Lecture 4
(Motion with Constant Acceleration, Free Fall)

Physics 160-02 Spring 2017

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Derivatives

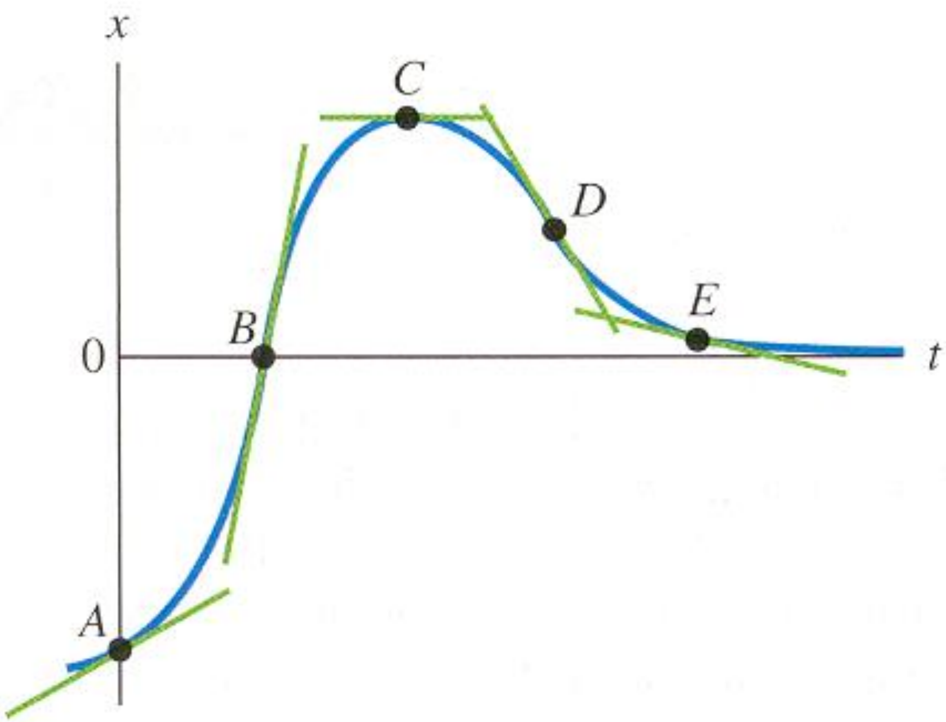
$$\frac{d}{dt} t^n = n t^{n-1}$$

$$\frac{d}{d\theta} \cos\theta = -\sin\theta$$

$$\frac{d}{d\theta} \sin\theta = \cos\theta$$

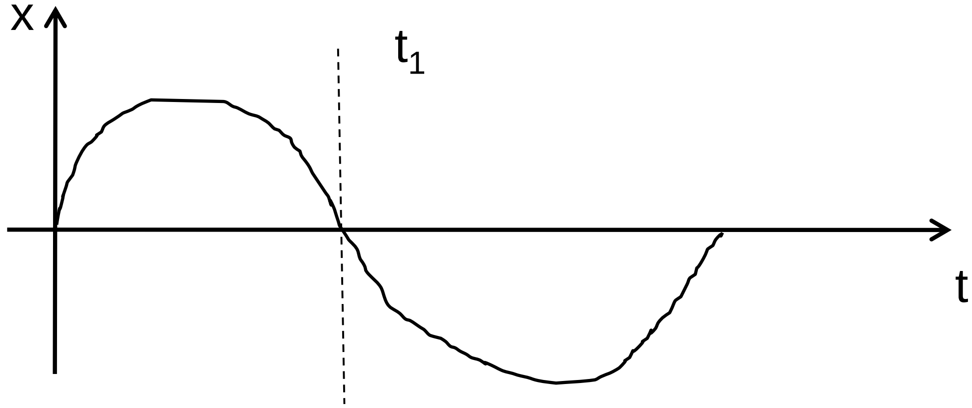
CPS Question 6-1

- At which point in the graph below is the velocity negative and acceleration positive?



