Lecture 24
(Periodic Motion)

Physics 160-02 Spring 2017

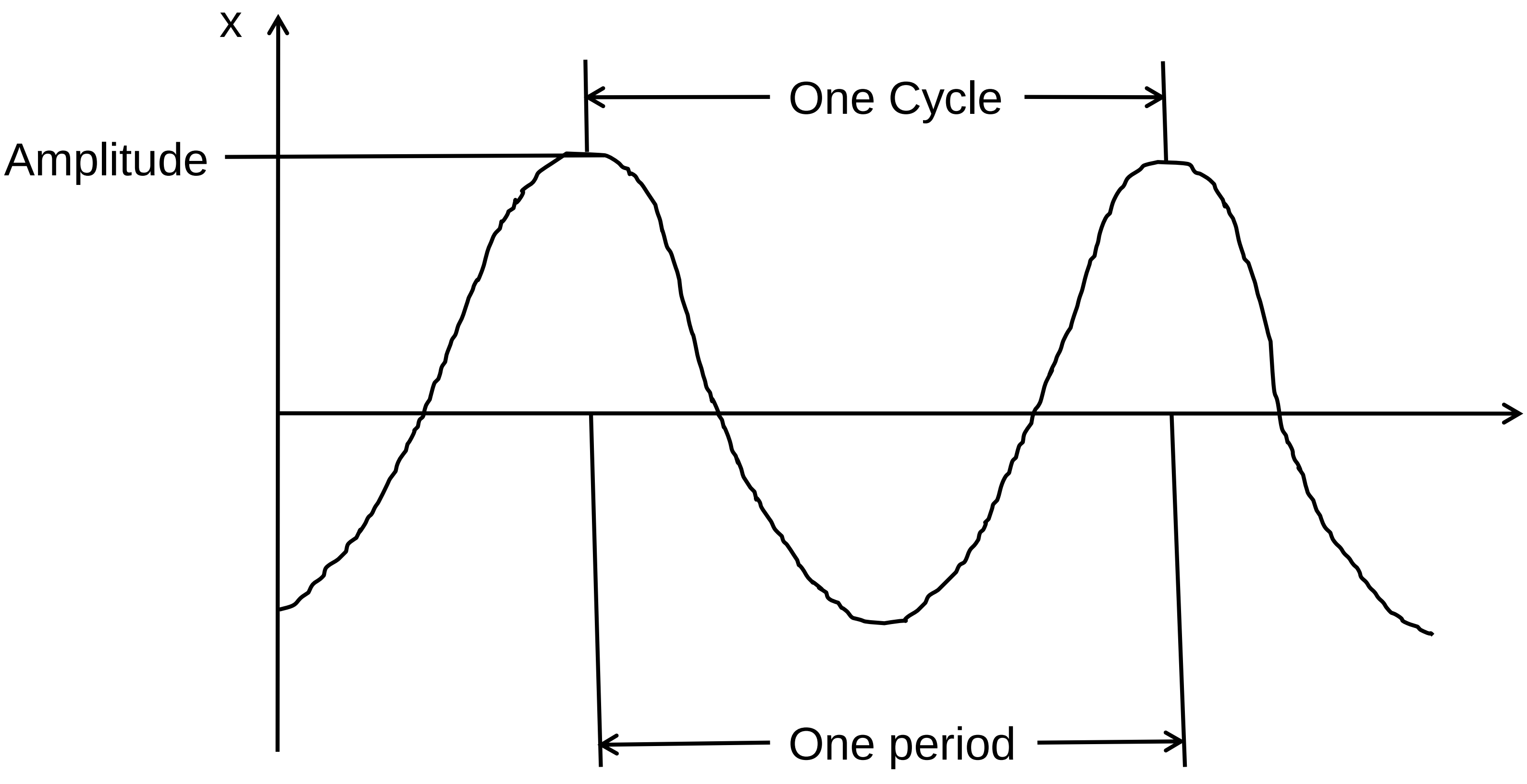
Douglas Fields

Periodic Motion

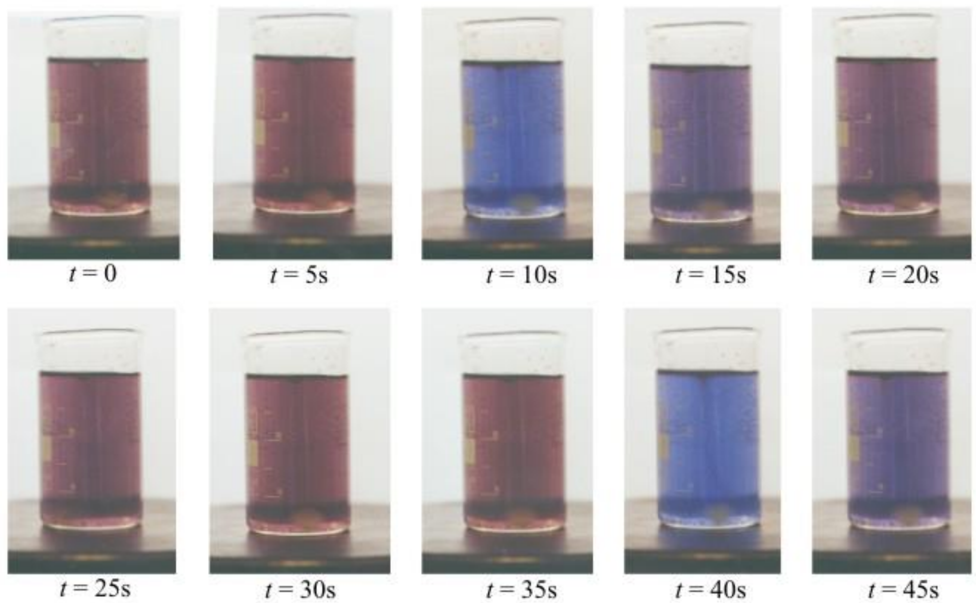
- Motion like:
  - Swinging pendulum
  - Sound vibrations
  - Vibrations of atoms
  - My pacing
- Any motion that is repeating (comes back to an original point and follows the same path again)
- Can be characterized by:
  - Amplitude of motion
  - Periodicity of motion
