

Lecture 3

(Scalar and Vector Multiplication & 1D Motion)

Physics 160-02 Spring 2017

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Multiplication of Vectors

- OK, adding and subtracting vectors seemed fairly straightforward, but how would one multiply vectors?
- There are two ways to multiply vectors and they give different answers...
 - Dot (or Scalar) Product
 - Cross (or Vector) Product
- You use the two ways for different purposes which will become clearer as you use them.

Dot (or Scalar) Product

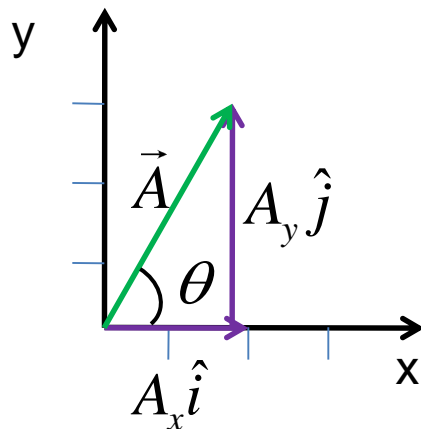
- The dot product of two vectors is written as: $\vec{A} \cdot \vec{B}$.
- The result of a dot product is a scalar (no direction).
- There are two ways to find the dot product:
 - $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB} \equiv AB \cos \theta$
 - or,
 - $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$
- But what IS the dot product – I mean what does it MEAN???

Dot (or Scalar) Product

- The dot product of two vectors $\vec{A} \cdot \vec{B}$ gives the length of \vec{A} in the direction of \vec{B} (projection of \vec{A} onto \vec{B}) times the length of \vec{B} .
- Example:

$$\vec{A} \cdot \hat{i} = |\vec{A}| |\hat{i}| \cos \theta_{A\hat{i}} \equiv A \times 1 \cos \theta = A \cos \theta$$

$$\vec{A} \cdot \hat{j} = |\vec{A}| |\hat{j}| \cos \theta_{A\hat{j}} \equiv A \times 1 \cos(90 - \theta) = A \sin \theta$$



$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$A_x = |\vec{A}| \cos \theta$$

$$A_y = |\vec{A}| \sin \theta$$

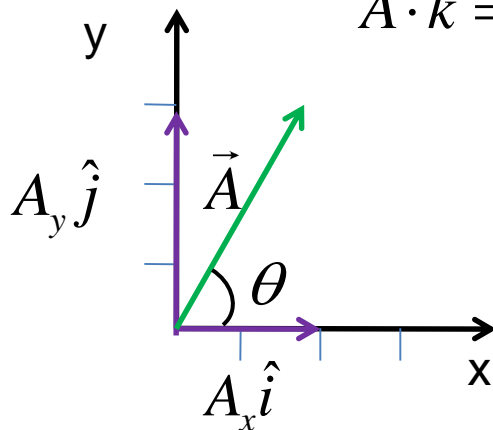
Dot (or Scalar) Product

- The dot product of two vectors $\vec{A} \cdot \vec{B}$ gives the length of \vec{A} in the direction of \vec{B} times the length of \vec{B} .
- Using the other method:

$$\vec{A} \cdot \hat{i} = A_x \cdot 1 + A_y \cdot 0 + A_z \cdot 0 = A_x$$

$$\vec{A} \cdot \hat{j} = A_x \cdot 0 + A_y \cdot 1 + A_z \cdot 0 = A_y$$

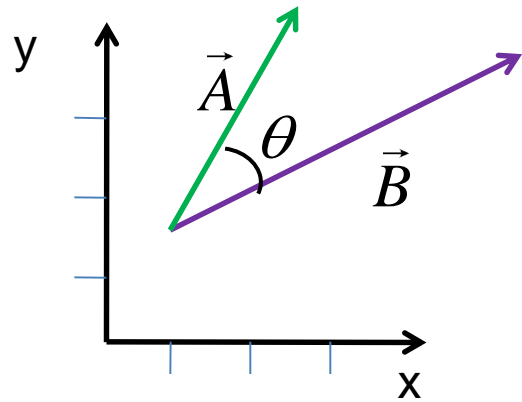
$$\vec{A} \cdot \hat{k} = A_x \cdot 0 + A_y \cdot 0 + A_z \cdot 1 = A_z$$



$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

Dot (or Scalar) Product

- The dot product of two vectors $\vec{A} \cdot \vec{B}$ gives the length of \vec{A} in the direction of \vec{B} times the length of \vec{B} .
- Another Example:



$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB} \equiv (A \cos \theta) B = A_B B \Rightarrow$$

$$A_B = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}; \quad B_A = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|}$$

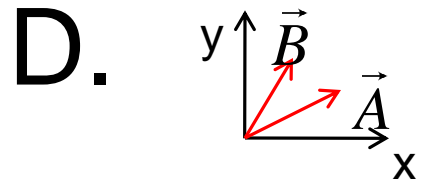
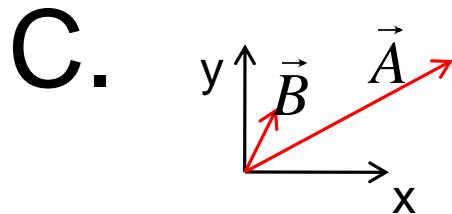
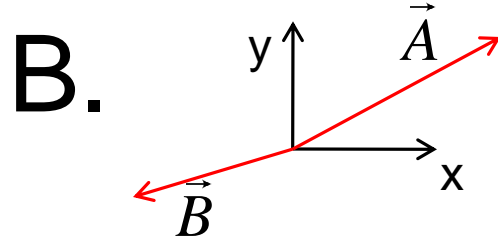
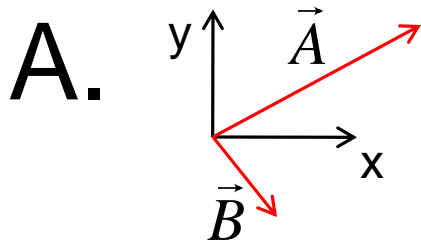
Dot (or Scalar) Product

- Physics Example:
- Work – force acting over a distance.

$$W = \vec{F} \cdot \vec{D}$$

CPS Question 3-1

- Which of the dot products $\vec{A} \cdot \vec{B}$ has the greatest *absolute* magnitude?



