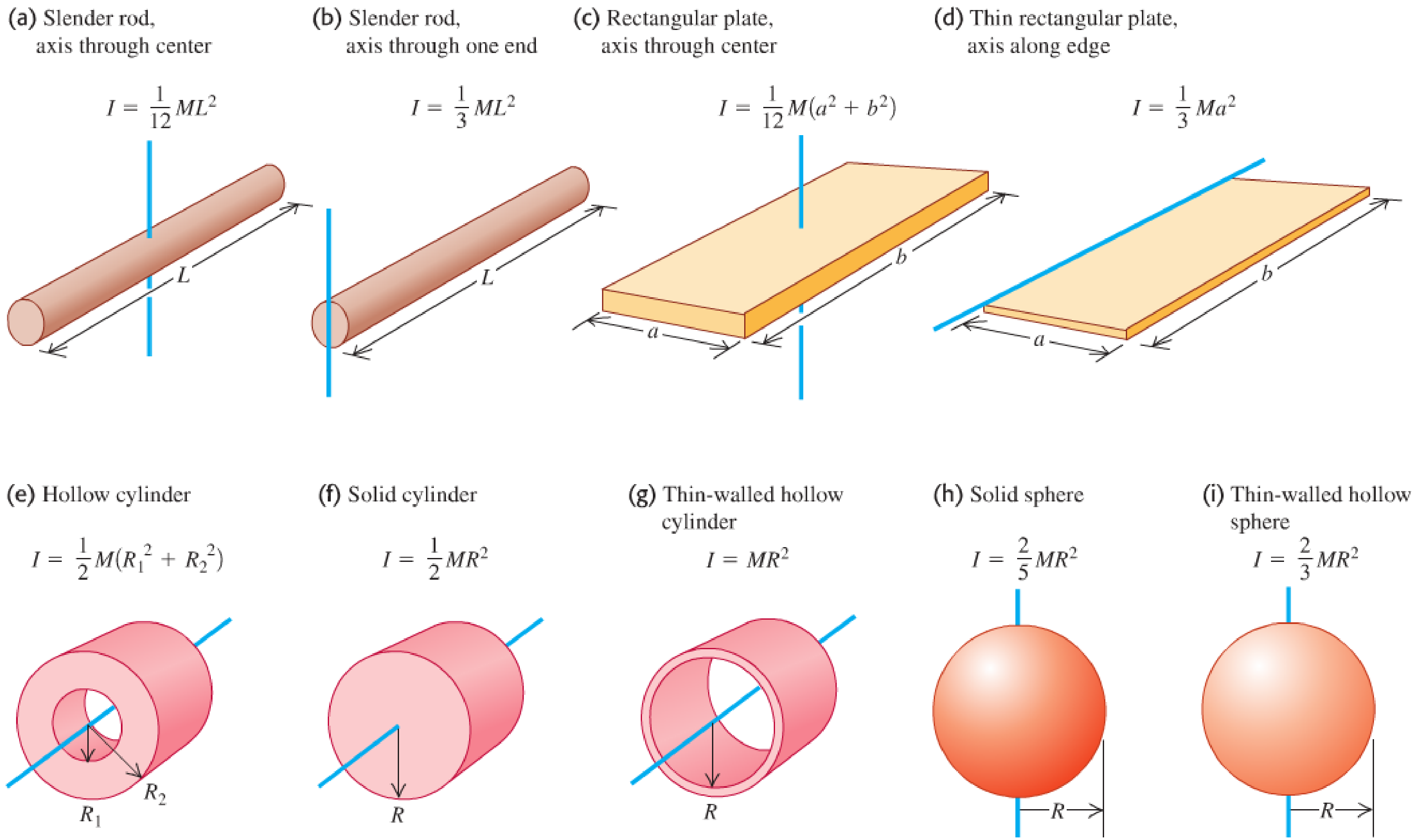
Lecture 20
(Torque)

Physics 160-02 Spring 2017

Douglas Fields

Moments of Inertia

Table 9.2 Moments of Inertia of Various Bodies



Review

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + a t$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

- Rotational
  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

CPS Question 26-1

- **Two objects of equal mass and diameter roll without slipping down an incline, one is a hollow cylinder, the other a solid cylinder. Which one wins the race to the bottom of the incline?**

A) The solid cylinder.

B) The hollow cylinder.

