

Lecture 3

(Addition and Subtraction of Vectors)

Physics 160-01 Spring 2017

Douglas Fields

Free (to you) Tutoring

- The Tutoring Table provides over 25 hours of TA availability every week to answer your questions - all with the convenience of a single location, Rm 111, Regener Hall, not far from here.
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- The Tutoring Table schedule is easily accessed on the department's web page under the Quick Links tab, and is also posted outside RH 111.

CPS Practice Question 1

- What is your class standing at UNM?
 - A. Freshman
 - B. Junior
 - C. Sophomore
 - D. Senior
 - E. Non-traditional

CPS Practice Question 2

- My mathematics background can be best described as:
 - A. I am currently enrolled in Calculus I
 - B. I have already taken and passed Calculus I
 - C. I had Calculus in High School
 - D. I have had no Calculus, and I am not currently enrolled in it
 - E. What is Calculus?

CPS Practice Question 3

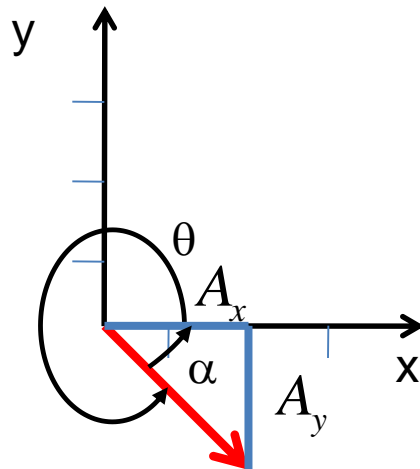
- What is $2 + 2 = ?$
 - A. 3
 - B. 5
 - C. 4
 - D. 2
 - E. No, really, what is Calculus?

Angles

- Angles can be specified in two ways:
 - Degrees (360° in one complete rotation)
 - Radians (2π radians in one complete rotation)
- It is best for you to get in the habit of using radians, since it will simplify calculations later in the semester.
- But even better is to use both so that you build up an intuitive feel for what radians mean.

Direction of Vectors

- The direction of a vector can be found using trigonometry:



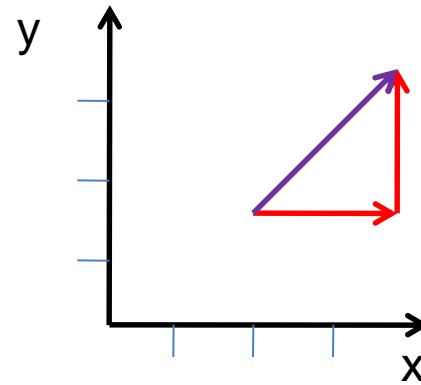
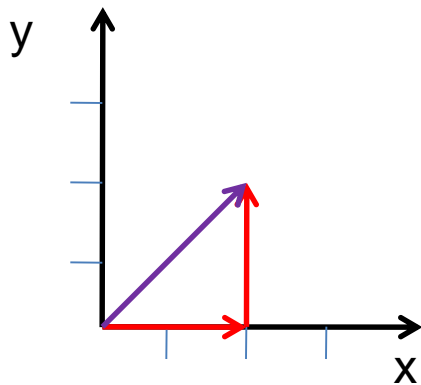
$$\tan \alpha = \frac{A_y}{A_x} \Rightarrow$$

$$\alpha = \tan^{-1} \frac{A_y}{A_x}$$

$$\theta = ?$$

Vector Addition

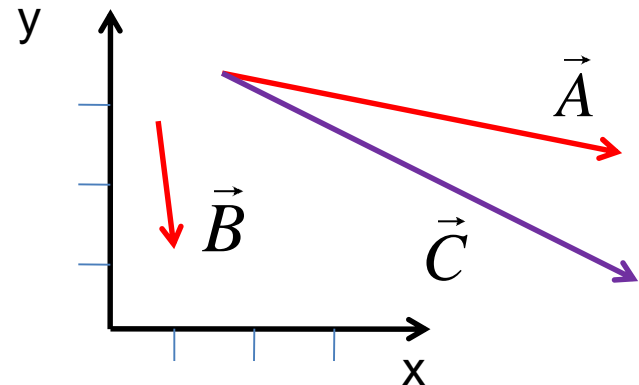
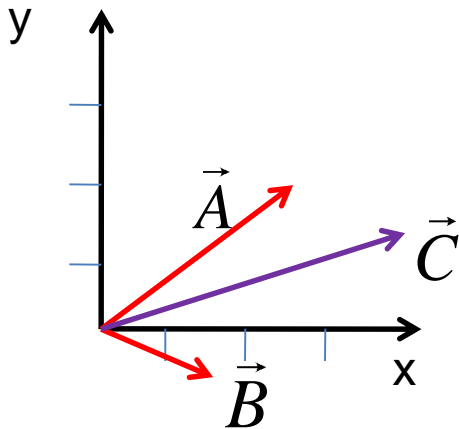
- Let's give a "real-life" situation to demonstrate how one adds vectors.
- Displacement (change in position) is a vector.
- Let's say I walk 2m in the +x-direction.
- Then I walk 2m in the +y-direction.
- What is my net displacement?
- It is the vector that points from where I started to where I end up.