Dot (or Scalar) Product

- Commutative and Distributive Laws are obeyed by the dot product:

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

Dot (or Scalar) Product

- This explains the second method then:

$$\begin{aligned}\vec{A} \cdot \vec{B} &= \left(A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \right) \cdot \left(B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \right) \\ &= A_x \hat{i} \cdot B_x \hat{i} + A_x \hat{i} \cdot B_y \hat{j} + A_x \hat{i} \cdot B_z \hat{k} \\ &\quad + A_y \hat{j} \cdot B_x \hat{i} + A_y \hat{j} \cdot B_y \hat{j} + A_y \hat{j} \cdot B_z \hat{k} \\ &\quad + A_z \hat{k} \cdot B_x \hat{i} + A_z \hat{k} \cdot B_y \hat{j} + A_z \hat{k} \cdot B_z \hat{k} \\ &= A_x B_x + A_y B_y + A_z B_z\end{aligned}$$

Dot (or Scalar) Product

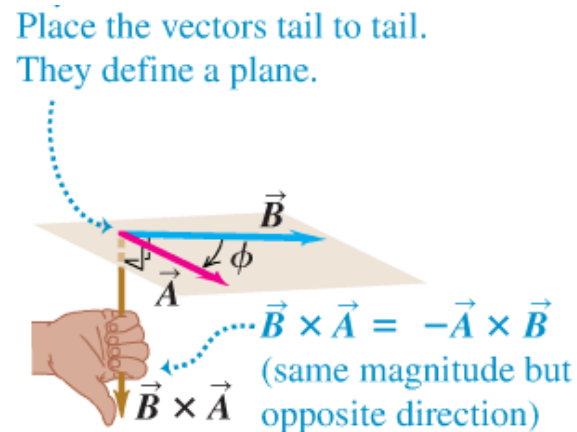
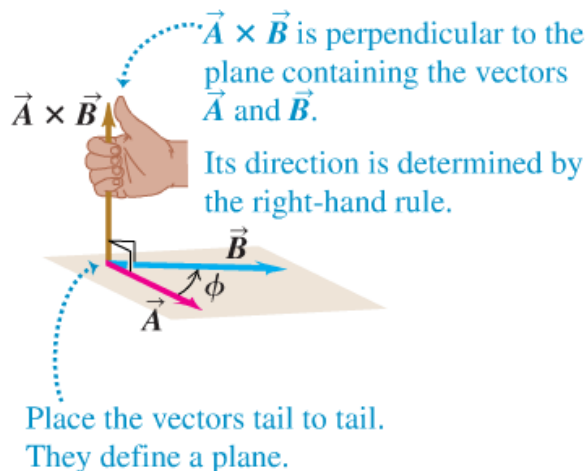
- Usefulness of combining the two methods:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = |\vec{A}| |\vec{B}| \cos \theta_{AB} \Rightarrow$$

$$\cos \theta_{AB} = \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}| |\vec{B}|}$$

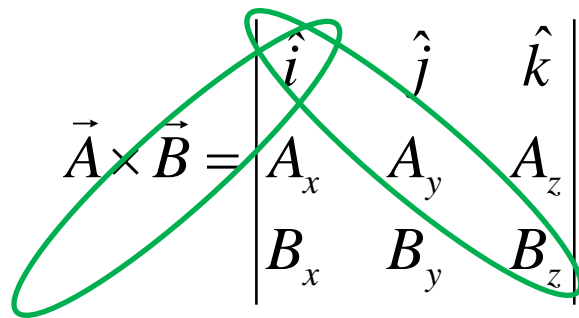
Cross (or Vector) Product

- The cross product of two vectors is written as: $\vec{A} \times \vec{B}$.
- The result of a vector product is a vector (has direction).
- To find the magnitude of a cross product:
 - $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin \theta_{AB} \equiv AB \sin \theta$
 - Its direction is perpendicular to both \vec{A} and \vec{B} , and given by the Right-Hand-Rule:



Cross (or Vector) Product

- Another way of finding $\vec{A} \times \vec{B}$:
 - $\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$
 - Good way to remember using determinant:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$


Cross (or Vector) Product

- Commutative law is NOT obeyed by the cross product:

$$- \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

- Distributive law is obeyed:

$$- \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

Cross (or Vector) Product

- Let's use this to get the second method:

$$- \vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x \hat{i} \times B_x \hat{i} + A_x \hat{i} \times B_y \hat{j} + A_x \hat{i} \times B_z \hat{k}$$

$$+ A_y \hat{j} \times B_x \hat{i} + A_y \hat{j} \times B_y \hat{j} + A_y \hat{j} \times B_z \hat{k}$$

$$+ A_z \hat{k} \times B_x \hat{i} + A_z \hat{k} \times B_y \hat{j} + A_z \hat{k} \times B_z \hat{k}$$

$$- \text{with } \hat{i} \times \hat{j} = \hat{k}; \quad \hat{j} \times \hat{k} = \hat{i}; \quad \hat{k} \times \hat{i} = \hat{j}$$

– then

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

Cross (or Vector) Product

- But what IS the vector product – I mean what does it MEAN???
- It gives a sense of the perpendicularity and length of two vectors.

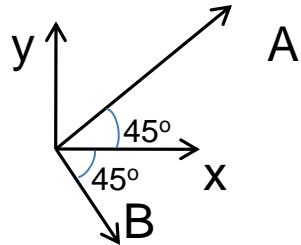
Cross (or Vector) Product

- Physics Example:
- Torque – what does it take to turn a sticky bolt?

$$\vec{\tau} = \vec{r} \times \vec{F}$$

CPS Question 4-1

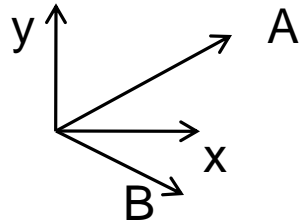
- $\vec{A} \times \vec{B} = ?$, \vec{A}, \vec{B} in the x-y plane



- A. 0
- B. $|\vec{A}||\vec{B}|\sin 45^\circ$
- C. $|\vec{A}||\vec{B}|$ in the positive z-direction
- D. $|\vec{A}||\vec{B}|$ in the negative z-direction
- E. $|\vec{A}||\vec{B}|\sin 45^\circ$ in the negative z-direction

CPS Question 4-2

- What is $\vec{A} \cdot (\vec{A} \times \vec{B}) = ?$, \vec{A}, \vec{B} in the x-y plane