CPS Question 6-2

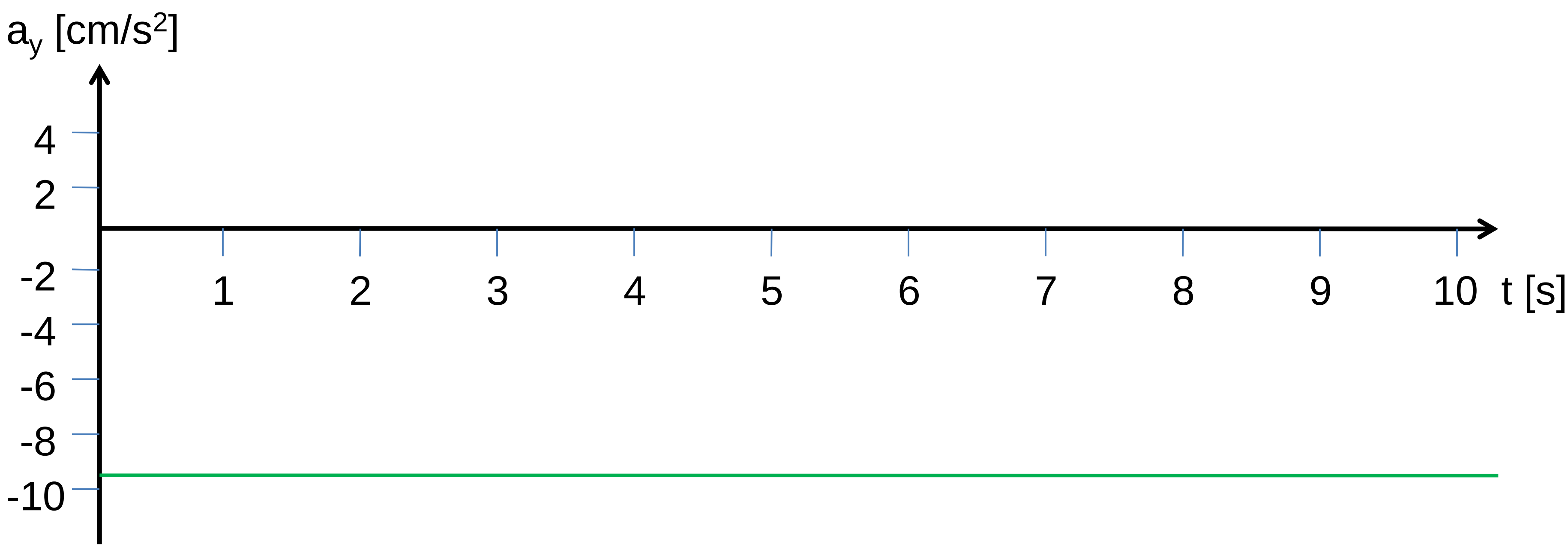
- In the position versus time graph shown below, which answer best describes the instantaneous velocity and acceleration at time t_1?



A. velocity is positive, acceleration is zero
B. velocity is positive, acceleration is positive
C. velocity is negative, acceleration is negative
D. velocity is negative, acceleration is zero
E. velocity is zero, acceleration is zero

