Lecture 16
(Momentum and Impulse, Collisions and Conservation of Momentum)

Physics 160-02 Spring 2017

Douglas Fields

Newton's Laws & Energy

- The work-energy theorem is related to Newton's 2nd Law

$$W = \Delta KE \Rightarrow$$

$$F \cdot d = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 \Rightarrow$$

$$2 \frac{F}{m} d = v_f^2 - v_i^2 \Rightarrow$$

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$$

Another Consequence of Newton’s Laws

- Notice that for Energy, we are talking about a force applied over a distance.
- In many cases, what we know is not the distance over which a force is applied, but a time during which a force is applied.
- This will lead us to a new quantity, called momentum, that we will also find very useful.

Momentum

$$\sum \vec{F} = m\vec{a}$$

- This is not the way that Newton wrote this famous equation. He actually wrote it as:

$$\sum \vec{F} = \frac{d}{dt}(\vec{p})$$

- Where p is the momentum…

Momentum

$$\vec{p} = m\vec{v}$$

- Then

$$\sum \vec{F} = \frac{d}{dt}\left(\vec{p}\right) = \frac{d}{dt}\left(m\vec{v}\right) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

- If the mass is constant, then the first term is zero, and we get back our version of Newton's 2[nd] Law:

$$\sum \vec{F} = \cancelto{0}{\frac{dm}{dt}}\vec{v} + m\frac{d\vec{v}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Impulse

$$\vec{F}_{Net} = \sum \vec{F} = \frac{d}{dt}\left(\vec{p}\right) \Rightarrow$$

$$\vec{F}_{Net} dt = d\vec{p} \Rightarrow$$

$$\int_{t_1}^{t_2} \vec{F}_{Net} dt = \int_{p_1}^{p_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

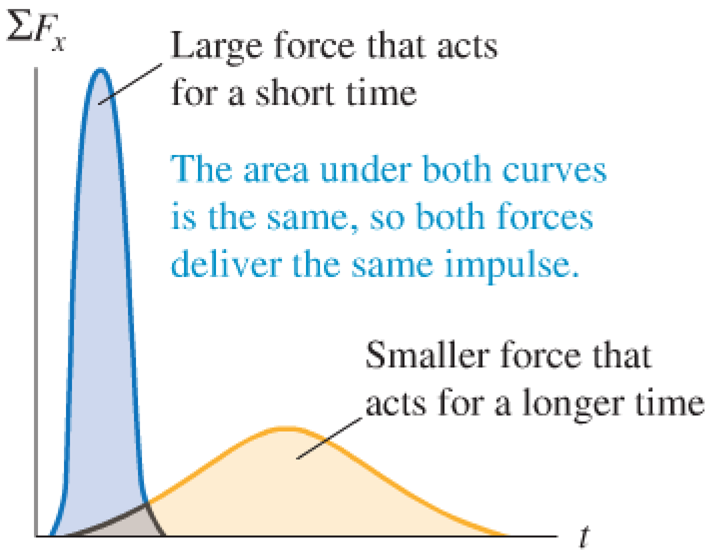
$$\vec{J} \equiv \int_{t_1}^{t_2} \vec{F}_{Net} dt = \vec{p}_2 - \vec{p}_1$$

- The impulse is defined as the integral of the force over time, and is therefore equal to the change in momentum over that same time period.