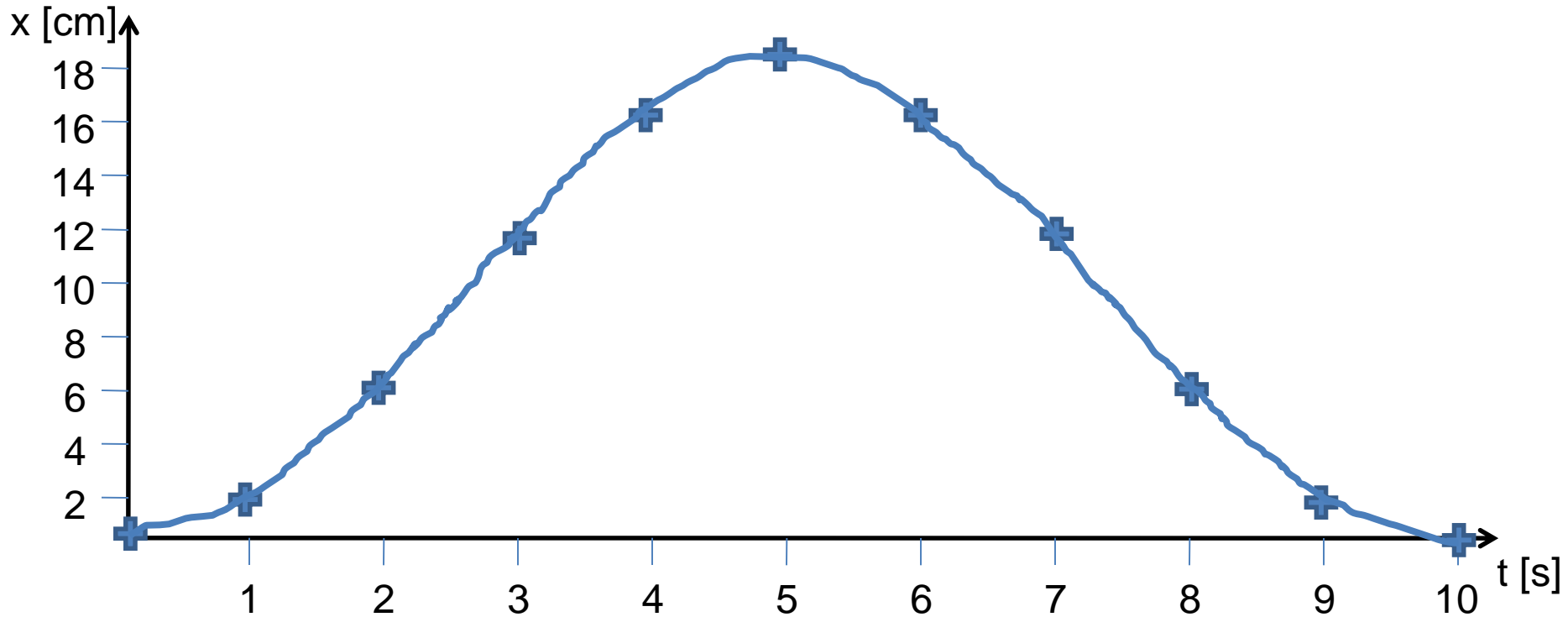
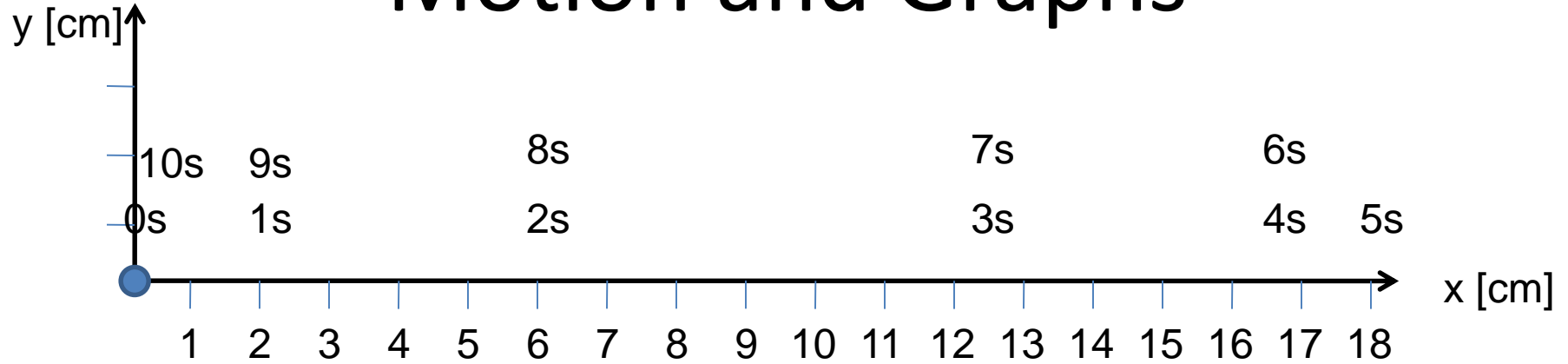


- A. 0
- B. $|\vec{A}|^2 |\vec{B}| \sin 45^\circ$
- C. $|\vec{A}|^2 |\vec{B}|$ in the positive z-direction
- D. $|\vec{A}|^2 |\vec{B}|$ in the negative z-direction
- E. not enough information

Motion in One Dimension

- We need to define some terms:
 - Distance (scalar) [m]
 - Displacement (vector) [m]
 - Speed (scalar) [m/s]
 - Velocity (vector) [m/s]
 - Acceleration (vector) [m/s²]
 - Time (scalar) [s]