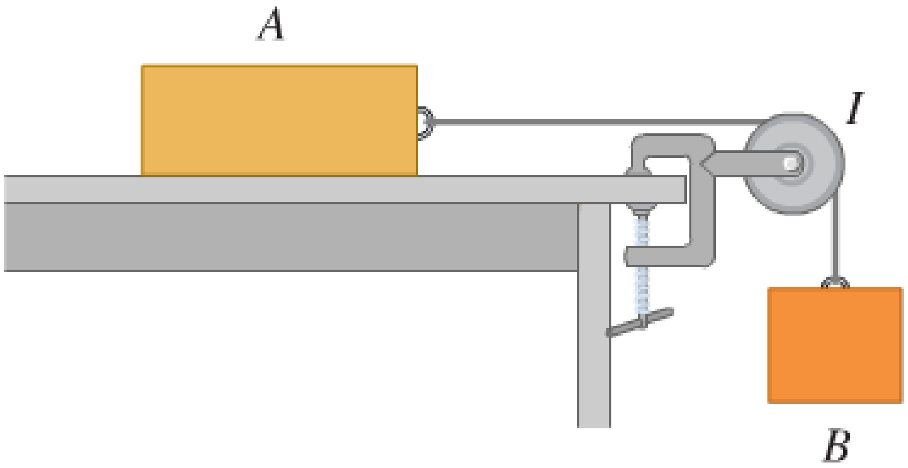
C) It's a tie.

D) Not enough information to solve.

Problem 9.85

9.85. The pulley in Fig. 9.35 has radius R and a moment of inertia I. The rope does not slip over the pulley, and the pulley spins on a frictionless axle. The coefficient of kinetic friction between block A and the tabletop is μ_k. The system is released from rest, and block B descends. Block A has mass m_A and block B has mass m_B. Use energy methods to calculate the speed of block B as a function of the distance d that it has descended.

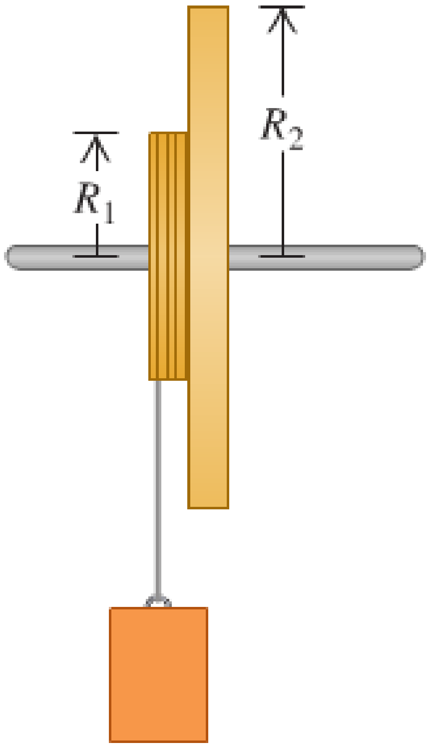
Figure 9.35 Problem 9.85.



Problem 9.89

9.89. Two metal disks, one with radius $R_1 = 2.50$ cm and mass $M_1 = 0.80$ kg and the other with radius $R_2 = 5.00$ cm and mass $M_2 = 1.60$ kg, are welded together and mounted on a frictionless axis through their common center (Fig. 9.37). (a) What is the total moment of inertia of the two disks? (b) A light string is wrapped around the edge of the smaller disk, and a 1.50-kg block is suspended from the free end of the string. If the block is released from rest at a distance of 2.00 m above the floor, what is its speed just before it strikes the floor? (c) Repeat the calculation of part (b), this time with the string wrapped around the edge of the larger disk. In which case is the final speed of the block the greatest? Explain why this is so.

Figure 9.37 Problem 9.89.



Review

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

  - Comes from:

$$W = \int \vec{F} \cdot d\vec{s}$$

- Rotational
  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

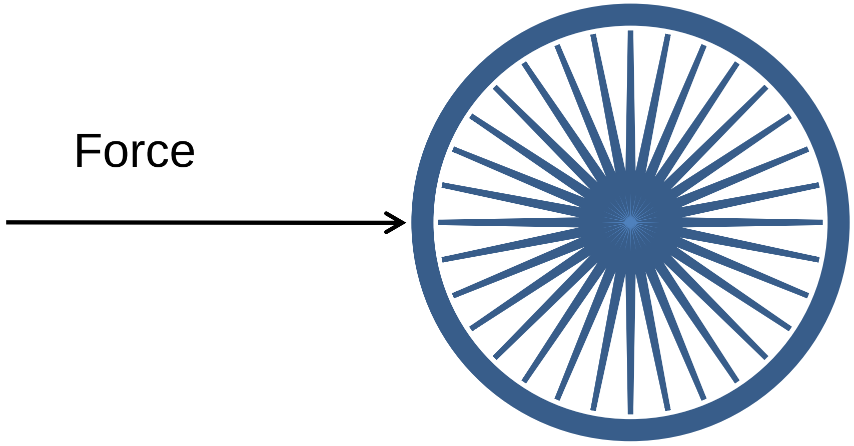
$$KE = \frac{1}{2} I \omega^2$$

  - Comes from:

???

How Do You Get Rotational KE?