Impulse

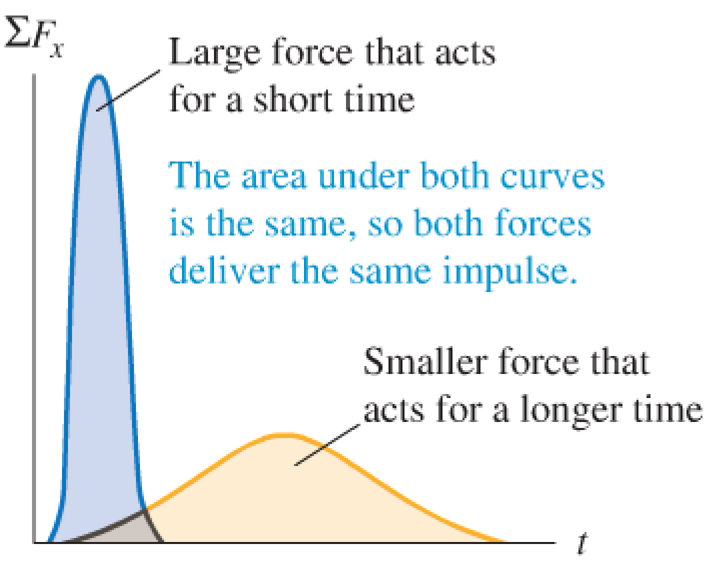
- How is this useful?
- There are many cases when the force or the time isn't known, but the change in momentum is…

$$\vec{J} \equiv \int_{t_1}^{t_2} \vec{F}_{Net} dt = \vec{p}_2 - \vec{p}_1$$



Car Crashes

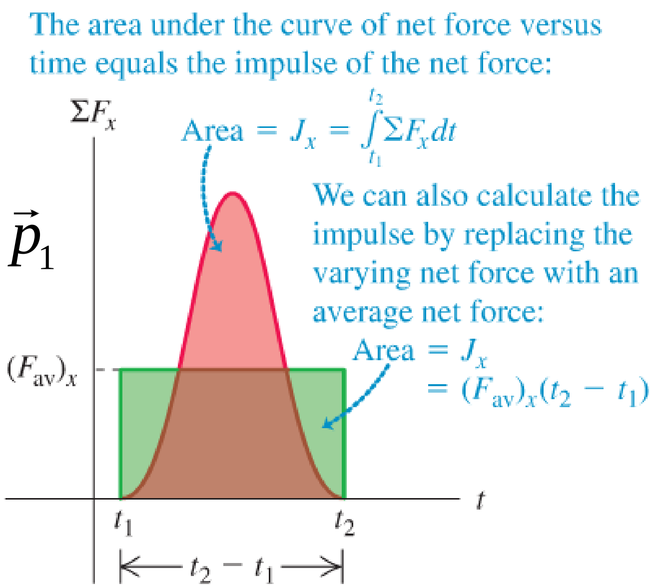
- In an automobile crash, the driver's momentum immediately before the accident and immediately after are determined, but the force acting on them is not. It depends upon the amount of time the change in momentum occurs.



Impulse

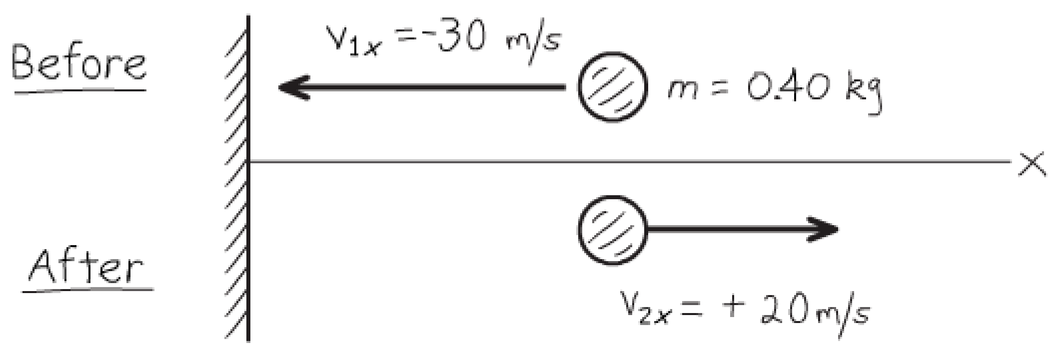
- In many cases, we may know the time period that a force acts, but not the exact form of the force as a function of time.
- In these cases, we can examine the average force that acted.