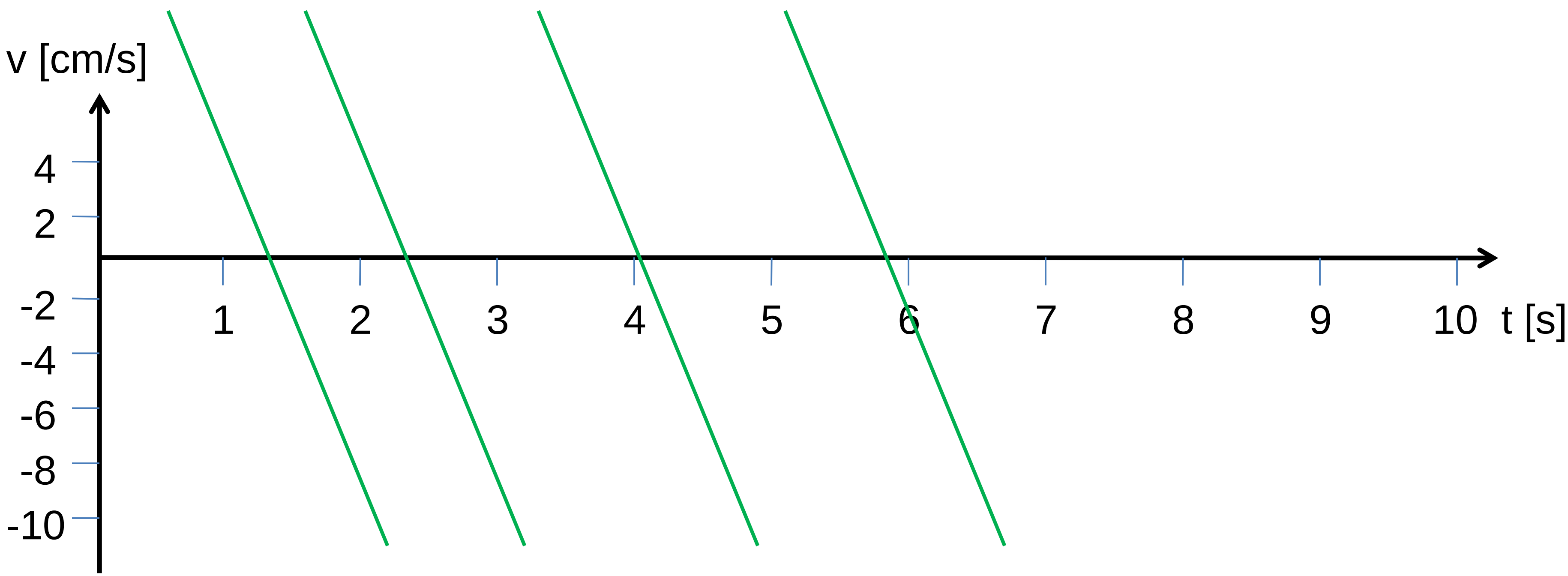
Motion With Constant Acceleration

- For many applications, we can consider the acceleration to be constant.
- Acceleration of a falling object near the earth's surface ≈ constant (if you ignore air resistance).



Motion With Constant Acceleration

- Now, from our discussions last class, what type of velocity dependence with time gives a constant acceleration?
- Linear behavior: $v_y(t) = a_y t \quad v_y(t) = v_{0y} + a_y t$
- But information is missing, the same slope gives the correct acceleration regardless of where the line starts.



Velocity From Acceleration

- From calculus, we can also get this result by using anti-differentiation (integration):

$$a_y(t) = \frac{dv_y}{dt} \Rightarrow$$

$$a_y(t)\,dt = dv_y \Rightarrow$$

$$\int a_y(t)\,dt = \int dv_y$$

- And if a_y is a constant,

$$\int a_y(t)\,dt = a_y \int dt = \int dv_y \Rightarrow$$

$$a_y t + C = v_{fy} + C \Rightarrow$$

$$v_{fy} = v_{oy} + a_y t$$

Position From Velocity

- We can do this again because we know that the position function is also related to the velocity function:

$$v_y(t) = \frac{dy}{dt} \Rightarrow$$

$$v_y(t)\,dt = dy \Rightarrow$$

$$\int v_y(t)\,dt = \int dy \Rightarrow$$

$$\int \left(a_y t + v_{0y}\right) dt = \int dy \Rightarrow$$

$$a_y \int t\,dt + v_{0y} \int dt = \int dy \Rightarrow$$

$$\frac{1}{2} a_y t^2 + v_{0y} t = y(t) + y_0 \Rightarrow$$

$$y_f = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$$

Equations of Motion (constant a)

- We then have the following equations of motion:

$$y_f = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_{fy} = v_{0y} + a_y t$$