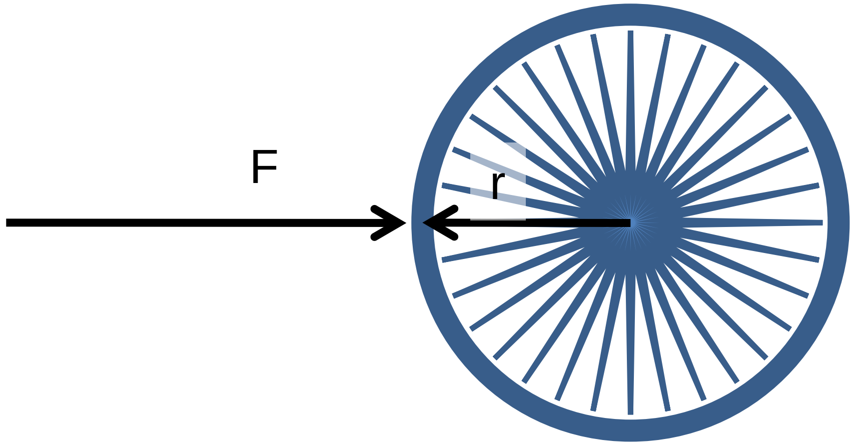
- You know it can't just be a force:



- Getting the wheel to rotate depends upon *where* you apply the force!!

Torque

- The concept of torque takes into account both the magnitude of a force applied, and where it is applied.



$$\vec{\tau} = \vec{r} \times \vec{F}$$

Direction from RHR

- Torque is maximum when the perpendicular distance to the rotation axis is maximum.

CPS Question 25-2

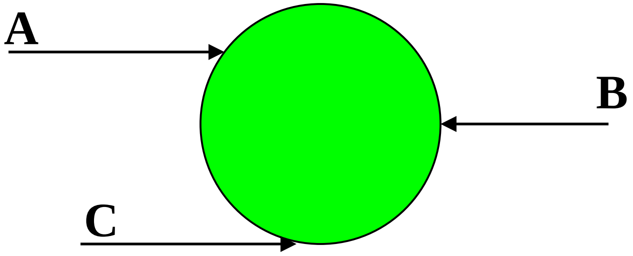
- **Three forces of equal magnitude act on a sphere. Which force supplies the largest torque on the object?**

A) A

B) B

C) C

D) Not enough information to solve.



Then…

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

  - Comes from:

$$W = \int \vec{F} \cdot d\vec{s}$$

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  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

  - Comes from:

$$W = \int \tau_z d\theta$$

What Causes Acceleration?

- Linear
  - For const a:

$$x_f = x_i + v_i t + \frac{1}{2} a t^2$$

$$v_f = v_i + at$$

  - Kinetic energy:

$$KE = \frac{1}{2} M v^2$$

  - Comes from:

$$W = \int \vec{F} \cdot d\vec{s}$$

  - Newton's 2nd Law

$$\sum \vec{F} = m\vec{a}$$

- Rotational
  - For const α:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f = \omega_i + \alpha t$$

  - Kinetic energy:

$$KE = \frac{1}{2} I \omega^2$$

  - Comes from:

$$W = \int \tau_z d\theta$$

  - Newton's 2nd Law

$$\sum \vec{\tau} = I\vec{\alpha}$$

No New Physics!