$$\vec{J} \equiv \int_{t_1}^{t_2} \vec{F}_{Net} dt = \vec{F}_{Avg} \int_{t_1}^{t_2} dt = \vec{F}_{Avg} \left(t_2 - t_1\right) = \vec{p}_2 - \vec{p}_1$$



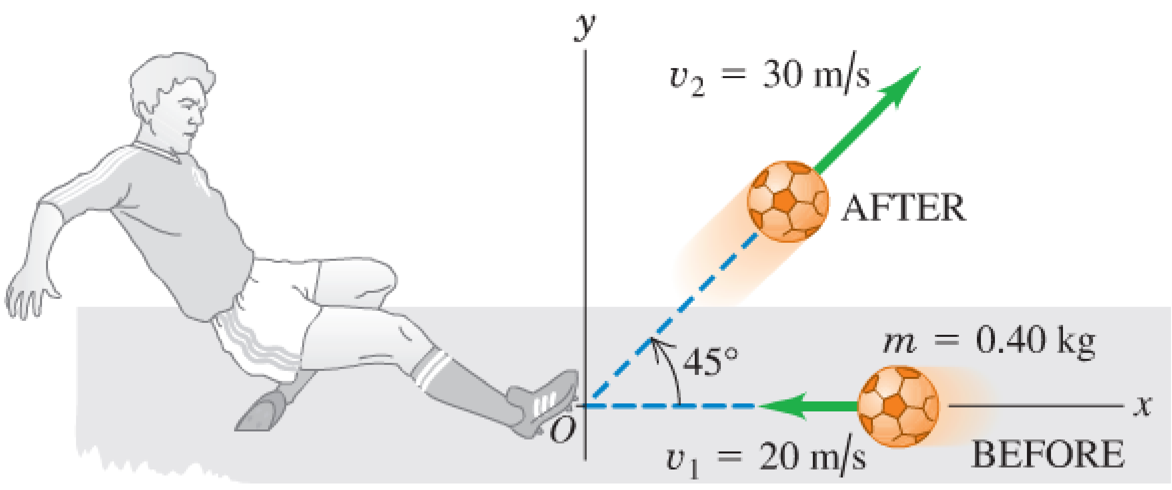
Example 1

Suppose you throw a ball with a mass of 0.40 kg against a brick wall. It hits the wall moving horizontally to the left at 30 m/s and rebounds horizontally to the right at 20 m/s. (a) Find the impulse of the net force on the ball during its collision with the wall. (b) If the ball is in contact with the wall for 0.010 s, find the average horizontal force that the wall exerts on the ball during the impact.



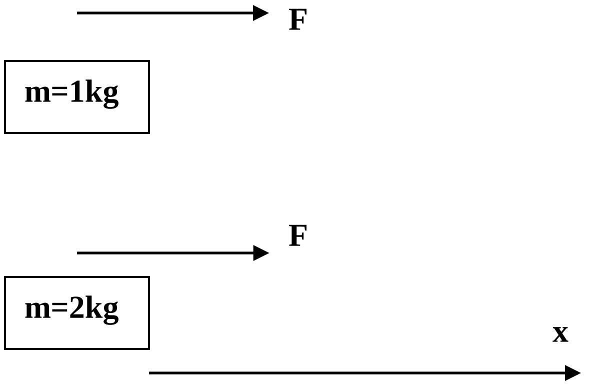
Example 2

A soccer ball has a mass of 0.40 kg. Initially, it is moving to the left at 20 m/s, but then it is kicked and given a velocity at 45° upward and to the right, with a magnitude of 30 m/s (Fig. 8.7a). Find the impulse of the net force and the average net force, assuming a collision time $\Delta t = 0.010$ s.



CPS Question 19-1

- **Two masses experience the same force, F, over the same distance, x. Describe the difference in the kinetic energy between the two masses at the end of the path.**



A) The heavier mass has more kinetic energy.

B) The lighter mass has more kinetic energy.

C) They have the same kinetic energy.