Graphical Vector Addition

- To add vectors graphically, put the tail of one vector to the point of the other.
- The resultant vector starts from the tail of the first and goes to the point of the second:

$$\vec{A} + \vec{B} = \vec{C}$$

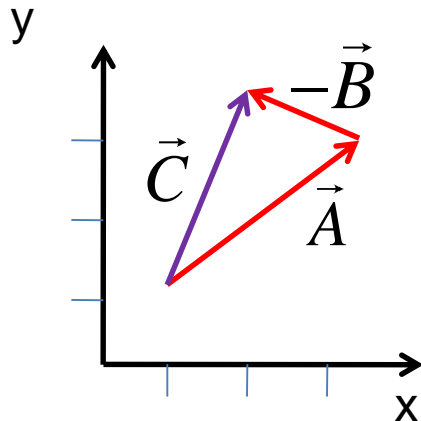


Subtraction

- The negative of a vector is a vector with the same length, but in the opposite direction:



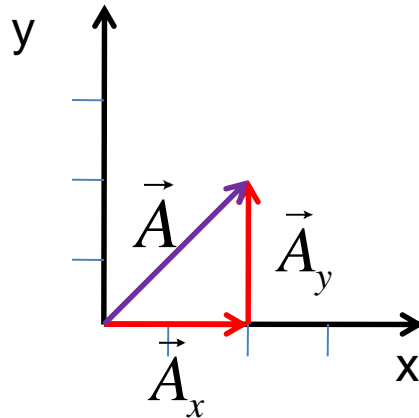
- To subtract vectors, just add the negative.



$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{C}$$

Vector Components

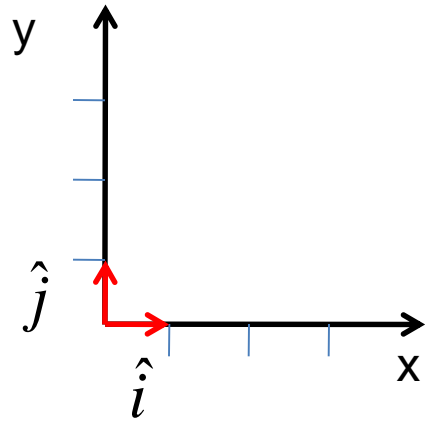
- In many cases, it is better to deal with the components of vectors:



$$\vec{A} = \vec{A}_x + \vec{A}_y$$

Unit Vectors

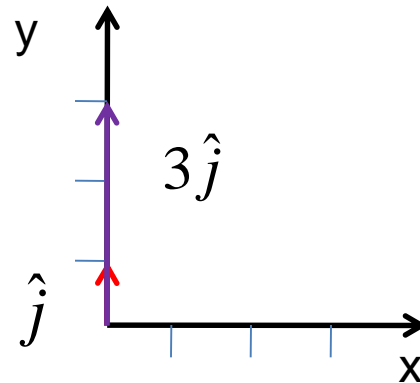
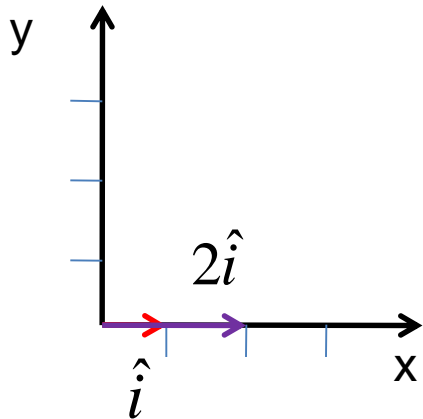
- Unit vectors are vectors in the directions of the axes of the coordinate system whose length is one unit.



