Lecture 26
(Damped and Forced Oscillations)

Physics 160-02 Spring 2017

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Damped Oscillations

- So far, we have ignored resistance in our discussion – energy is conserved because none is lost to heating.
- Let's not ignore poor friction…

$$F = -kx - bv_x = ma_x = m\frac{d^2x}{dt^2} \Rightarrow$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x - \frac{b}{m}\frac{dx}{dt}$$

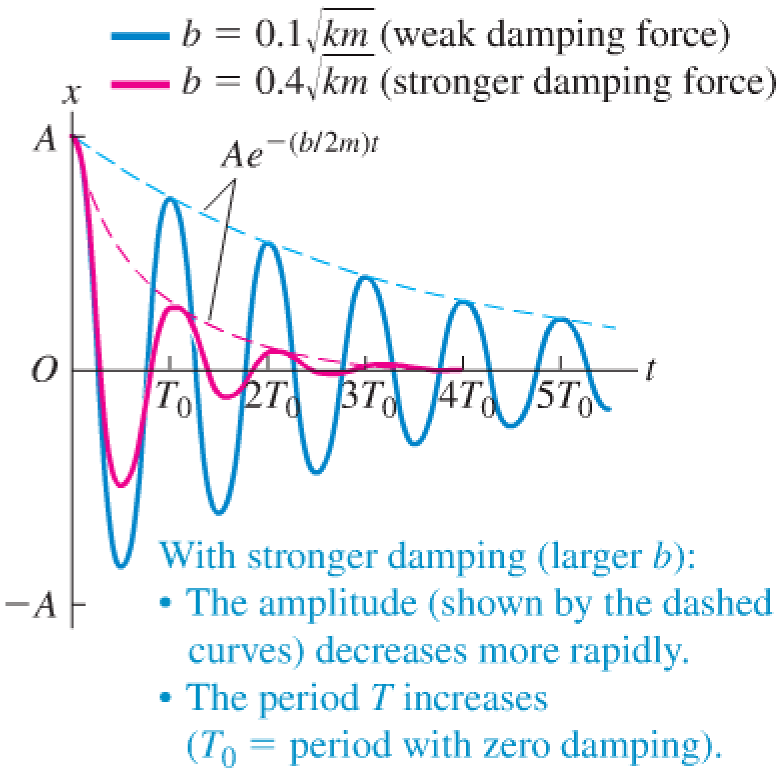
- Differential equation relating the changing acceleration to the position **and** velocity. Solution (when damping isn't large):

$$x(t) = Ae^{-\left(b/2m\right)t}\cos\left(\omega' t + \phi\right), \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Damped Oscillations

- With increasing value of b, the amplitude of the oscillations decrease and the frequency decreases.
- At a certain value of b (critical damping), the frequency goes to zero:

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = 0, \quad b = 2\sqrt{km}$$



Damped Oscillations

- https://phet.colorado.edu/sims/resonance/resonance_en.html

Forced Oscillations

- What about our swing example? Left to one's self, you will eventually stop swinging, unless…

$$F = -kx - bv_x + F_{\max}\cos\left(\omega_d t + \phi_d\right) = ma_x = m\frac{d^2x}{dt^2} \Rightarrow$$

$$\frac{d^2x}{dt^2} = +F_{\max}\cos\left(\omega_d t + \phi_d\right) - \frac{k}{m}x - \frac{b}{m}\frac{dx}{dt}$$

- Solving this differential equation is beyond us (for now), but we can understand the result in terms of the amplitude of oscillations as a function of the magnitude of the driving force and the damping force:

$$A = \frac{F_{\max}}{\sqrt{\left(k - m\omega_d^2\right) + b^2\omega_d^2}}$$

Forced Oscillations

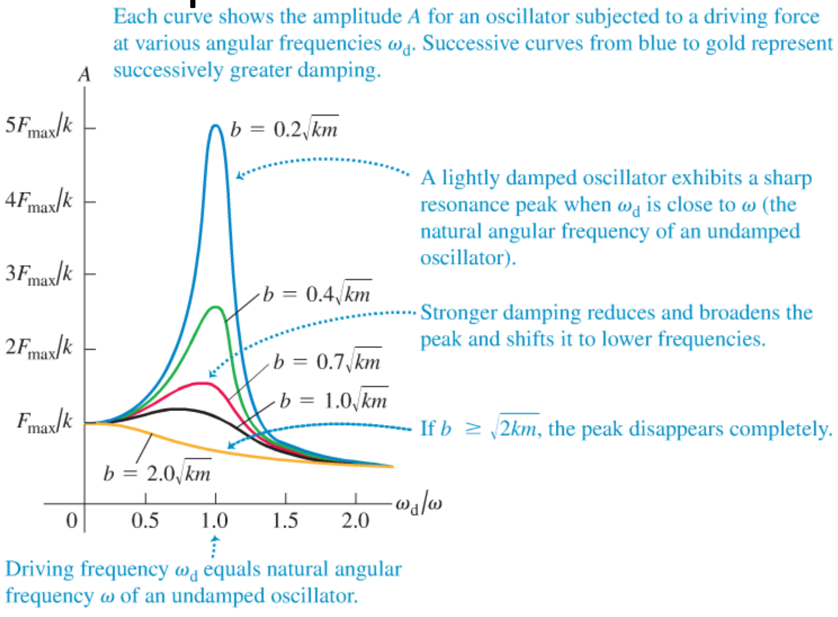
- A forced oscillator has the same frequency as the driving force, but with a varying amplitude.

$$A = \frac{F_{\max}}{\sqrt{\left(k - m\omega_d^2\right) + b^2\omega_d^2}}$$

- When the driving force has a frequency that is near the "natural frequency" of the body, the amplitude of oscillations is at a maximum.

$$k = m\omega_d^2 \Rightarrow$$

$$\omega_d = \sqrt{\frac{k}{m}}$$



Is This Important?

- Some shrewd reporter asked Dr. Tesla at this point what he would need to destroy the Empire State Building and the doctor replied:—“Five pounds of air pressure. If I attached the proper oscillating machine on a girder that is all the force I would need, five pounds. Vibration will do anything. It would only be necessary to step up the vibrations of the machine to fit the natural vibration of the building and the building would come crashing down. That's why soldiers always break step crossing a bridge.”

- Video http://www.youtube.com/watch?v=j-zczJXSxnw