Motion and Graphs



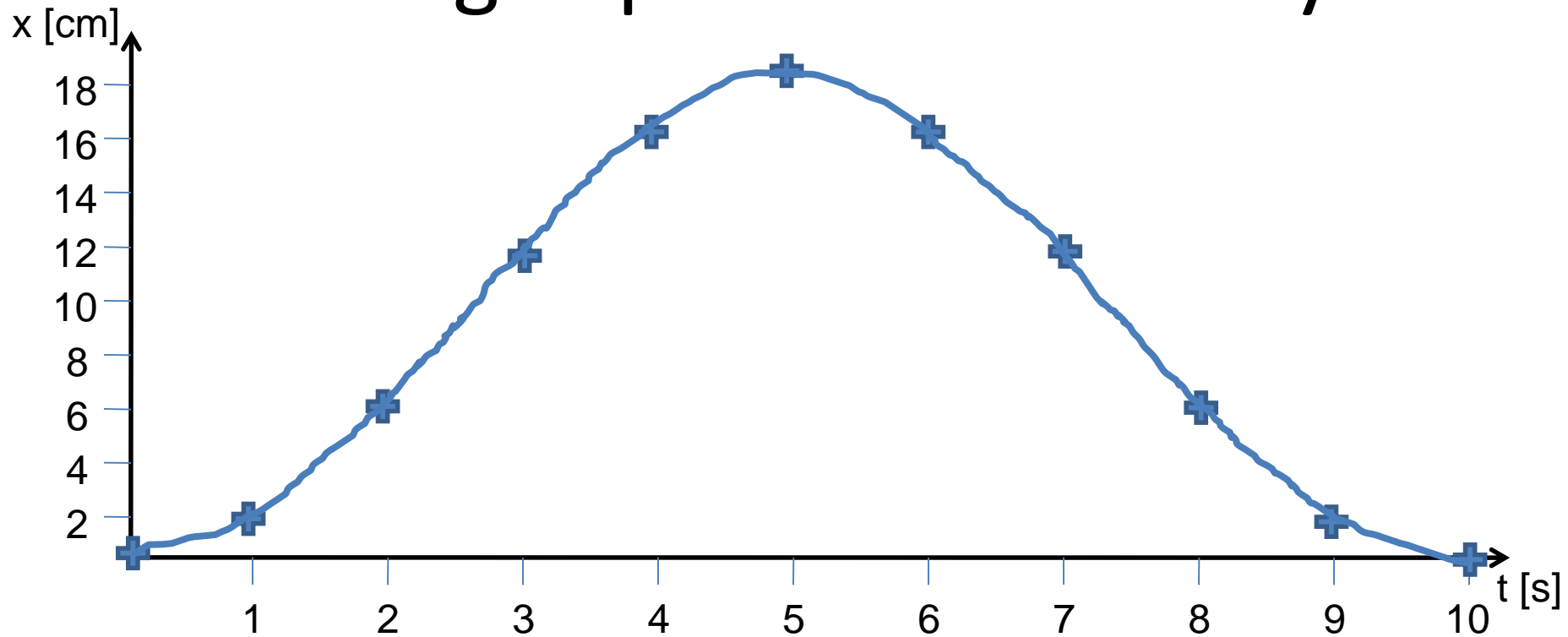
Average Speed and Velocity

- When we talk about speed and velocity, we are referring to changes in distance and displacement over a change in time.
- Important to be specific about these changes:

$$s_{\text{avg}} = \frac{\text{total distance}}{\text{total time}}$$

$$\vec{v}_{\text{avg}} = \frac{\text{total displacement}}{\text{total time}} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i}$$

Average Speed and Velocity

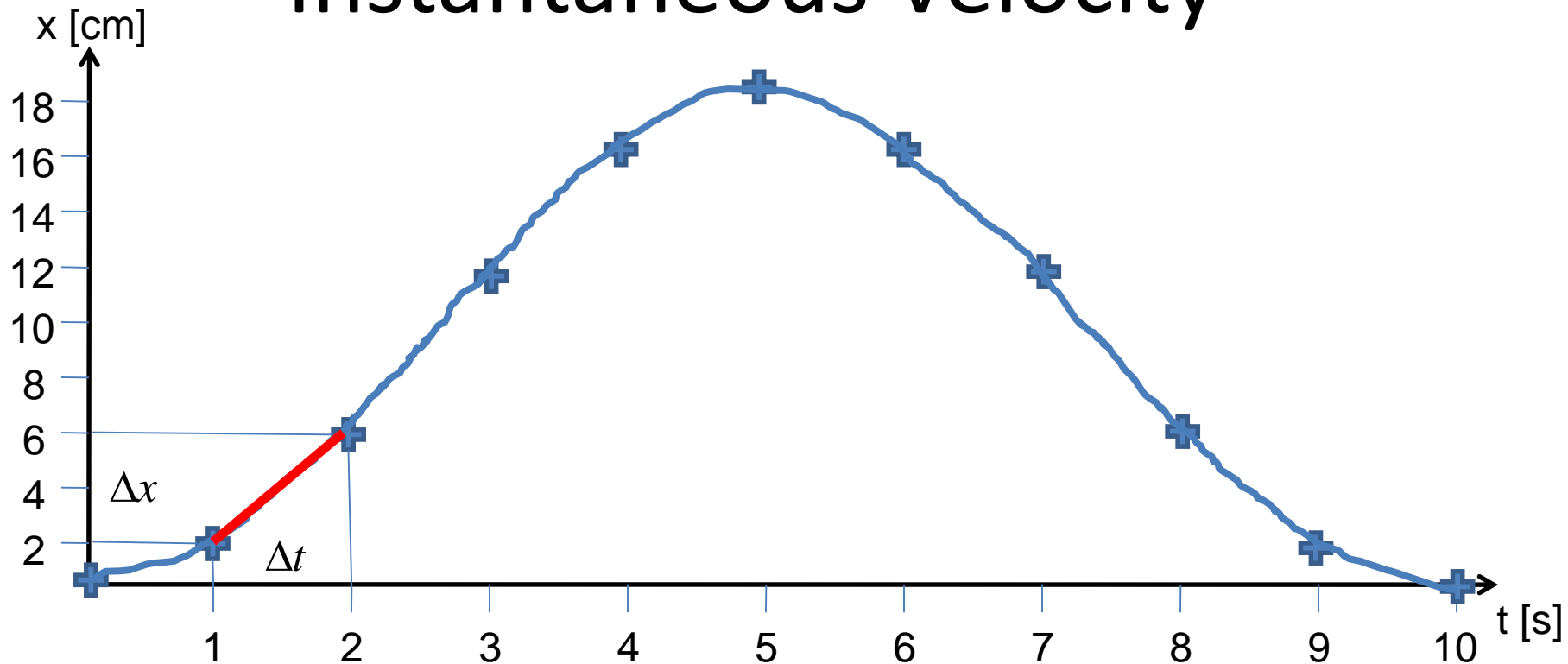


- What is the average speed from 0 – 5s? From 0 – 10s?
- What is the average velocity from 0 – 5s? From 0 – 10s?

CPS Question 5-1

- Someone walks 100m in what we will call the negative-x direction, then turns around and walks half way back, all at the same pace (speed). It takes 100s to walk the entire path. What is their average velocity?
 - A. 1.5m/s
 - B. 1.5m/s in the +x direction
 - C. 0.5m/s
 - D. 0.5m/s in the -x direction
 - E. 1.0m/s in the -x direction