- Start with F=ma…

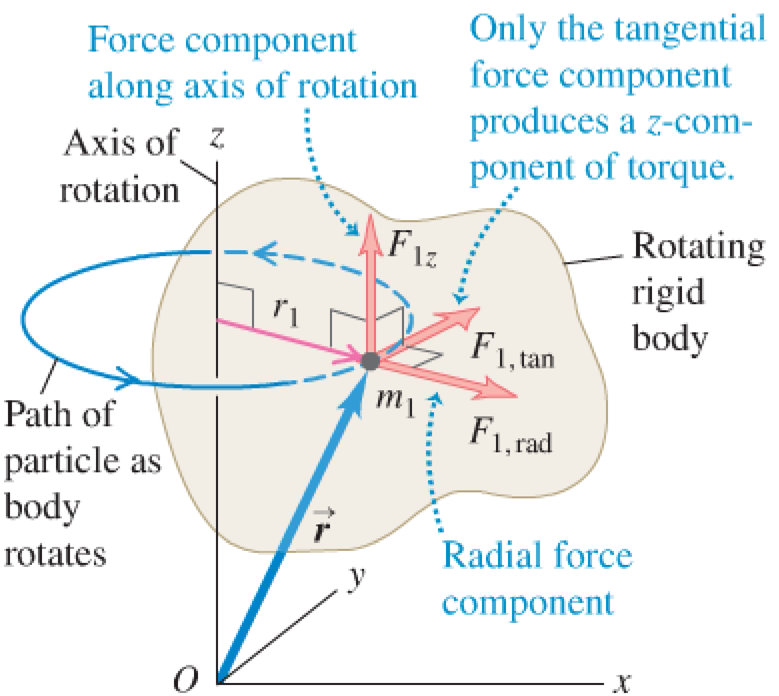
$$F_{1\tan} = m_1 a_{1\tan}$$

- But,

$$a_{1\tan} = r_1 \alpha_z$$

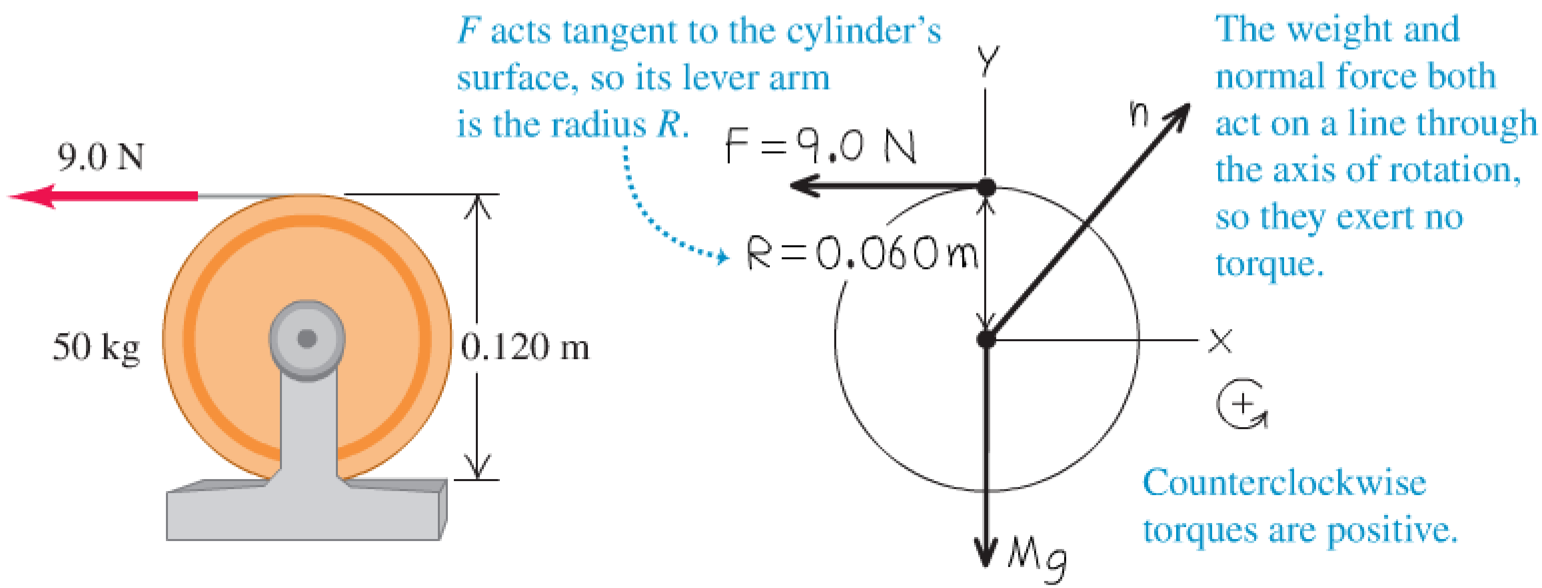
- And multiply both sides by r_1:

$$F_{1\tan} r_1 = m_1 r_1^2 \alpha_z$$



Example

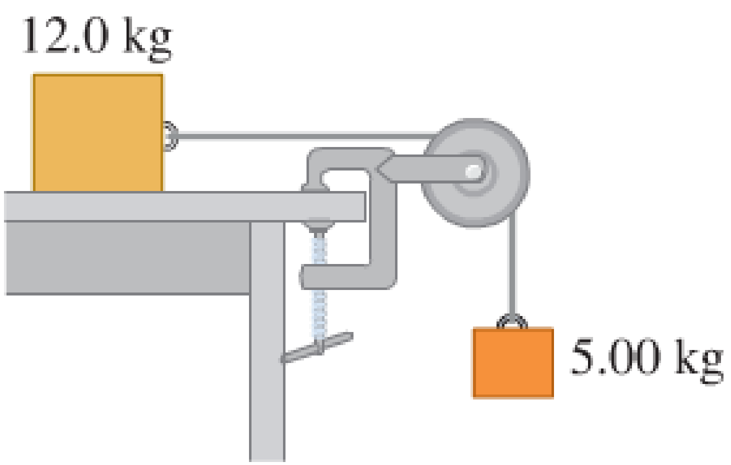
Figure 10.9a shows the same situation that we analyzed in Example 9.8 (Section 9.4) using energy methods. A cable is wrapped several times around a uniform solid cylinder that can rotate about its axis. The cylinder has diameter 0.120 m and mass 50 kg. The cable is pulled with a force of 9.0 N. Assuming that the cable unwinds without stretching or slipping, what is its acceleration?