- We will learn how to make a unit vector soon.

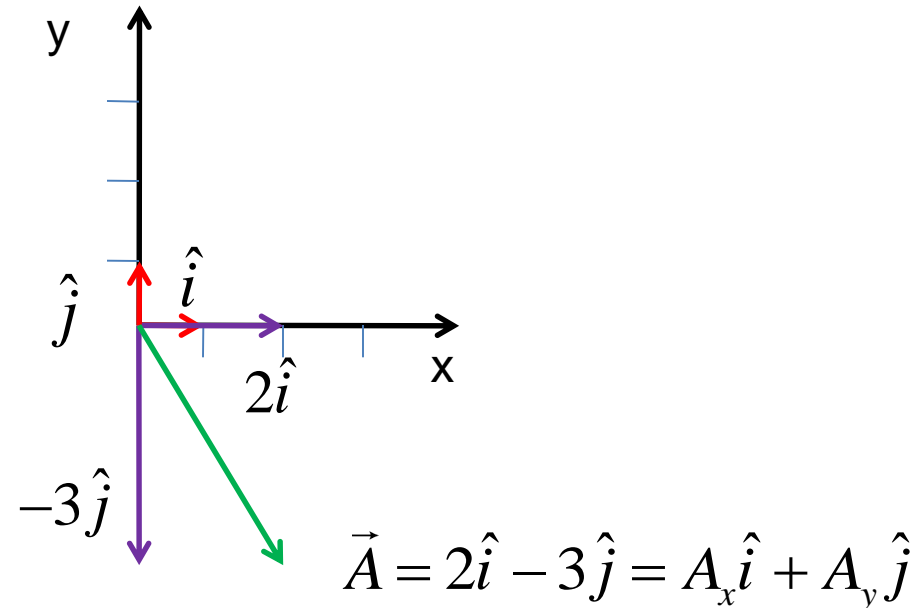
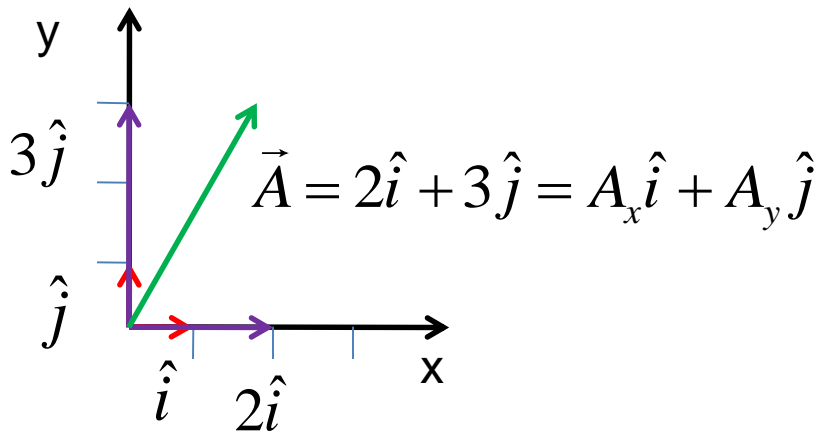
Multiplication of a Vector by a Number

- When multiplying a vector by a number, its length is increased by a factor of that number:



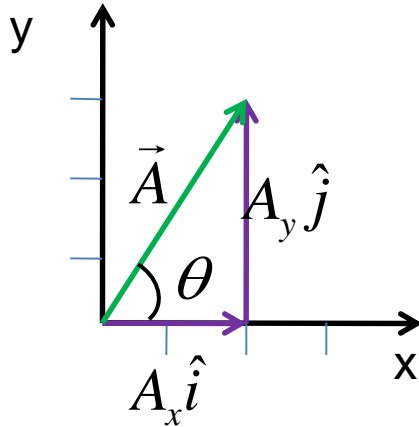
Component Notation

- A vector can then be represented in component notation as:



Finding Components

- Use trigonometry:



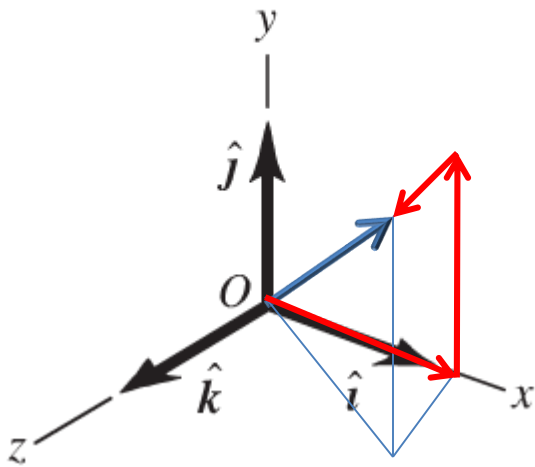
$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$A_x = |\vec{A}| \cos \theta$$

$$A_y = |\vec{A}| \sin \theta$$

In 3-dimensions

The unit vectors \hat{i} , \hat{j} , and \hat{k} .