Instantaneous Velocity

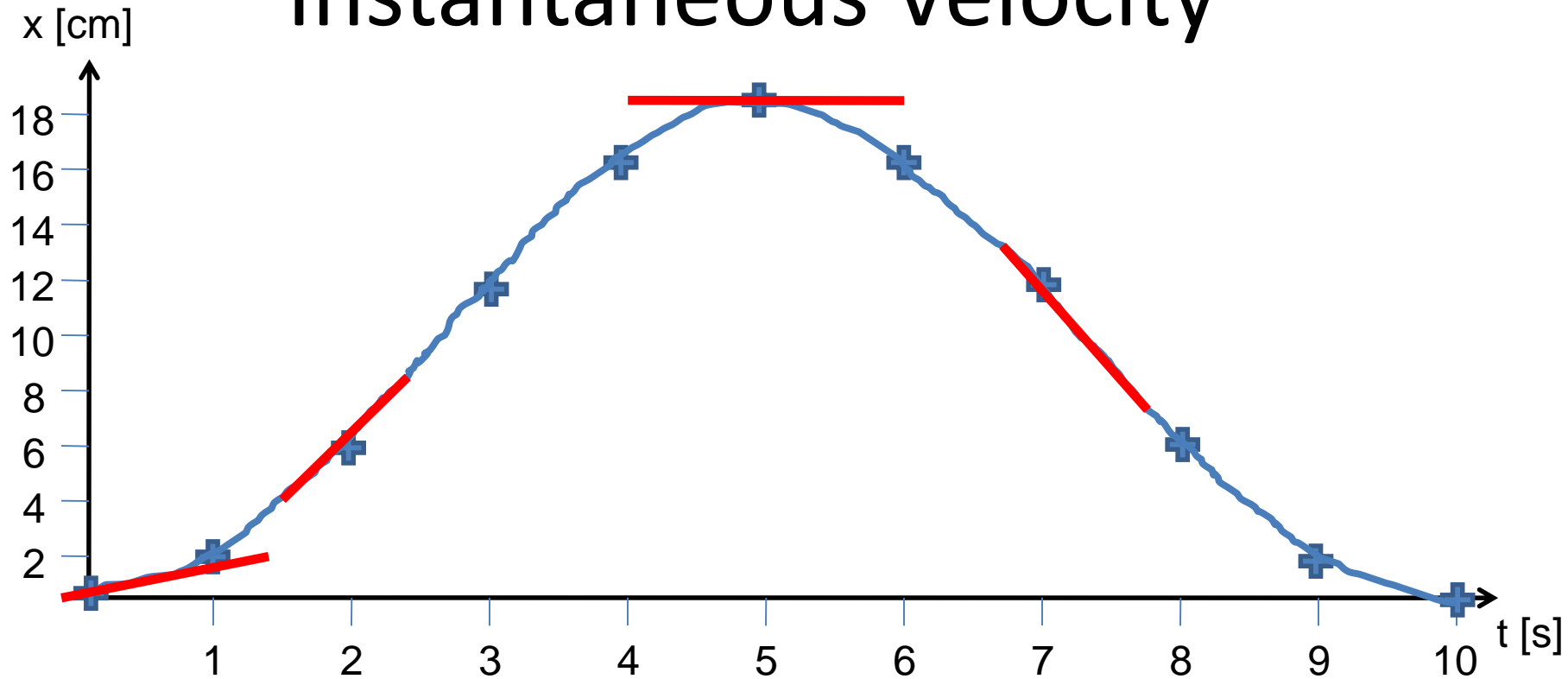


- What is the average velocity from 1 – 2s?

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} = \frac{(6\text{cm})\hat{i} - (2\text{cm})\hat{i}}{2\text{s} - 1\text{s}} = \left(4 \frac{\text{cm}}{\text{s}}\right)\hat{i} = \left(4 \frac{\text{cm}}{\text{s}}\right)\left(\frac{1\text{m}}{100\text{cm}}\right)\hat{i} = \left(0.04 \frac{\text{m}}{\text{s}}\right)\hat{i}$$

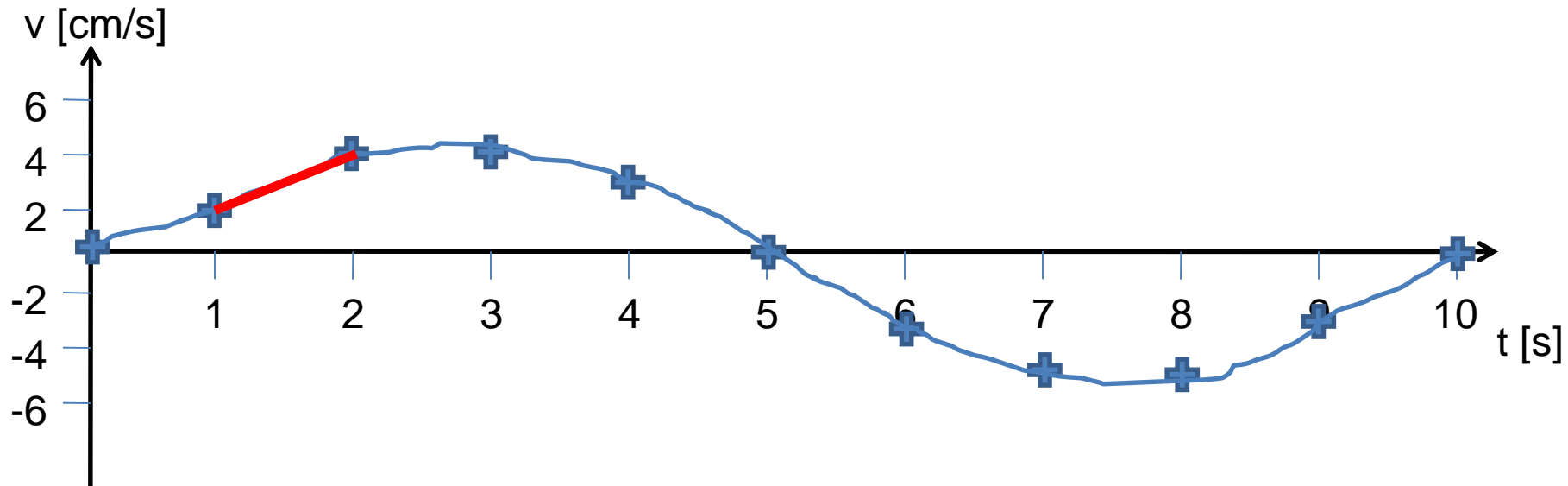
- Note that this is the slope of the line connecting these two points.

Instantaneous Velocity



- What is the velocity at 2s? $\vec{v}(t) = \lim_{\Delta t \rightarrow 0} \vec{v}_{\text{avg}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t} \equiv \frac{d\vec{x}}{dt}$
- This is the definition of the derivative of the function that describes the position as a function of time.
- It gives a the slope of the tangent line to the function at any point.