- From these, we can solve for t in the second, and substitute into the first:

$$v_y = v_{0y} + a_y t \Rightarrow t = \frac{v_y - v_{0y}}{a_y} \Rightarrow$$

$$y_f = y_0 + v_{0y}\left(\frac{v_y - v_{0y}}{a_y}\right) + \frac{1}{2}a_y\left(\frac{v_y - v_{0y}}{a_y}\right)^2$$

$$2a_y\left(y_f - y_0\right) = \left(v_y - v_{0y}\right)^2 + 2v_{0y}\left(v_y - v_{0y}\right) = v_y^2 - 2v_y v_{oy} + v_{0y}^2 + 2v_y v_{oy} - 2v_{0y}^2 \Rightarrow$$

$$v_y^2 - v_{0y}^2 = 2a_y\left(y_f - y_0\right)$$

Equations of Motion (constant a)

- We then have the following equations of motion:

$$y_f = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$v_{fy} = v_{0y} + a_y t$$

$$v_{fy}^2 - v_{0y}^2 = 2a_y\left(y_f - y_0\right)$$

$$y_f - y_0 = \left(\frac{v_{fy} - v_{0y}}{2}\right)t$$

- Remember:
  - the “y” is just a name for a direction, I could have used x (like the book) or Nancy…

Example

- A car starts from rest and accelerates with constant acceleration = 2m/s^2 for 5s. What is it's velocity after this time?
- What is it's position after at t=4s?

CPS 6-3

- A car starts from rest and accelerates with constant acceleration = $4 m/s^2$ through a distance of 100m. How much time does it take the car to reach this distance?
- A. 25s
- B. 50s
- C. 7s
- D. 10s

Free Fall

- Near the earth's surface, objects fall with nearly constant acceleration of $9.8 m/s^2$ pointing down (due to the force of gravity).
- Objects that have significant air resistance compared to the force of gravity (feathers, parachutes) fall with lower acceleration.
- If you take away the air…

Free Fall in Vacuum

- Demonstration

Timed Free Fall

- Demonstration

CPS Demonstration Question
Timed Free Fall

- **The timer starts when the ball is dropped, and ends when it hits a plate 2m below. How much time does it take?**

