the theoretical plots. However, it was found that in the case of the 10-G magnetic field it is necessary to calculate an

exact splitting of Zeeman sublevels in order to fit the theoretical calculation to the observed spectra and that saturation spectroscopy can be used as a method for observing the Zeeman splitting.

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