Instantaneous Velocity

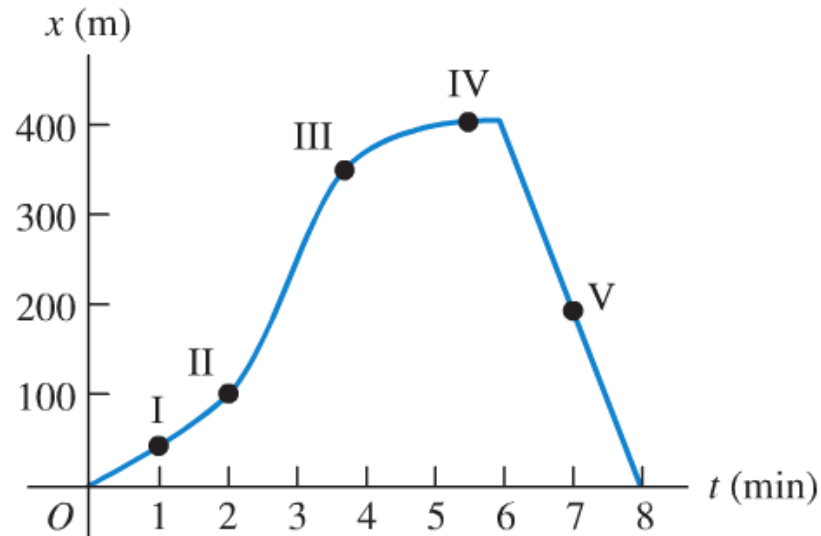


- We can now plot the velocity as a function of time.
- Notice that the velocity also changes.
- So, we can ask, what is the change in velocity as a function of time?
- This is known as the acceleration.

Exercise 2.10

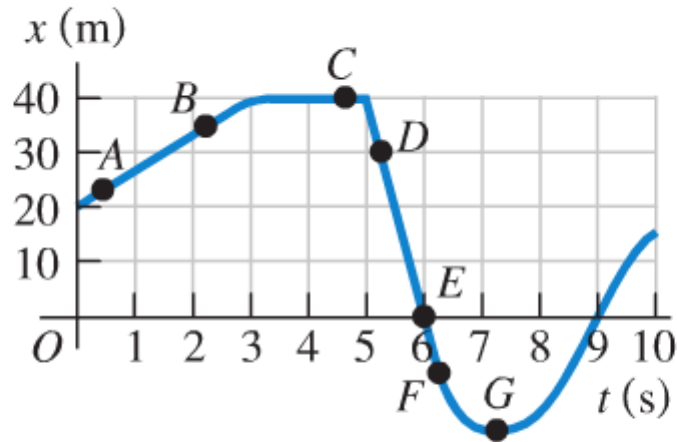
2.10 • A physics professor leaves her house and walks along the sidewalk toward campus. After 5 min it starts to rain and she returns home. Her distance from her house as a function of time is shown in Fig. E2.10. At which of the labeled points is her velocity (a) zero? (b) constant and positive? (c) constant and negative? (d) increasing in magnitude? (e) decreasing in magnitude?

Figure **E2.10**



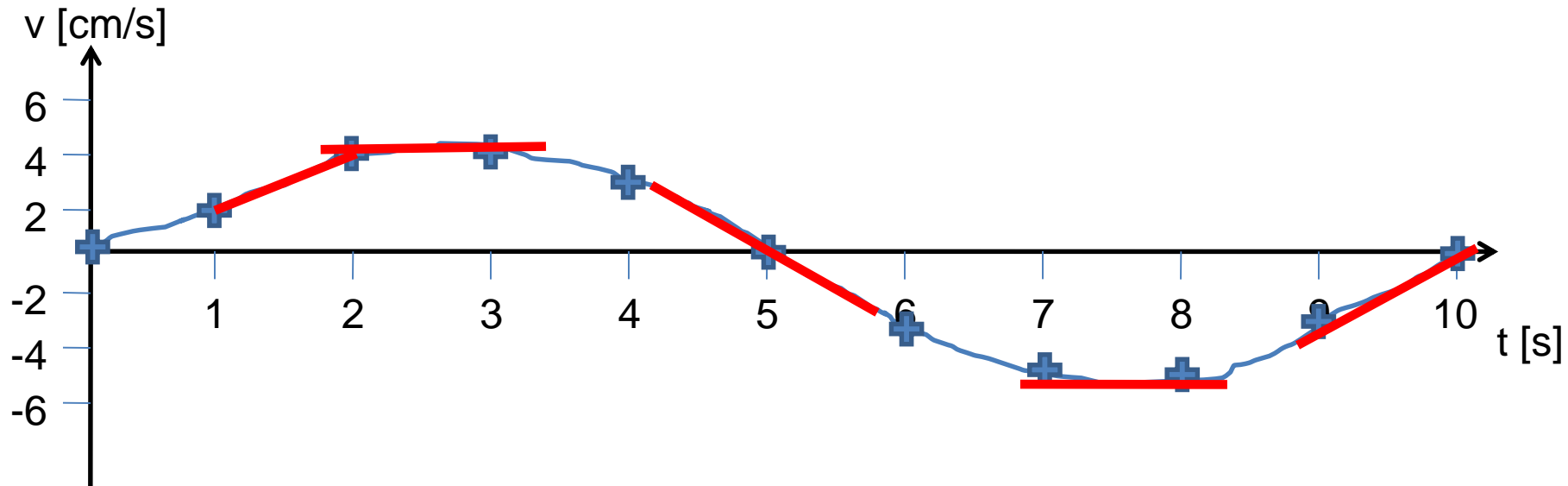
Exercise 2.11

Figure **E2.11**



2.11 •• A test car travels in a straight line along the x -axis. The graph in Fig. E2.11 shows the car's position x as a function of time. Find its instantaneous velocity at points A through G .