    - Or, the frequency
    - Or, the angular frequency

One-dimensional Period Motion

- My Pacing



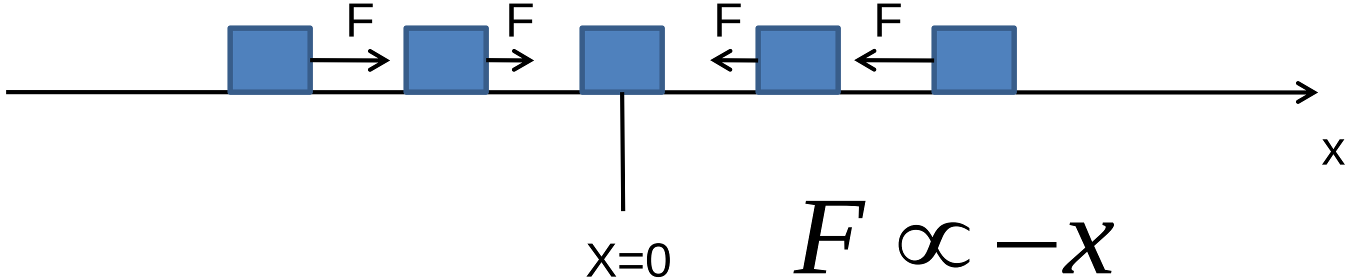
Periodic Behavior



Belousov-Zhabotinsky reaction

Simple Harmonic Motion

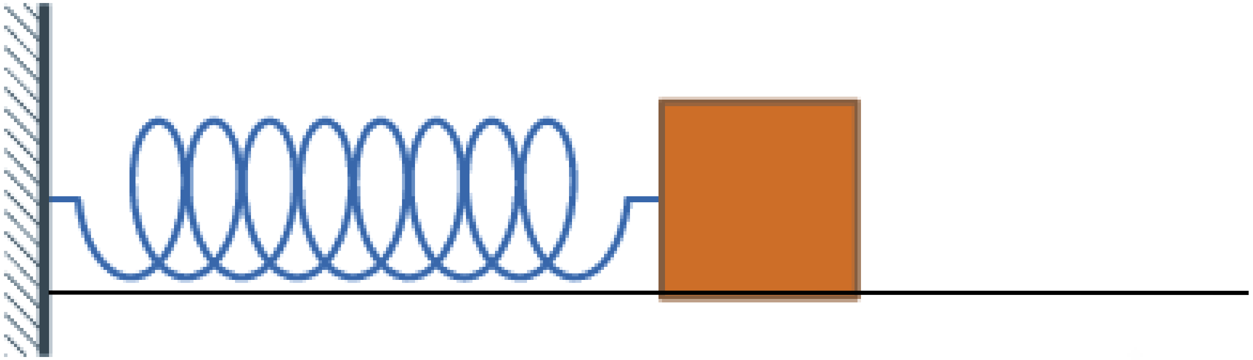
- A type of periodic motion with a very explicit definition:
- Motion about an equilibrium point with a restoring force proportional to the distance away from the equilibrium point.