D) Not enough information to solve.

Proof

- Let's use our equations of motion…

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 2ax_f \Rightarrow$$

$$v_f^2 = 2\frac{F}{m}x_f$$

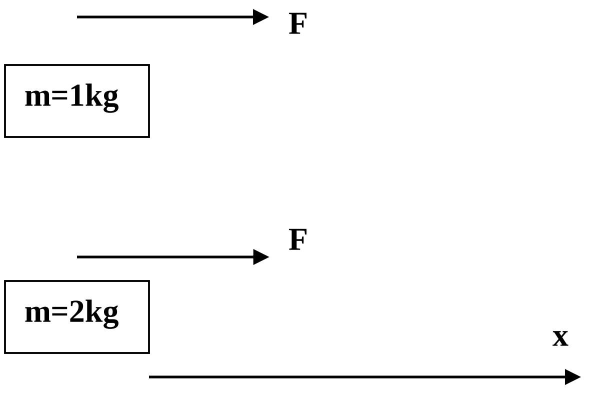
$$but,$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(2\frac{F}{m}x_f\right) = F \cdot x_f$$

- Independent of mass!

CPS Question 19-2

- **Two masses experience the same force, F, over the same distance, x. Describe the difference in the momentum between the two masses at the end of the path.**



A) The heavier mass has more momentum.

B) The lighter mass has more momentum.

C) They have the same momentum.

D) Not enough information to solve.