Instantaneous Acceleration

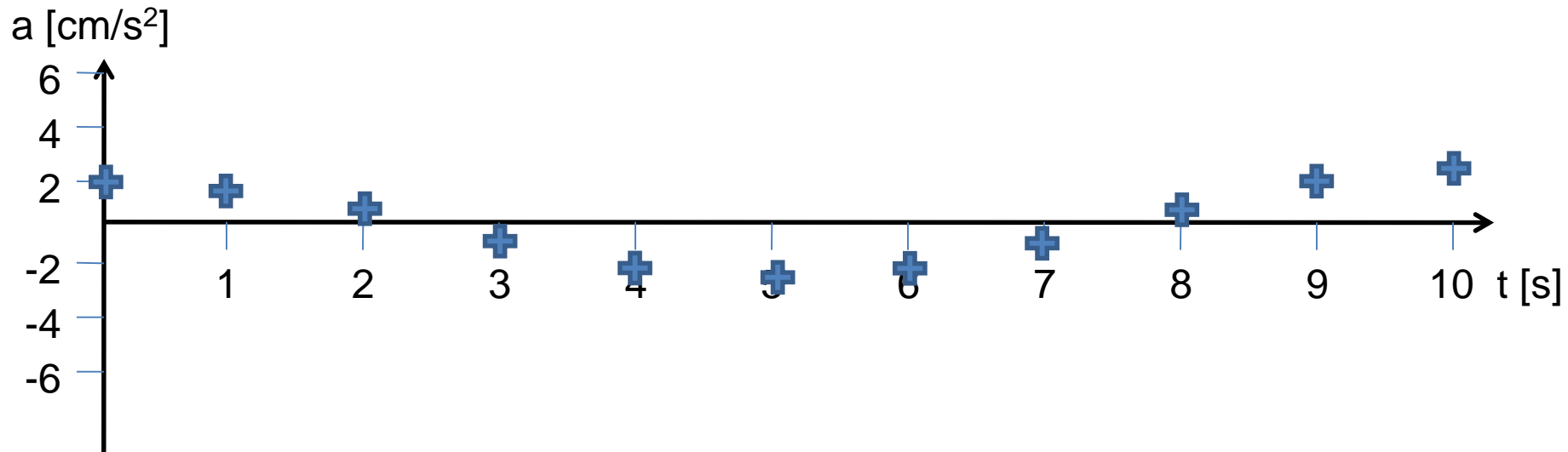
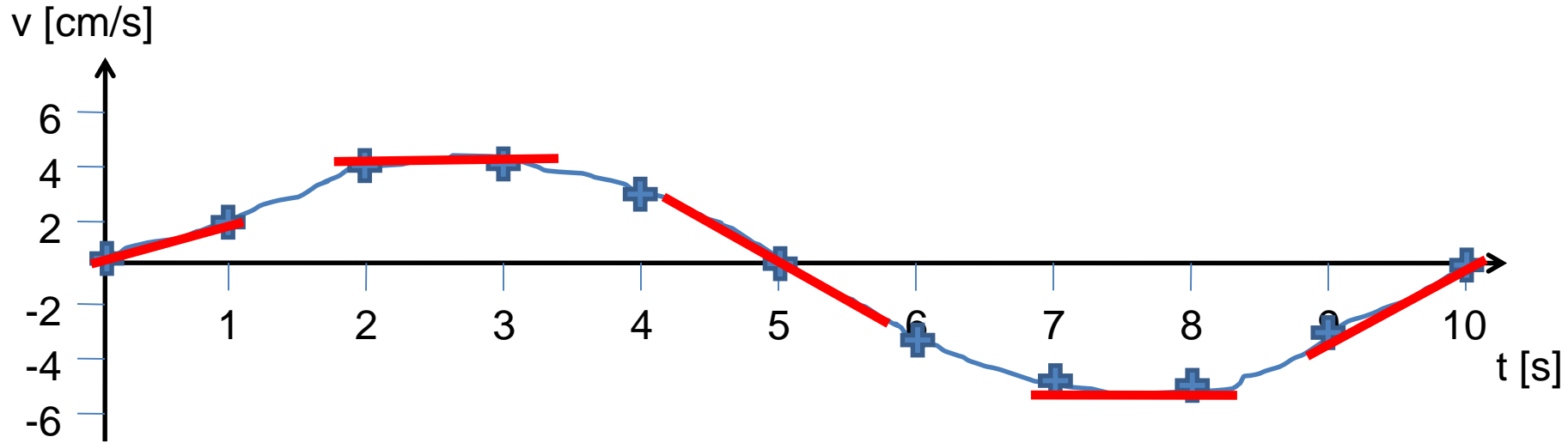


- The acceleration is just given by the derivative of the velocity function.

$$\vec{a}(t) = \lim_{\Delta t \rightarrow 0} \vec{a}_{\text{avg}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} \equiv \frac{d\vec{v}}{dt} = \frac{d}{dt} \left(\frac{d\vec{x}}{dt} \right) \equiv \frac{d^2 \vec{x}}{dt^2}$$

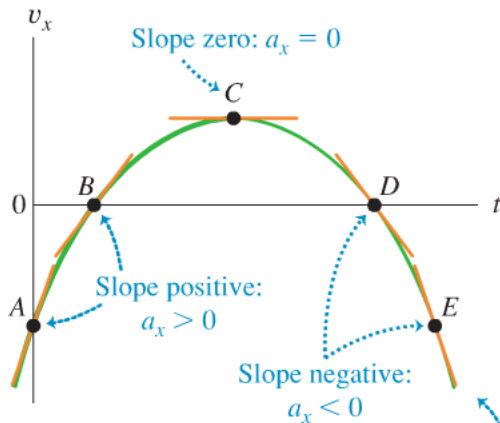
- So, it is also the second derivative of the position function.

Instantaneous Acceleration



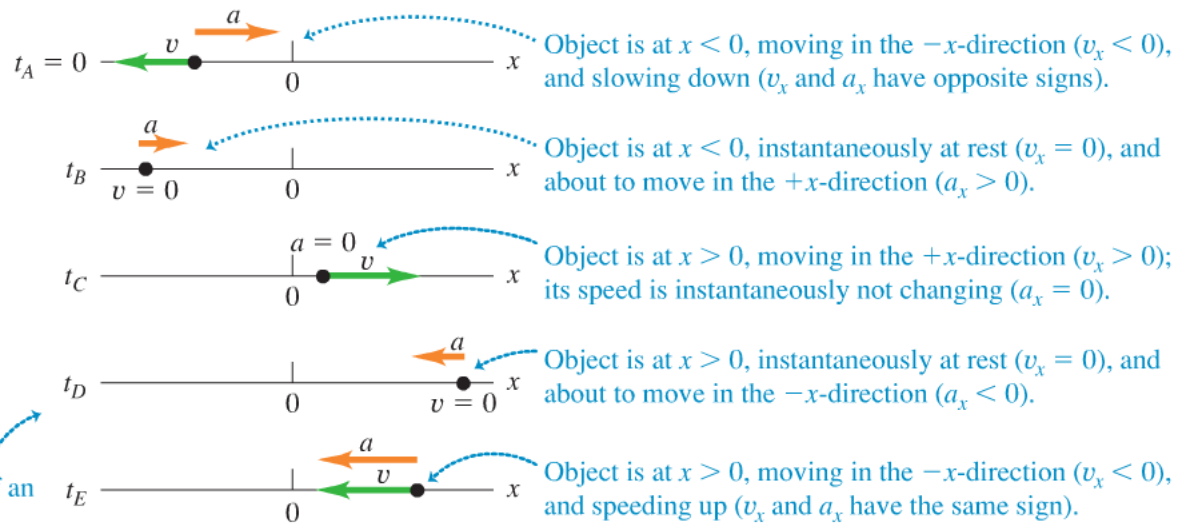
Velocity and acceleration

(a) v_x - t graph for an object moving on the x -axis



The steeper the slope (positive or negative) of an object's v_x - t graph, the greater is the object's acceleration in the positive or negative x -direction.