$$F \propto -x$$

Simple Harmonic Motion

$$F = -kx$$



Simple Harmonic Motion

- Analyze: $F = -kx = ma = m\frac{d^2x}{dt^2} \Rightarrow$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

- Differential equation relating the changing acceleration to the position. Try non-periodic solutions:

$$x(t) = C \Rightarrow \frac{d^2x}{dt^2} = 0 \neq -\frac{k}{m}x(t) \ \text{ unless } C = 0$$

$$x(t) = e^{\sqrt{\frac{k}{m}}t} \Rightarrow \frac{dx}{dt} = \sqrt{\frac{k}{m}}e^{\sqrt{\frac{k}{m}}t} \Rightarrow \frac{d^2x}{dt^2} = \frac{k}{m}e^{\sqrt{\frac{k}{m}}t} \neq -\frac{k}{m}e^{\sqrt{\frac{k}{m}}t}$$

Simple Harmonic Motion

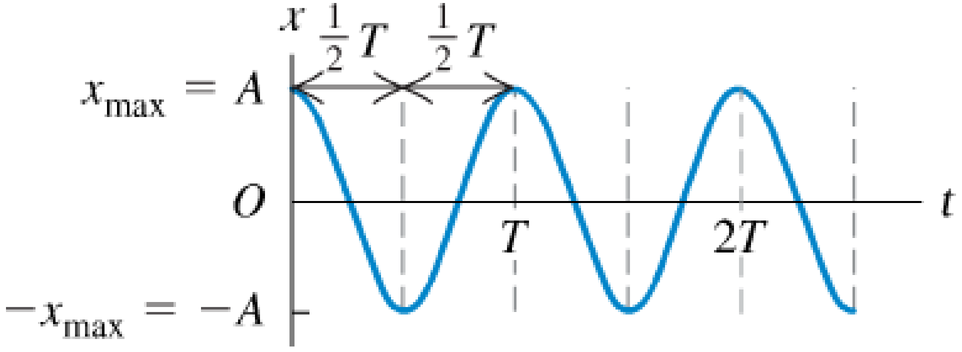
- Try a periodic solution:

$$x(t) = \cos(ct) \Rightarrow \frac{dx}{dt} = -c\sin(ct) \Rightarrow \frac{d^2x}{dt^2} = -c^2\cos(ct) = -\frac{k}{m}\cos(ct)$$

$$\text{if } c^2 = \frac{k}{m}$$

- The general solution is:

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t + \phi\right)$$

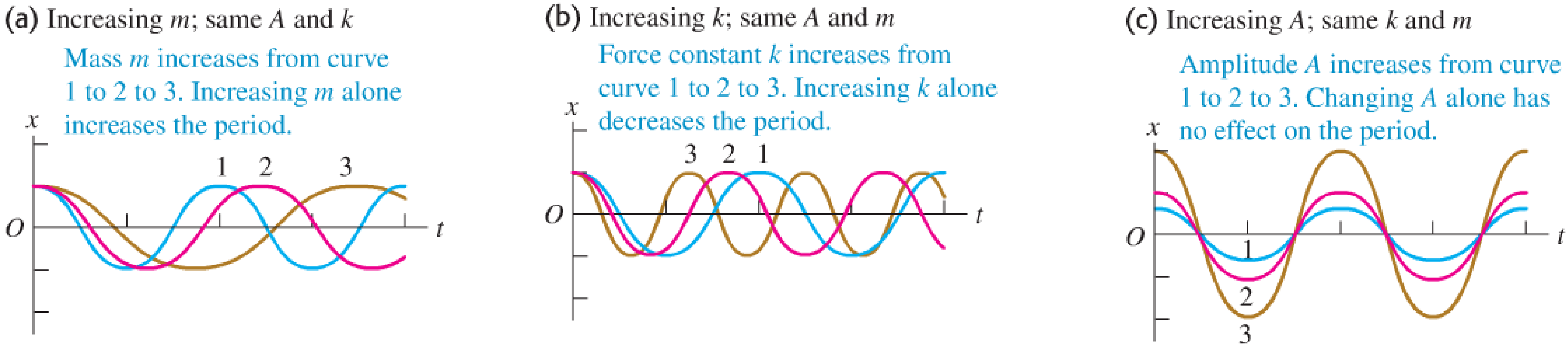


For φ = 0

Simple Harmonic Motion

- The factor in front of time sets the frequency of oscillations, so:

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t + \phi\right) = A\cos(\omega t + \phi), \quad \omega = \sqrt{\frac{k}{m}}$$



For φ = 0

Frequency, Angular Frequency and Period

- There is sometimes confusion about these quantities.

$$x(t) = A\cos(\omega t + \phi), \quad \omega = \sqrt{\frac{k}{m}}$$

- ω is called the angular frequency.
- x returns to its starting point when ωt = 2π, so the period (amount of time to complete one cycle), is:

$$T = \frac{2\pi}{\omega}$$

- The frequency (number of cycles per second) is just:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

CPS 34-1

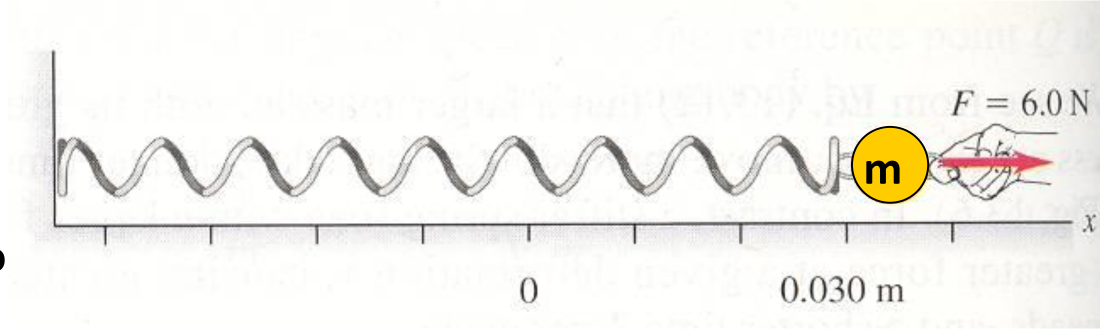
- **A mass m = 1.0kg is attached to a massless spring. The spring is stretched with a force of 6.0N to a distance of 0.03m and then released. What is the frequency of oscillations of the mass?**

A) 1.25 Hz.

B) 2.25 Hz.

C) 3.25 Hz.

D) Not enough information to so

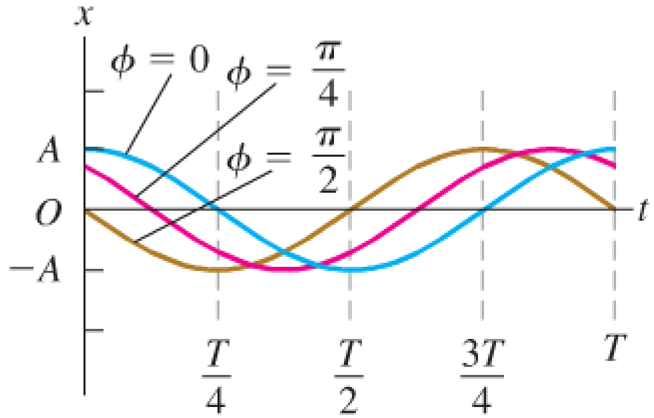


Simple Harmonic Motion

- The phase factor determines the value of x at t=0:

$$x(t) = A\cos(\omega t + \phi), \quad \omega = \sqrt{\frac{k}{m}}$$

These three curves show SHM with the same period T and amplitude A but with different phase angles ϕ.