Problem 10.16

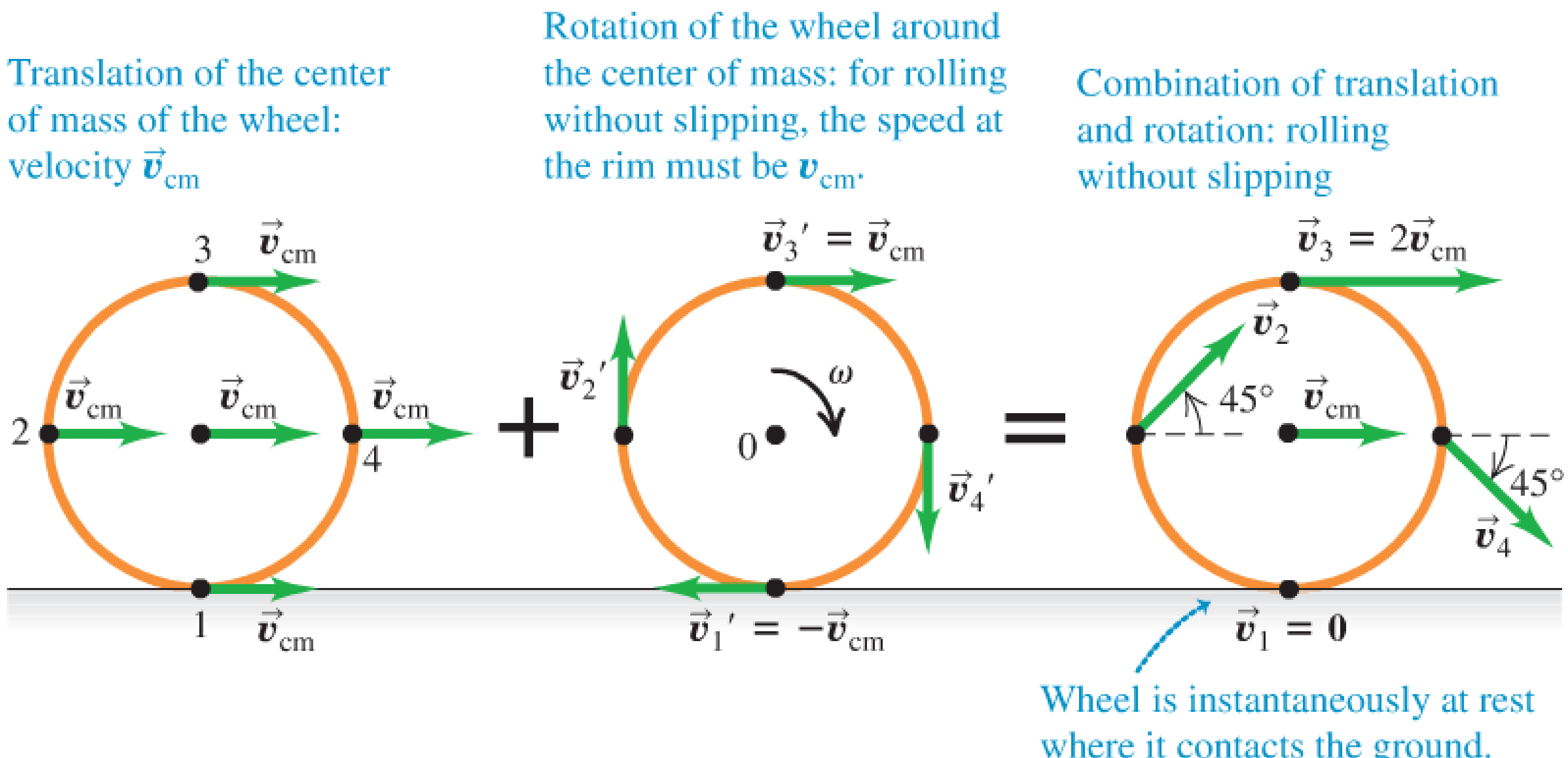
10.16. A 12.0-kg box resting on a horizontal, frictionless surface is attached to a 5.00-kg weight by a thin, light wire that passes over a frictionless pulley (Fig. 10.44). The pulley has the shape of a uniform solid disk of mass 2.00 kg and diameter 0.500 m. After the system is released, find (a) the tension in the wire on both sides of the pulley, (b) the acceleration of the box, and (c) the horizontal and vertical components of the force that the axle exerts on the pulley.

Figure 10.44 Exercise 10.16.