Proof

- Let's use our equations of motion…

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 2ax_f \Rightarrow$$

$$v_f^2 = 2\frac{F}{m}x_f \Rightarrow$$

$$v_f = \sqrt{2\frac{F}{m}x_f}$$

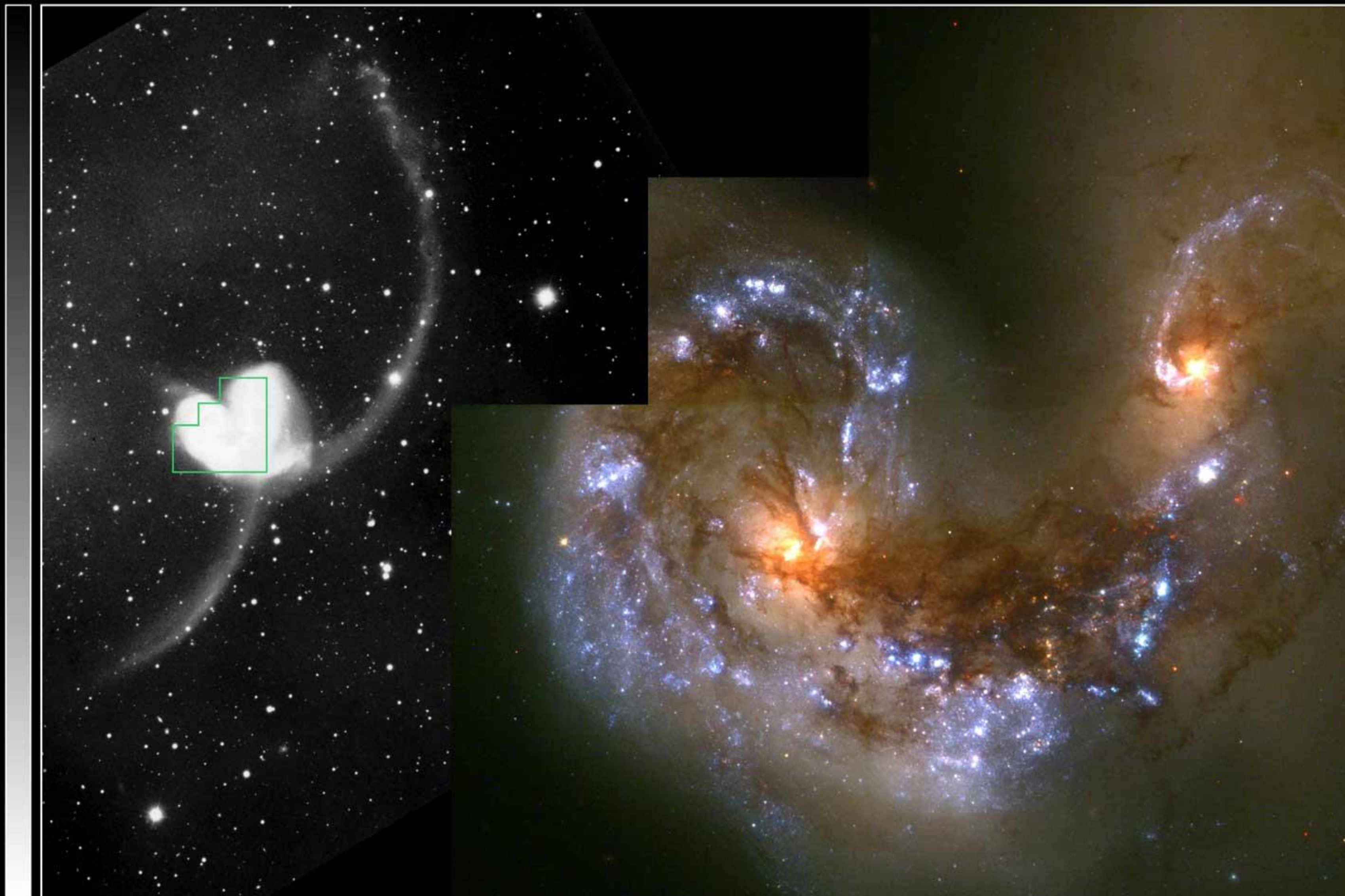
$$but,$$

$$p = mv = m\sqrt{2\frac{F}{m}x_f} = \sqrt{2mFx_f}$$

- NOT independent of mass!

Collisions

- Collisions are interactions between bodies.
- Generally, there is a large force acting over a short period of time:
  - Pool balls, or bat and ball.
  - Bullet strikes a wooden target.
  - Meteor strikes the earth.
  - Cosmic ray hits an atom in the atmosphere.
- Sometimes collisions take a longer period of time:
  - Space probe “sling-shots” around a planet or sun.
  - Galaxies collide.



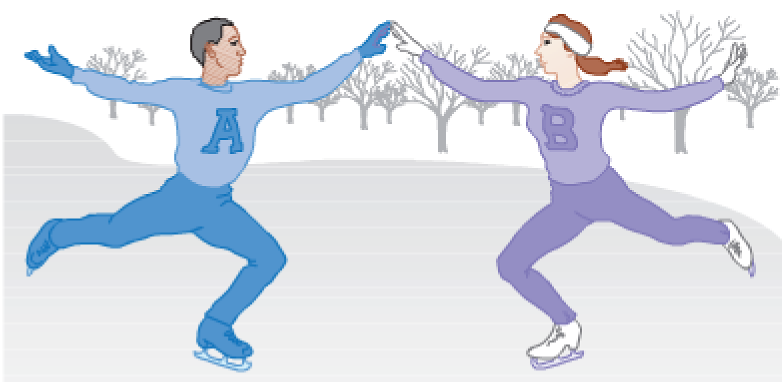
Colliding Galaxies NGC 4038 and NGC 4039