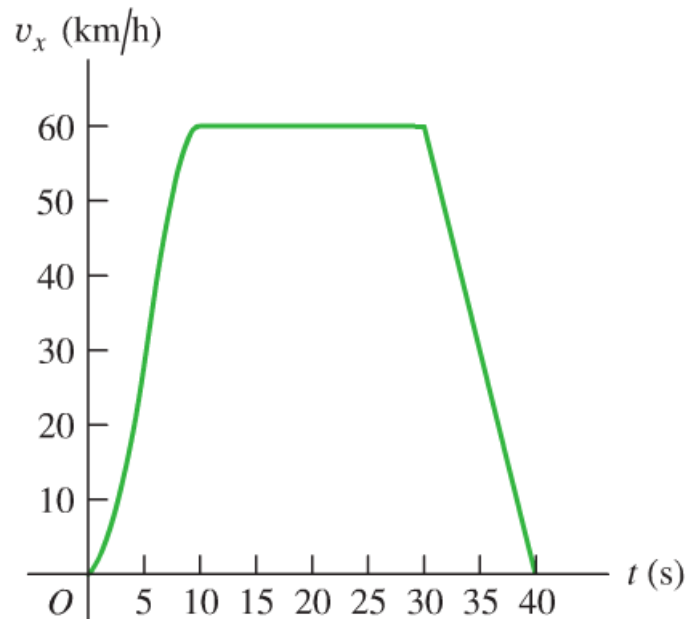
(b) Object's position, velocity, and acceleration on the x -axis



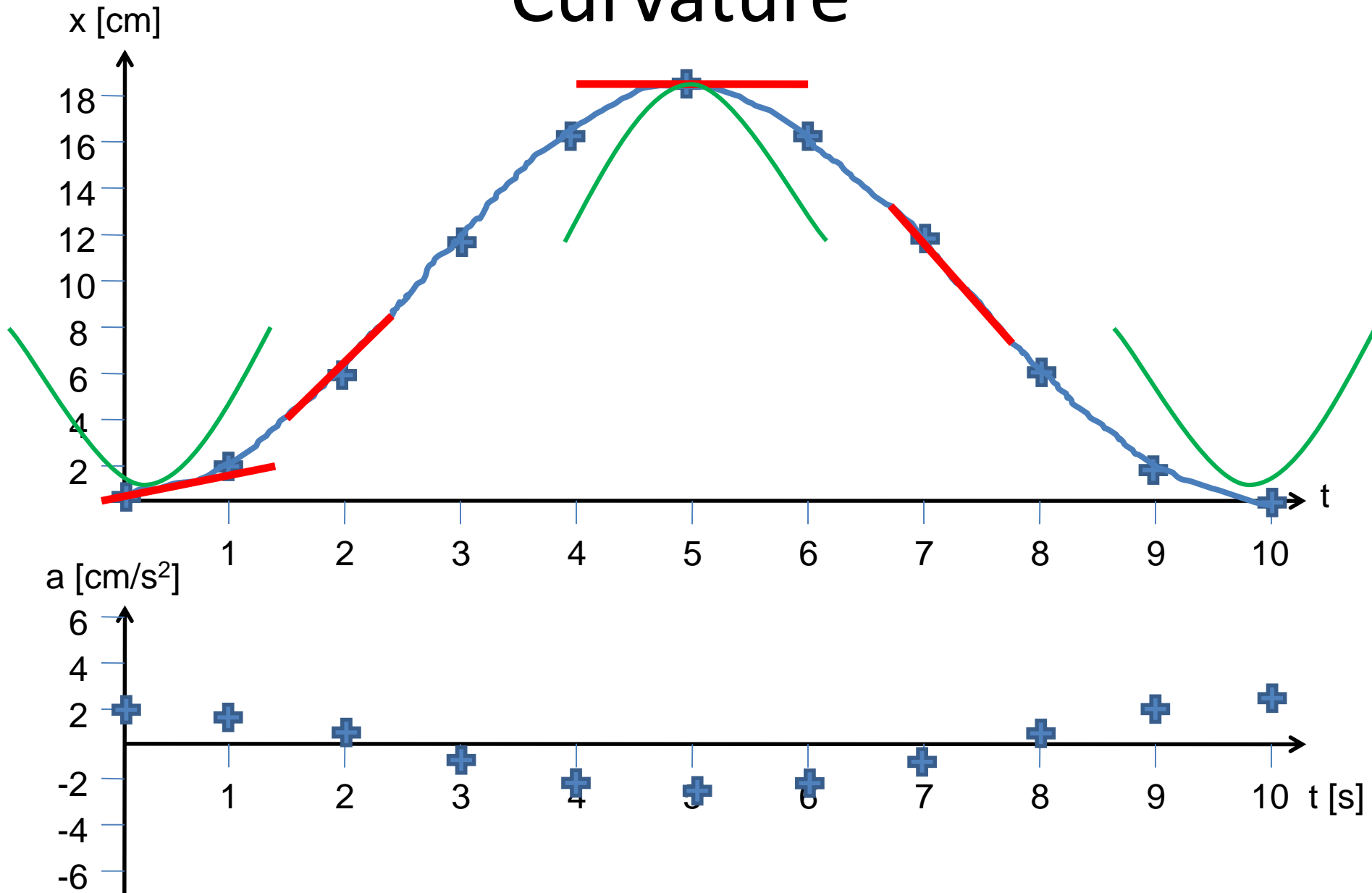
Exercise 2.12

2.12 • Figure E2.12 shows the velocity of a solar-powered car as a function of time. The driver accelerates from a stop sign, cruises for 20 s at a constant speed of 60 km/h, and then brakes to come to a stop 40 s after leaving the stop sign. (a) Compute the average acceleration during the following time intervals: (i) $t = 0$ to $t = 10$ s; (ii) $t = 30$ s to $t = 40$ s; (iii) $t = 10$ s to $t = 30$ s; (iv) $t = 0$ to $t = 40$ s. (b) What is the instantaneous acceleration at $t = 20$ s and at $t = 35$ s?

Figure **E2.12**



Curvature



Spherical Cows

- http://en.wikipedia.org/wiki/Spherical_cow
- Milk production at a dairy farm was low, so the farmer wrote to the local university, asking for help from academia. A multidisciplinary team of professors was assembled, headed by a theoretical physicist, and two weeks of intensive on-site investigation took place. The scholars then returned to the university, notebooks crammed with data, where the task of writing the report was left to the team leader. Shortly thereafter the physicist returned to the farm, saying to the farmer "I have the solution, but it only works in the case of spherical cows in a vacuum."