Rolling Without Slipping

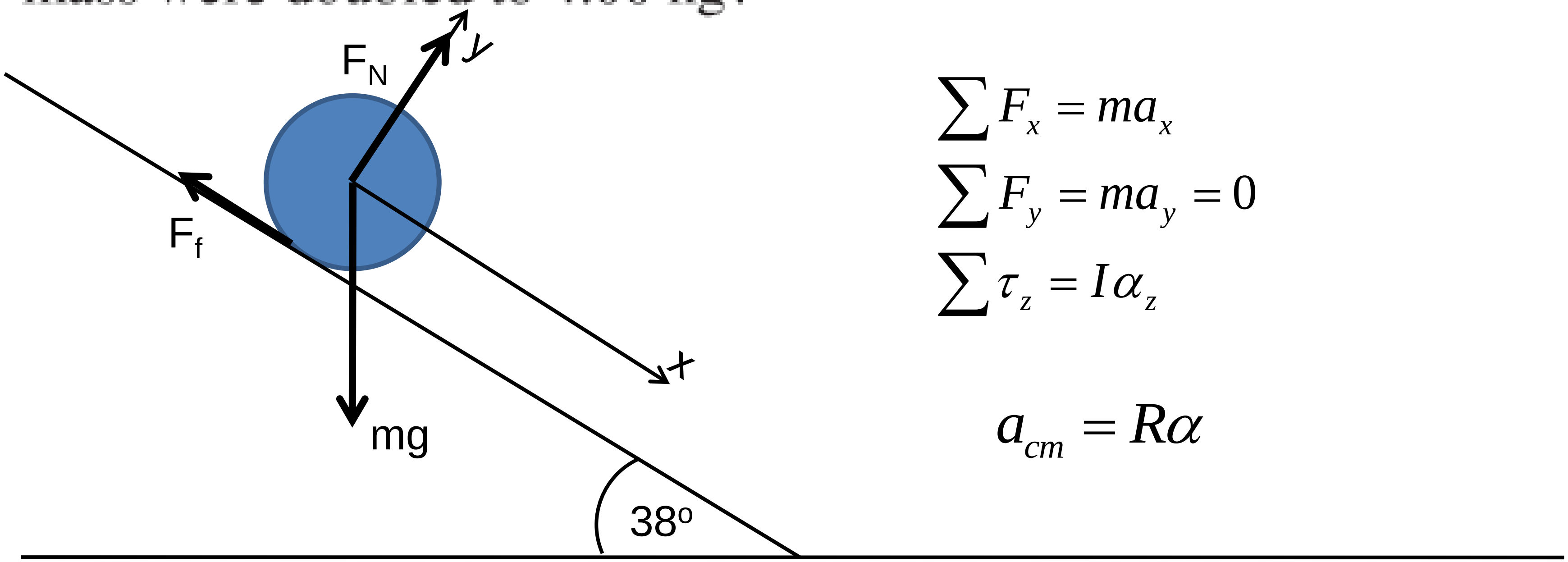
- An important case of a combination of translational and rotational motion:



- So, must have: $v_{cm} = R\omega$
- If v_{cm} changes with time, then also must have: $a_{cm} = R\alpha$

Problem 10.22

10.22. A hollow, spherical shell with mass 2.00 kg rolls without slipping down a 38.0° slope. (a) Find the acceleration, the friction force, and the minimum coefficient of friction needed to prevent slipping. (b) How would your answers to part (a) change if the mass were doubled to 4.00 kg?



