## Lecture 5

(2-D Motion and Relative Motion)
Physics 160-02 Spring 2017
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## Problem 2.83

- Sam heaves a shot with weight $16-\mathrm{lb}$ straight upward, giving it a constant upward acceleration from rest of $46.0 \mathrm{~m} / \mathrm{s}^{2}$ for a height 62.0 cm . He releases it at height 2.17 m above the ground. You may ignore air resistance.
- What is the speed of the shot when he releases it?
- How high above the ground does it go?
- How much time does he have to get out of its way before it returns to the height of the top of his head, a distance 1.84 m above the ground?


## MasteringPhysics Problem

- As you look out of your dorm window, a flower pot suddenly falls past. The pot is visible for a time $t$, and the vertical length of your window is $L_{w}$. Take down to be the positive direction, so that downward velocities are positive and the acceleration due to gravity is the positive quantity $g$.
- Assume that the flower pot was dropped by someone on the floor above you (rather than thrown downward).

$$
h=\frac{\left(\frac{L_{w}}{t}+\frac{g t}{2}\right)^{2}}{2 g}
$$

## Two- and Three-Dimensional Motion

- For motion in more than one dimension, we need to extend the ideas we developed earlier.
- How do we define a position?


$$
\begin{aligned}
\vec{r}(t) & =r_{x}(t) \hat{i}+r_{y}(t) \hat{j}+r_{z}(t) \hat{k} \\
\vec{r} & =x \hat{i}+y \hat{j}+z \hat{k}
\end{aligned}
$$

## Path and Velocity

- The path, or trajectory of the object is defined as the set of points given by $\vec{r}(t)$.
- The velocity is time derivative of the position function.


The velocity at any point is tangent to the trajectory and is given by:

$$
\begin{aligned}
\vec{v}(t) & =\frac{d \vec{r}}{d t}=\frac{d}{d t}[x(t) \hat{i}+y(t) \hat{j}+z(t) \hat{k}] \\
& =\frac{d x}{d t} \hat{i}+\frac{d y}{d t} \hat{j}+\frac{d z}{d t} \hat{k} \\
& =v_{x} \hat{i}+v_{y} \hat{j}+v_{z} \hat{k}
\end{aligned}
$$

$$
\vec{r}_{1} \equiv \vec{r}\left(t=t_{1}\right)
$$

## Acceleration

- Then, the acceleration is just given by the time derivative of the velocity vector function.

$$
\begin{aligned}
\vec{a}(t) & =\frac{d \vec{v}}{d t}=\frac{d}{d t}\left[v_{x}(t) \hat{i}+v_{y}(t) \hat{j}+v_{z}(t) \hat{k}\right] \\
& =\frac{d v_{x}}{d t} \hat{i}+\frac{d v_{y}}{d t} \hat{j}+\frac{d v_{z}}{d t} \hat{k} \\
& =a_{x} \hat{i}+a_{y} \hat{j}+a_{z} \hat{k}
\end{aligned}
$$

## Examples

- 1-dimensional motion with zero acceleration:
y [m]

- 1-dimensional motion with non-zero acceleration:
y [m]


$$
\begin{array}{ll}
\longrightarrow & \begin{array}{c}
\text { Blue arrow }=\text { Position } \\
\text { Green arrow }=\text { Velocity }
\end{array} \\
\longrightarrow & \text { Red arrow }=\text { Acceleration }
\end{array}
$$

## Examples

- 2-dimensional motion with zero acceleration:

- 2-dimensional motion with non-zero acceleration but y [m] constant speed:




## CPS Question 7-1

A skier moves along a ski-jump ramp as shown in Fig. 3.14a. The ramp is straight from point $A$ to point $C$ and curved from point $C$ onward. The skier picks up speed as she moves downhill from point $A$ to point $E$, where her speed is maximum. She slows down after passing point $E$.

Which vector most closely represents the direction of her acceleration at point B ?


## CPS Question 7-2

A skier moves along a ski-jump ramp as shown in Fig. 3.14a. The ramp is straight from point $A$ to point $C$ and curved from point $C$ onward. The skier picks up speed as she moves downhill from point $A$ to point $E$, where her speed is maximum. She slows down after passing point $E$.

Which vector most closely represents the direction of her acceleration at point D ?


## CPS Question 7-3

A skier moves along a ski-jump ramp as shown in Fig. 3.14a. The ramp is straight from point $A$ to point $C$ and curved from point $C$ onward. The skier picks up speed as she moves downhill from point $A$ to point $E$, where her speed is maximum. She slows down after passing point $E$.

Which vector most closely represents the direction of her acceleration at point F?


## Independence of Components

- It is important to note that in all of this, the components of positions, velocity and acceleration are independent ( $\mathrm{x}, \mathrm{y}$, and z ):

$$
\begin{array}{cl}
\vec{r}(t)=x(t) \hat{i}+y(t) \hat{j}+z(t) \hat{k} \\
v_{x}(t)=\frac{d x(t)}{d t} & a_{x}(t)=\frac{d v_{x}(t)}{d t}=\frac{d^{2} x(t)}{d t^{2}} \\
v_{y}(t)=\frac{d y(t)}{d t} & a_{y}(t)=\frac{d v_{y}(t)}{d t}=\frac{d^{2} y(t)}{d t^{2}} \\
v_{z}(t)=\frac{d z(t)}{d t} & a_{z}(t)=\frac{d v_{z}(t)}{d t}=\frac{d^{2} z(t)}{d t^{2}}
\end{array}
$$

## Example

- Consider two objects:
- One which has zero initial velocity and zero acceleration in the $x$ direction, and with zero initial velocity but constant acceleration in the $y$-direction (free fall).
- The other has an initial velocity (but still zero acceleration) in the $x$ direction, and with zero initial velocity but constant acceleration in the $y$-direction (free fall).



## CPS Demonstration Question Simultaneous Ball Drop

- Simultaneously (at the same time), one ball is dropped straight down, one is projected horizontally. Which ball will hit the floor first?
A) The dropped ball.
B) The projected ball.
C) Both balls hit at the same time.
D) Cannot determine, insufficient information.


## Problem

- A daring 510 N swimmer dives off a cliff with a running horizontal leap, as shown in the figure. What must her minimum speed be just as she leaves the top of the cliff so that she will miss the ledge at the bottom, which is 1.75 m wide and 9.00 m below the top of the cliff?



## Relative Velocity

- For velocities much smaller than the speed of light, our intuition gives the right answer:

$$
\vec{v}_{P / A}=\vec{v}_{P / B}+\vec{v}_{B / A}
$$


https://youtu.be/BLul118nhzc

## Relative Velocity



Inertial Reference

- An inertial reference frame



## Inertial Reference

## - A non-inertial reference frame


(c) The vehicle rounds a turn at constant speed.


You tend to continue moving in a straight line as the vehicle turns.

## Problem

- A canoe has a velocity of $0.33 \mathrm{~m} / \mathrm{s}$ southeast relative to the earth. The canoe is on a river that is flowing $0.46 \mathrm{~m} / \mathrm{s}$ east relative to the earth. Find the magnitude and direction of the velocity of the canoe relative to the river.