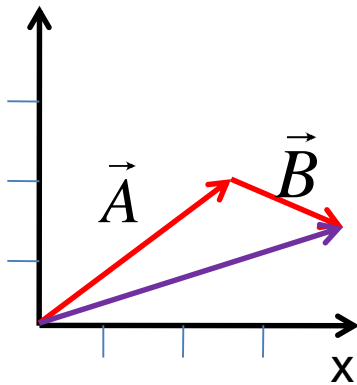
$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

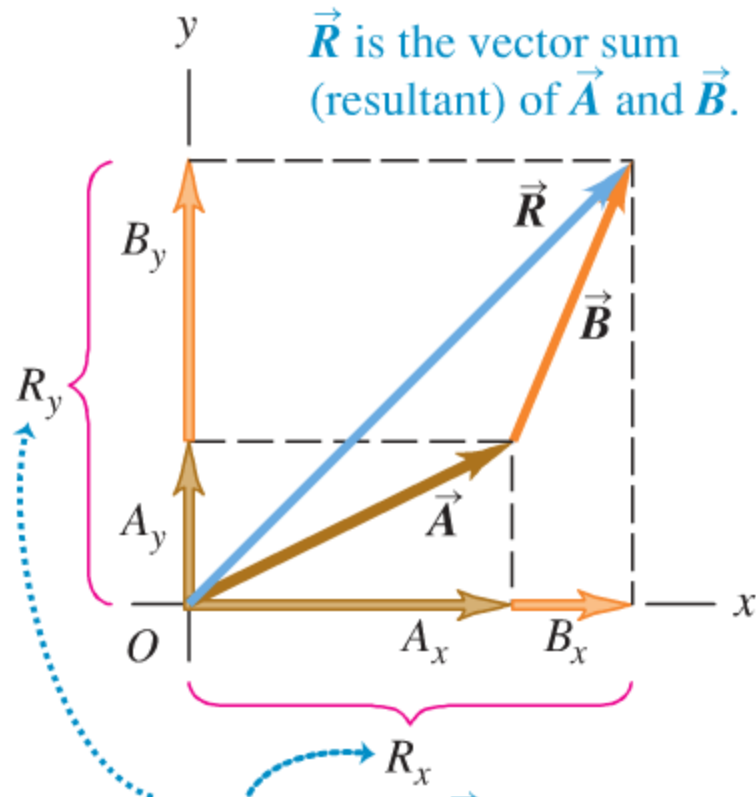
Adding Vectors Using Components

- Then, to add vectors using components, you just have to add the components:

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$



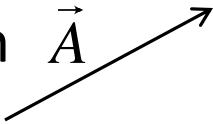
Adding Graphically

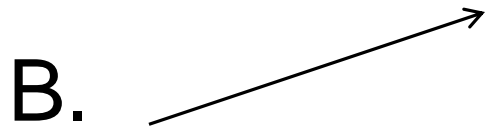
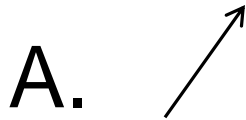


The components of \vec{R} are the sums
of the components of \vec{A} and \vec{B} :

$$R_y = A_y + B_y \quad R_x = A_x + B_x$$

CPS Question 2-1

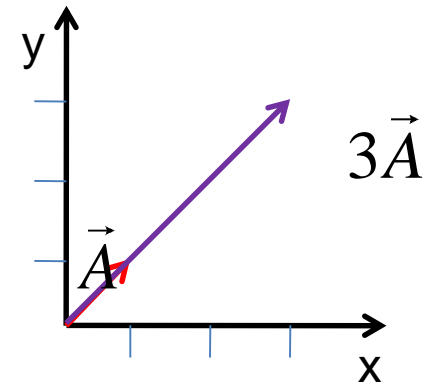
- Which vector most closely represents $\vec{A} - \vec{B}$ with \vec{A} and $\overleftarrow{\vec{B}}$? 