$$\sum F_x = ma_x$$

$$\sum F_y = ma_y = 0$$

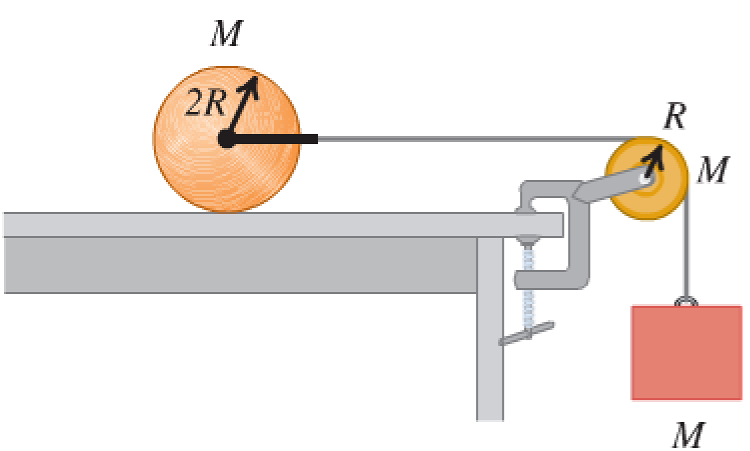
$$\sum \tau_z = I\alpha_z$$

$$a_{cm} = R\alpha$$

Problem 10.83

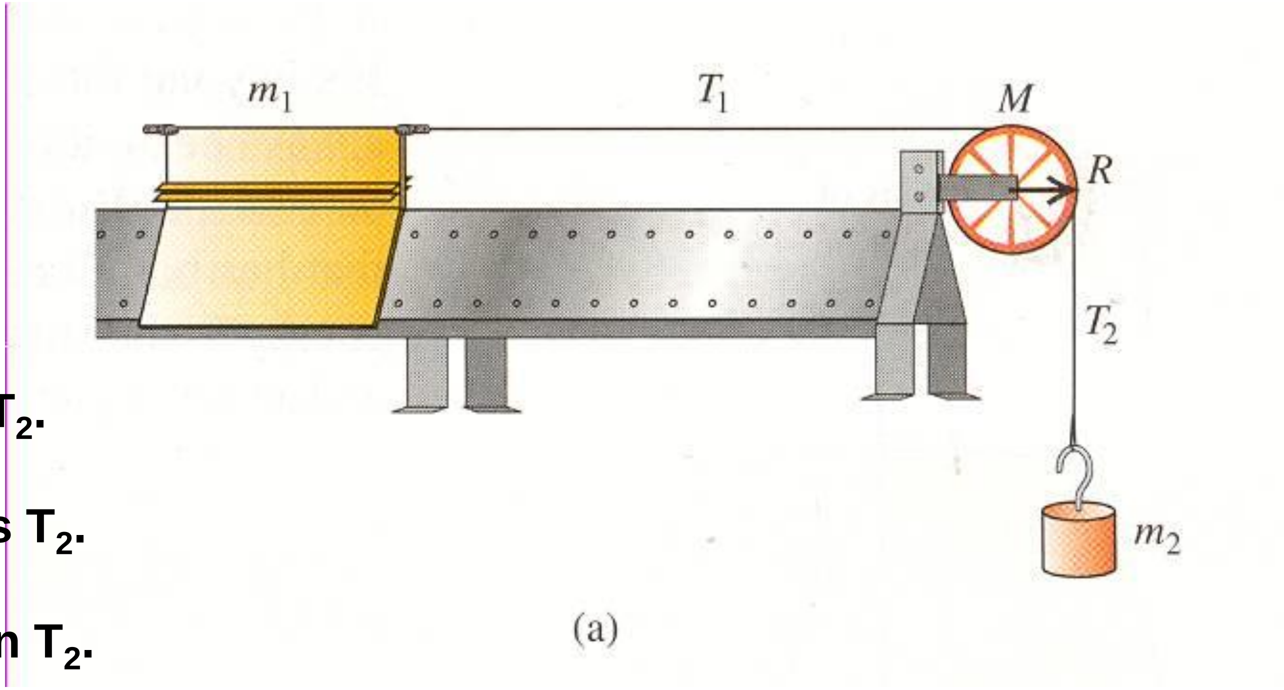
10.83. A uniform, solid cylinder with mass M and radius $2R$ rests on a horizontal tabletop. A string is attached by a yoke to a frictionless axle through the center of the cylinder so that the cylinder can rotate about the axle. The string runs over a disk-shaped pulley with mass M and radius R that is mounted on a frictionless axle through its center. A block of mass M is suspended from the free end of the string (Fig. 10.62). The string doesn't slip over the pulley surface, and the cylinder rolls without slipping on the tabletop. Find the magnitude of the acceleration of the block after the system is released from rest.

Figure 10.62 Problem 10.83.



CPS Question 26-1

- **What is the magnitude of the tension T_1 relative to the tension T_2 in the drawing below?**



A) T_1 **is larger than** T_2**.**

B) T_1 **is the same as** T_2**.**

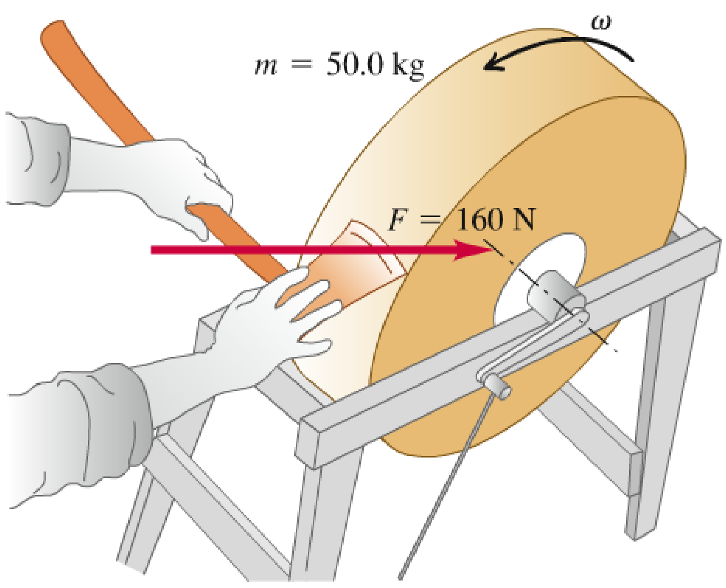
C) T_1 **is smaller than** T_2**.**

D) Not enough information to solve.

Problem 10.57

10.57 •• A 50.0-kg grindstone is a solid disk 0.520 m in diameter. You press an ax down on the rim with a normal force of 160 N (Fig. P10.57). The coefficient of kinetic friction between the blade and the stone is 0.60, and there is a constant friction torque of 6.50 N · m between the axle of the stone and its bearings. (a) How much force must be applied tangentially at the end of a crank handle 0.500 m long to bring the stone from rest to 120 rev/min in 9.00 s? (b) After the grindstone attains an angular speed of 120 rev/min, what tangential force at the end of the handle is needed to maintain a constant angular speed of 120 rev/min? (c) How much time does it take the grindstone to come from 120 rev/min to rest if it is acted on by the axle friction alone?