A) 0.41s

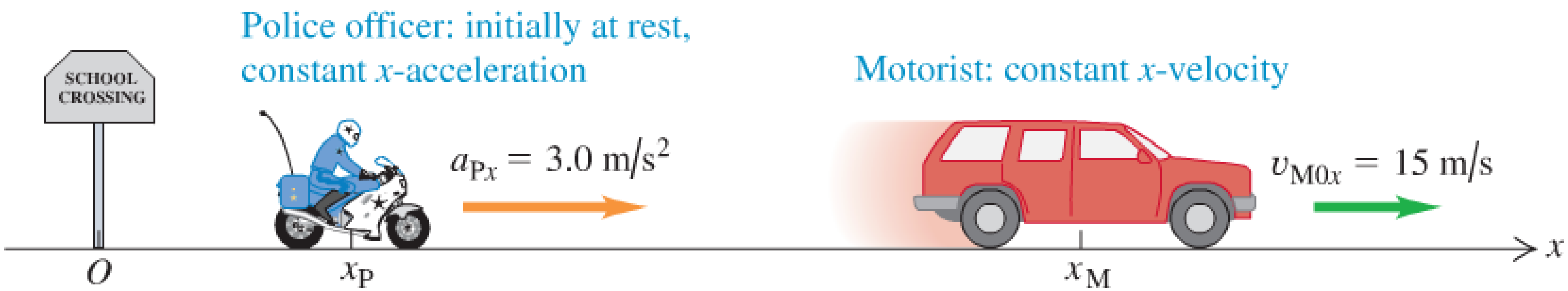
B) 1.5s

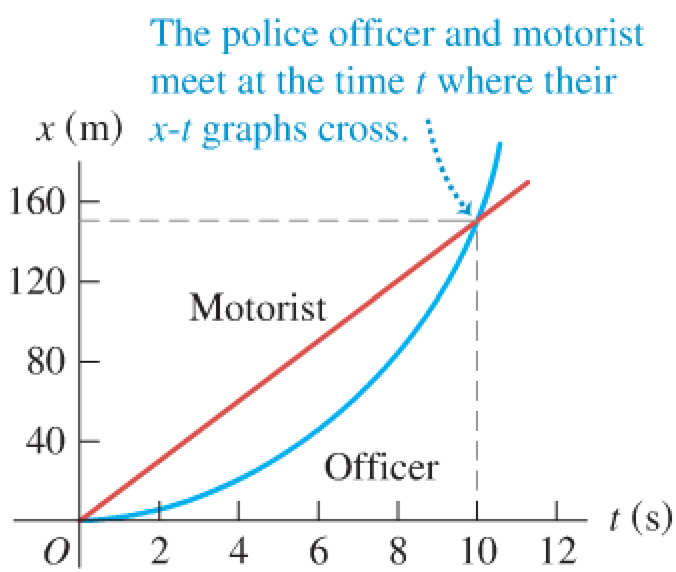
C) 0.64s

D) 2.2s

E) Cannot determine, insufficient information.

Two Objects





$$x_M = 0 + v_{M0x}t + \frac{1}{2}(0)t^2 = v_{M0x}t$$

$$x_P = 0 + (0)t + \frac{1}{2}a_{Px}t^2 = \frac{1}{2}a_{Px}t^2$$

$$v_{M0x}t = \frac{1}{2}a_{Px}t^2$$

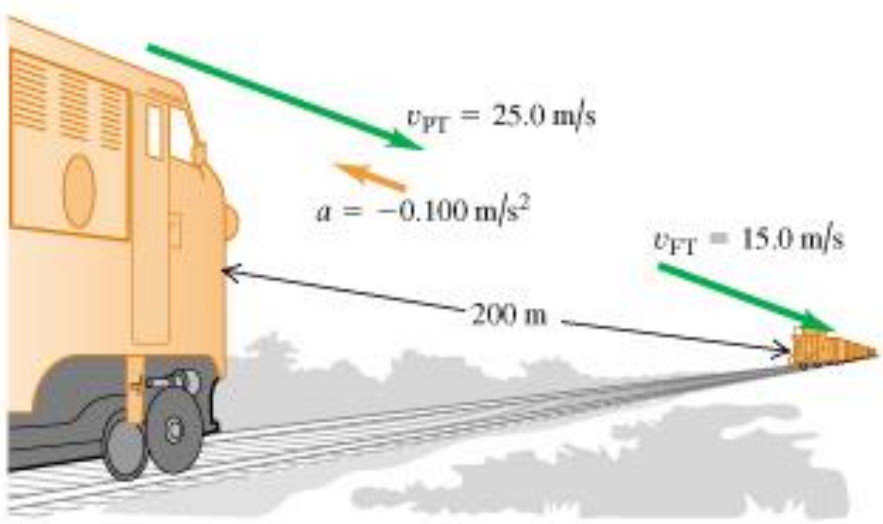
$$t = 0 \qquad \text{or} \qquad t = \frac{2v_{M0x}}{a_{Px}} = \frac{2(15\ \text{m/s})}{3.0\ \text{m/s}^2} = 10\ \text{s}$$

$$v_{Px} = v_{P0x} + a_{Px}t = 0 + (3.0\ \text{m/s}^2)t$$

$$x_M = v_{M0x}t = (15\ \text{m/s})(10\ \text{s}) = 150\ \text{m}$$

Problem 2.66

- The engineer of a passenger train traveling at 25.0m/s sights a freight train whose caboose is 200m ahead on the same track. The freight train is traveling at 15.0m/s in the same direction as the passenger train. The engineer of the passenger train immediately applies the brakes, causing a constant acceleration of -0.100m/s^2, while the freight train continues with constant speed. Take x=0 at the location of the front of the passenger train when the engineer applies the brakes. Is there a collision? If so, where?



Problem 2.83

- Sam heaves a shot with weight 16-lb straight upward, giving it a constant upward acceleration from rest of 46.0m/s^2 for a height 62.0cm. He releases it at height 2.17m above the ground. You may ignore air resistance.
- What is the speed of the shot when he releases it?
- How high above the ground does it go?
- How much time does he have to get out of its way before it returns to the height of the top of his head, a distance 1.84m above the ground?