Multiplying scalars and vectors

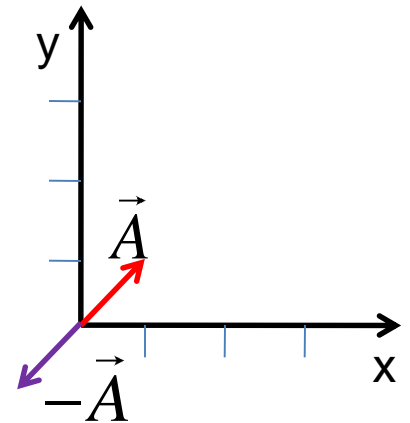
- How do we determine $3\vec{A}$?

$$3\vec{A} = 3 \cdot (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) = 3A_x \hat{i} + 3A_y \hat{j} + 3A_z \hat{k}$$



- How do we determine $-\vec{A}$?

$$-\vec{A} = -1 \cdot (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) = -A_x \hat{i} - A_y \hat{j} - A_z \hat{k}$$



Example

Example 1.7 Adding vectors using their components

Three players on a reality TV show are brought to the center of a large, flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements:

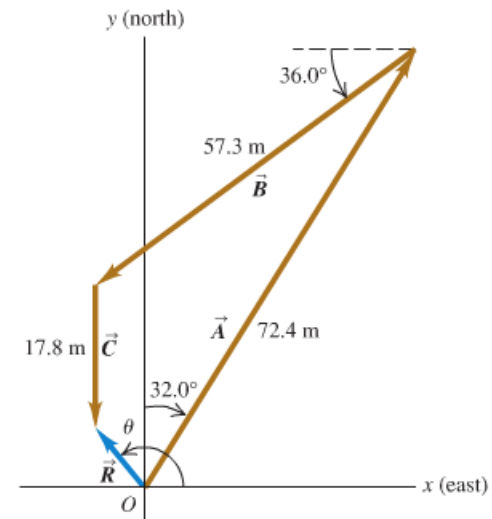
\vec{A} : 72.4 m, 32.0° east of north

\vec{B} : 57.3 m, 36.0° south of west

\vec{C} : 17.8 m due south

The three displacements lead to the point in the field where the keys to a new Porsche are buried. Two players start measuring immediately, but the winner first *calculates* where to go. What does she calculate?

1.22 Three successive displacements \vec{A} , \vec{B} , and \vec{C} and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



Example

EXECUTE: The angles of the vectors, measured from the $+x$ -axis toward the $+y$ -axis, are $(90.0^\circ - 32.0^\circ) = 58.0^\circ$, $(180.0^\circ + 36.0^\circ) = 216.0^\circ$, and 270.0° , respectively. We may now use Eqs. (1.6) to find the components of \vec{A} :

$$A_x = A \cos \theta_A = (72.4 \text{ m})(\cos 58.0^\circ) = 38.37 \text{ m}$$

$$A_y = A \sin \theta_A = (72.4 \text{ m})(\sin 58.0^\circ) = 61.40 \text{ m}$$

We've kept an extra significant figure in the components; we'll round to the correct number of significant figures at the end of our calculation. The table below shows the components of all the displacements, the addition of the components, and the other calculations.

Distance	Angle	x-component	y-component
$A = 72.4 \text{ m}$	58.0°	38.37 m	61.40 m
$B = 57.3 \text{ m}$	216.0°	-46.36 m	-33.68 m
$C = 17.8 \text{ m}$	270.0°	0.00 m	-17.80 m
		$R_x = -7.99 \text{ m}$	$R_y = 9.92 \text{ m}$

$$R = \sqrt{(-7.99 \text{ m})^2 + (9.92 \text{ m})^2} = 12.7 \text{ m}$$

$$\theta = \arctan \frac{9.92 \text{ m}}{-7.99 \text{ m}} = -51^\circ$$

Comparing to Fig. 1.22 shows that the calculated angle is clearly off by 180° . The correct value is $\theta = 180^\circ - 51^\circ = 129^\circ$, or 39° west of north.

EVALUATE: Our calculated answers for R and θ agree with our estimates. Notice how drawing the diagram in Fig. 1.22 made it easy to avoid a 180° error in the direction of the vector sum.

1.22 Three successive displacements \vec{A} , \vec{B} , and \vec{C} and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