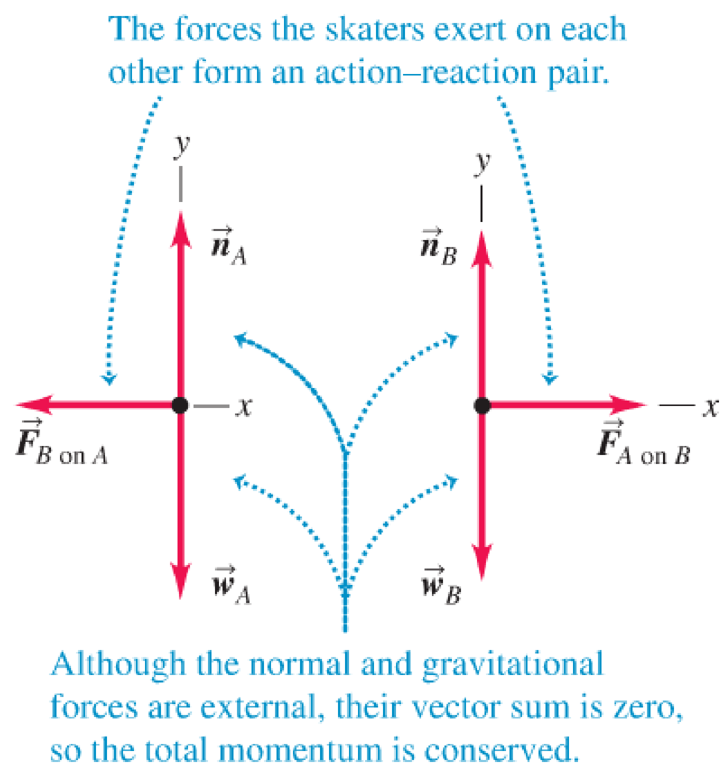
Hubble Space Telescope • Wide Field Planetary Camera 2

PRC97-34a • ST ScI OPO • October 21, 1997 • B. Whitmore (ST ScI) and NASA

Collisions

- In many circumstances, collisions of a "system" of bodies (can be more than two) has no NET forces acting on them from outside of the system.



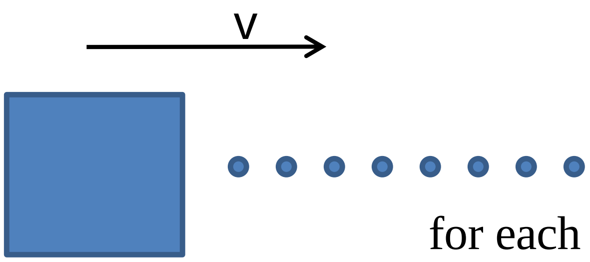


Collisions

- In that case, since $\vec{J} \equiv \int_{t_1}^{t_2} \vec{F}_{Net} dt = \vec{p}_2 - \vec{p}_1$
- Then $p_2 = p_1$, or better stated, the momentum *of the system* remains constant.
- It does NOT mean that the kinetic energy of the system is constant…

Drag Force

- Series of collisions:



for n particles hitting in time Δt,

$$\Delta\vec{p} = -nm\vec{v}$$

so,

$$\vec{J} = -nm\vec{v}$$

so,

$$\vec{F}_{Avg} = \frac{-nm\vec{v}}{\Delta t} = -\frac{n}{\Delta t} m\vec{v}$$

but,

$$\frac{nm}{\Delta t} = \rho A\left|\vec{v}\right|$$

so,

$$\vec{F}_{Avg} = -\rho A v^2$$

for each particle,

$$\vec{p}_i = 0$$

$$\vec{p}_f = m\vec{v}$$

$$\Delta\vec{p} = m\vec{v}$$

Collisions

- Two general categories of collisions:
- Elastic –
  - Both momentum and kinetic energy are conserved.
- Inelastic –
  - Only momentum is conserved.
- In general a collision is somewhere between these (not all kinetic energy is “lost” in inelastic collisions).

CPS 20-1

- Given one ball with initial velocity in the Newton's cradle, how many balls will have a non-zero final velocity on the other side?
- A) 1
- B) 2
- C) 3
- D) 4
- E) It depends

Problem 8.84

8.84. A 5.00-g bullet is shot *through* a 1.00-kg wood block suspended on a string 2.00 m long. The center of mass of the block rises a distance of 0.45 cm. Find the speed of the bullet as it emerges from the block if its initial speed is 450 m/s.

- For most students, the problem here is "Where is momentum conserved and where is energy conserved?"