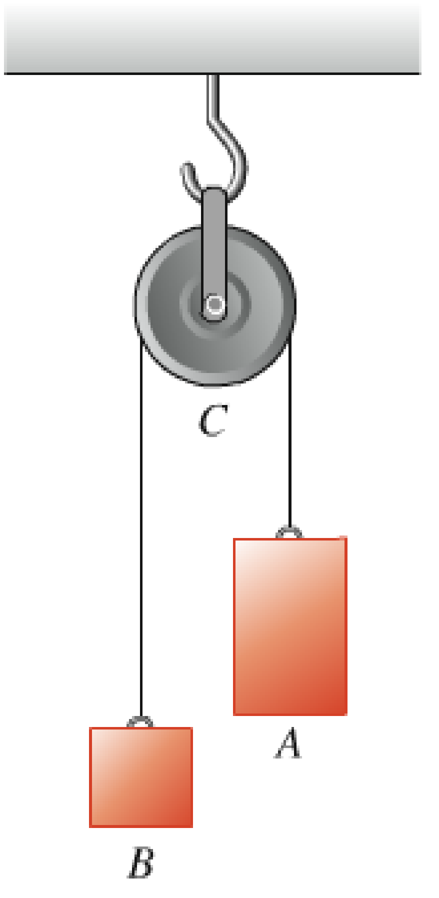
Figure **P10.57**



Problem 10.67

10.67 •• Atwood's Machine. Figure P10.67 illustrates an Atwood's machine. Find the linear accelerations of blocks *A* and *B*, the angular acceleration of the wheel *C*, and the tension in each side of the cord if there is no slipping between the cord and the surface of the wheel. Let the masses of blocks *A* and *B* be 4.00 kg and 2.00 kg, respectively, the moment of inertia of the wheel about its axis be $0.300\ \text{kg} \cdot \text{m}^2$, and the radius of the wheel be 0.120 m.

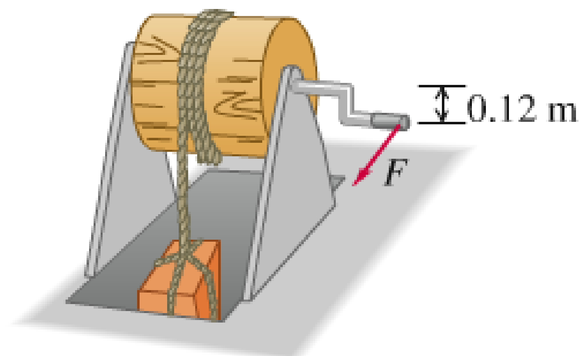
Figure **P10.67**



Problem 10.68

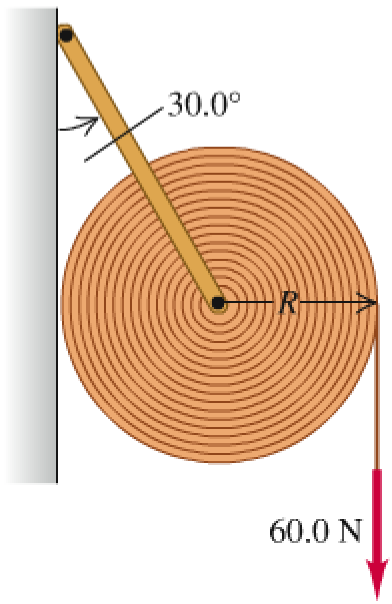
10.68 ••• The mechanism shown in Fig. P10.68 is used to raise a crate of supplies from a ship's hold. The crate has total mass 50 kg. A rope is wrapped around a wooden cylinder that turns on a metal axle. The cylinder has radius 0.25 m and moment of inertia $I = 2.9\ \text{kg} \cdot \text{m}^2$ about the axle. The crate is suspended from the free end of the rope. One end of the axle pivots on frictionless bearings; a crank handle is attached to the other end. When the crank is turned, the end of the handle rotates about the axle in a vertical circle of radius 0.12 m, the cylinder turns, and the crate is raised. What magnitude of the force $\vec{F}$ applied tangentially to the rotating crank is required to raise the crate with an acceleration of $1.40\ \text{m/s}^2$? (You can ignore the mass of the rope as well as the moments of inertia of the axle and the crank.)

Figure **P10.68**



Problem 10.69

Figure **P10.69**



10.69 •• A large 16.0-kg roll of paper with radius R = 18.0 cm rests against the wall and is held in place by a bracket attached to a rod through the center of the roll (Fig. P10.69). The rod turns without friction in the bracket, and the moment of inertia of the paper and rod about the axis is $0.260\ \text{kg} \cdot \text{m}^2$. The other end of the bracket is attached by a frictionless hinge to the wall such that the bracket makes an angle of 30.0° with the wall. The weight of the bracket is negligible. The coefficient of kinetic friction between the paper and the wall is $\mu_k = 0.25$. A constant vertical force F = 60.0 N is applied to the paper, and the paper unrolls. (a) What is the magnitude of the force that the rod exerts on the paper as it unrolls? (b) What is the magnitude of the angular acceleration of the roll?

Problem 10.70

10.70 •• A block with mass $m = 5.00$ kg slides down a surface inclined 36.9° to the horizontal (Fig. P10.70). The coefficient of kinetic friction is 0.25. A string attached to the block is wrapped around a flywheel on a fixed axis at O. The flywheel has mass 25.0 kg and moment of inertia 0.500 kg · m^2 with respect to the axis of rotation. The string pulls without slipping at a perpendicular distance of 0.200 m from that axis. (a) What is the acceleration of the block down the plane? (b) What is the tension in the string?

Figure **P10.70**