Position, Velocity and Acceleration

- We can differentiate to get the velocity

$$x(t) = A\cos(\omega t + \phi) \Rightarrow$$

$$v(t) = \frac{dx(t)}{dt} = -\omega A\sin(\omega t + \phi)$$

- And again to get acceleration

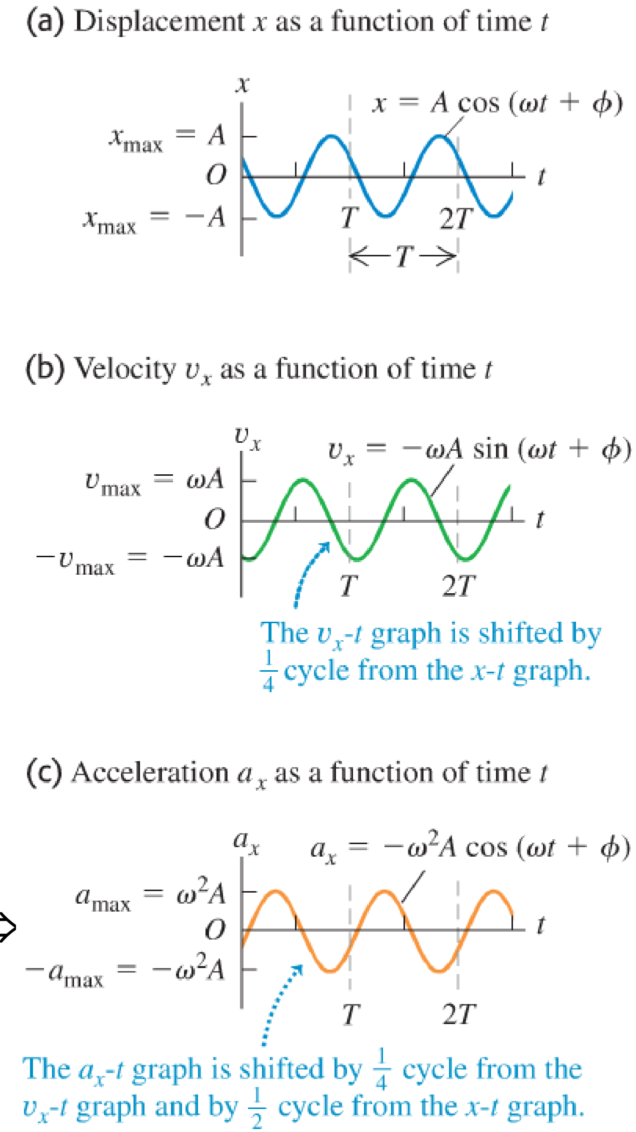
$$v(t) = -\omega A\sin(\omega t + \phi) \Rightarrow$$

$$a(t) = \frac{dv(t)}{dt} = -\omega^2 A\cos(\omega t + \phi)$$

- Note that:

$$a(t) = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x(t) = -\frac{k}{m}x(t) \Rightarrow$$

$$ma(t) = -kx(t)$$



Energy in Simple Harmonic Motion

- Without any other forces (friction), we can describe the energy of a spring-mass system by the kinetic energy:

$$KE = \frac{1}{2}mv^2\left(t\right) = \frac{1}{2}m\omega^2 A^2 \sin^2\left(\omega t + \phi\right)$$

$$= \frac{1}{2}m\left(\sqrt{\frac{k}{m}}\right)^2 A^2 \sin^2\left(\omega t + \phi\right) = \frac{1}{2}kA^2 \sin^2\left(\omega t + \phi\right)$$

- And the potential energy is:

$$U_{el} = \frac{1}{2}kx^2\left(t\right) = \frac{1}{2}kA^2 \cos^2\left(\omega t + \phi\right)$$

Energy in Simple Harmonic Motion

- So, the total energy is the sum of these:

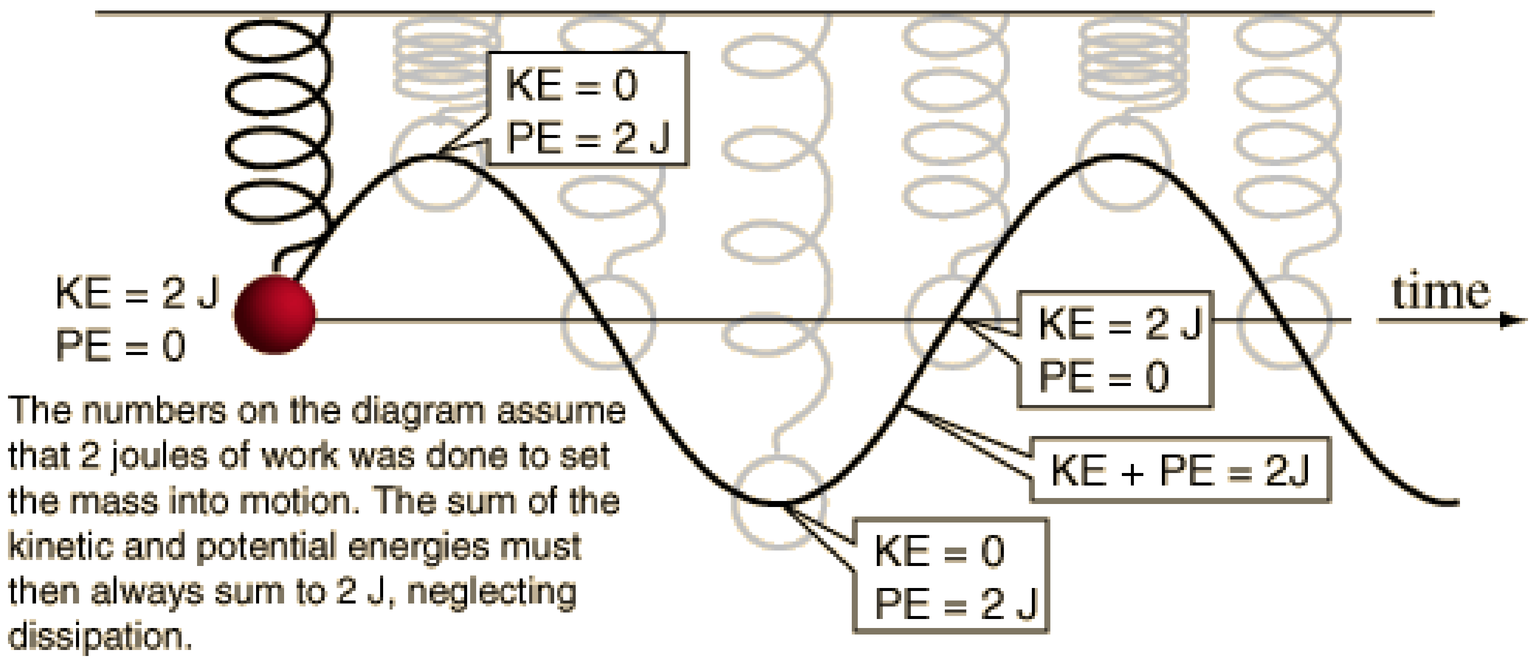
$$E_{total} = KE + U_{el} = \frac{1}{2}kA^2 \sin^2(\omega t + \phi) + \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

$$= \frac{1}{2}kA^2 \left(\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right)$$

$$= \frac{1}{2}kA^2$$

- But there is no time dependence here – conservation of energy!

Energy in Simple Harmonic Motion



No gravity…

14.27 • A 0.150-kg toy is undergoing SHM on the end of a horizontal spring with force constant $k = 300$ N/m. When the toy is 0.0120 m from its equilibrium position, it is observed to have a speed of 0.400 m/s. What are the toy's (a) total energy at any point of its motion; (b) amplitude of motion; (c) maximum speed during its motion?

14.37 • A 175-g glider on a horizontal, frictionless air track is attached to a fixed ideal spring with force constant 155 N/m. At the instant you make measurements on the glider, it is moving at 0.815 m/s and is 3.00 cm from its equilibrium point. Use *energy conservation* to find (a) the amplitude of the motion and (b) the maximum speed of the glider. (c) What is the angular frequency of the oscillations?

14.38 • A proud deep-sea fisherman hangs a 65.0-kg fish from an ideal spring having negligible mass. The fish stretches the spring 0.180 m. (a) Find the force constant of the spring. The fish is now pulled down 5.00 cm and released. (b) What is the period of oscillation of the fish? (c) What is the maximum speed it will reach?

14.83 ••• **CP** A rifle bullet with mass 8.00 g and initial horizontal velocity 280 m/s strikes and embeds itself in a block with mass 0.992 kg that rests on a frictionless surface and is attached to one end of an ideal spring. The other end of the spring is attached to the wall. The impact compresses the spring a maximum distance of 15.0 cm. After the impact, the block moves in SHM. Calculate the period of this motion.