Lecture 6
(Projectile Motion & Circular Motion)

Physics 160-02 Spring 2017

Douglas Fields

CPS Question 8-1

- **For a trajectory of** $\begin{aligned} x &= 1.0m + \left(1.0\,m/s\right)t \\ y &= 2.0m + \left(1.0\,m/s\right)t + \left(1.0\,m/s^2\right)t^2 \end{aligned}$ **, what is the acceleration at time = 3s?**

A) 18.0 m/s^2 in the +y direction

B) 2.0 m/s^2 in the +y direction

C) 9.0 m/s^2 in the +y direction

D) 1.0 m/s^2 in the +y direction

E) 9.8 m/s^2 in the -y direction

Two- and Three-Dimensional Motion

- From last time, we have:

$$\vec{r}(t) = r_x(t)\hat{i} + r_y(t)\hat{j} + r_z(t)\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Projectile Motion

- Projectile motion is motion in two dimensions, where there is (ideally) zero acceleration in one dimension, and non-zero acceleration in the other dimension.
- Near the earth's surface, an object thrown has an acceleration in the dimension pointing towards the center of the earth, and (ignoring air resistance) no acceleration in the dimension parallel to the earth's surface.

$$\vec{r}(t) = r_x(t)\hat{i} + r_y(t)\hat{j} = x\hat{i} + y\hat{j}$$

$$\vec{v}(t) = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a}(t) = -g\hat{j}$$

Projectile Motion

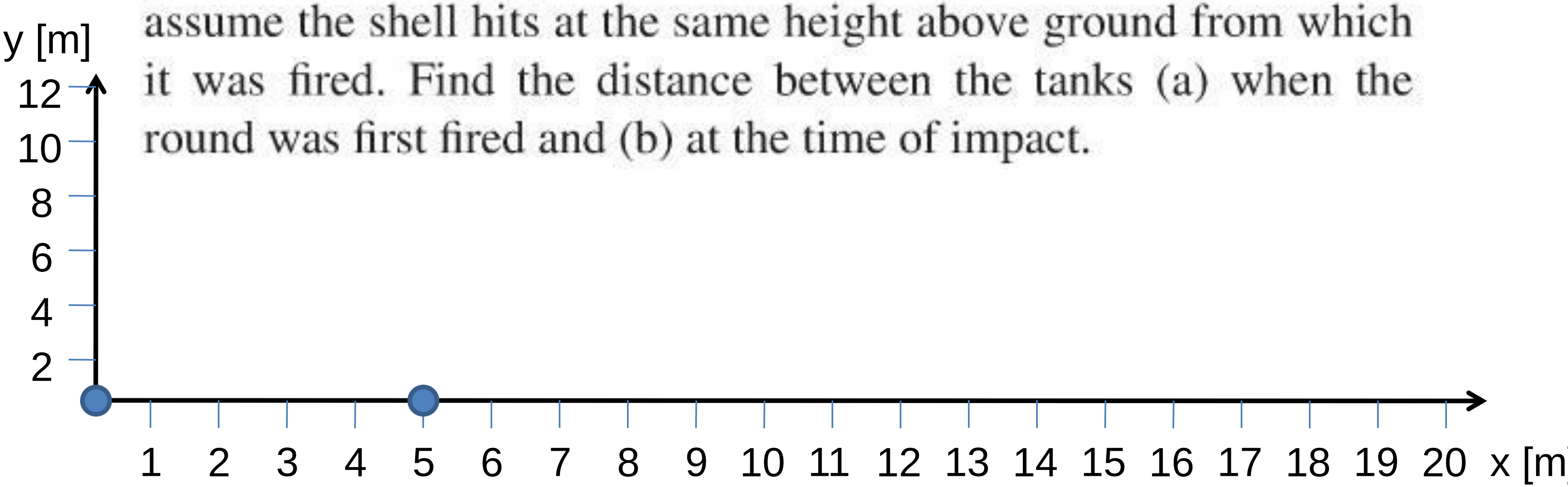
- Then, for projectile motion we have:

$$x_f = x_0 + v_{x0}t$$

$$y_f = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

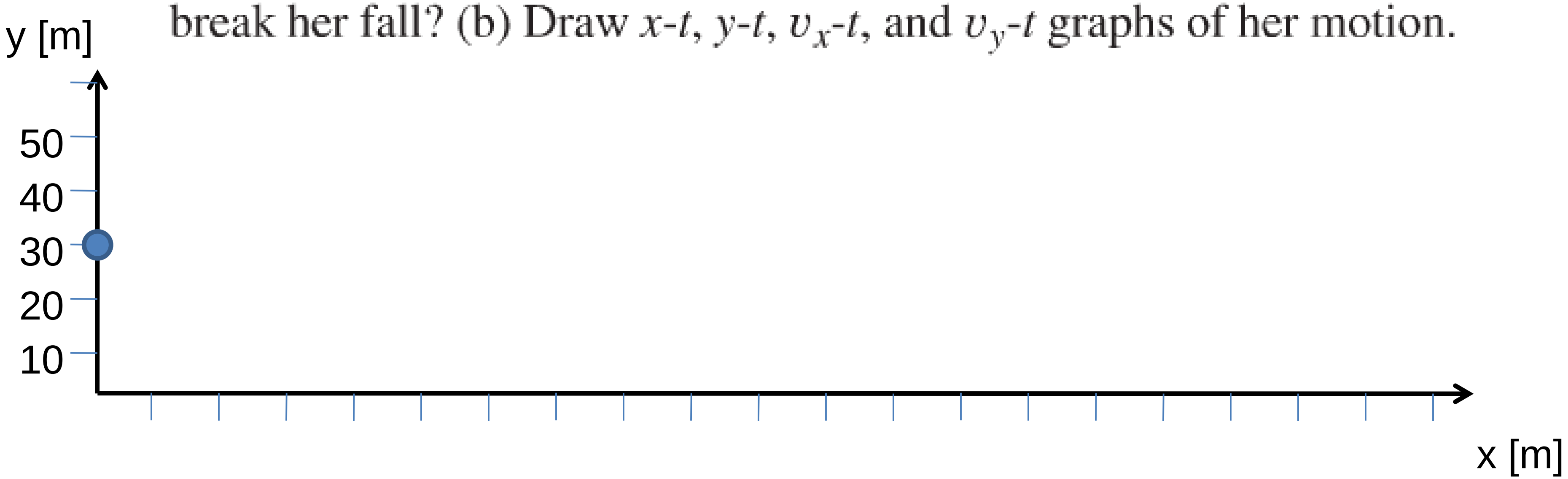
Problem 3.69

3.69. Two tanks are engaged in a training exercise on level ground. The first tank fires a paint-filled training round with a muzzle speed of 250 m/s at 10.0° above the horizontal while advancing toward the second tank with a speed of 15.0 m/s relative to the ground. The second tank is retreating at 35.0 m/s relative to the ground, but is hit by the shell. You can ignore air resistance and assume the shell hits at the same height above ground from which it was fired. Find the distance between the tanks (a) when the round was first fired and (b) at the time of impact.



Problem 3.52

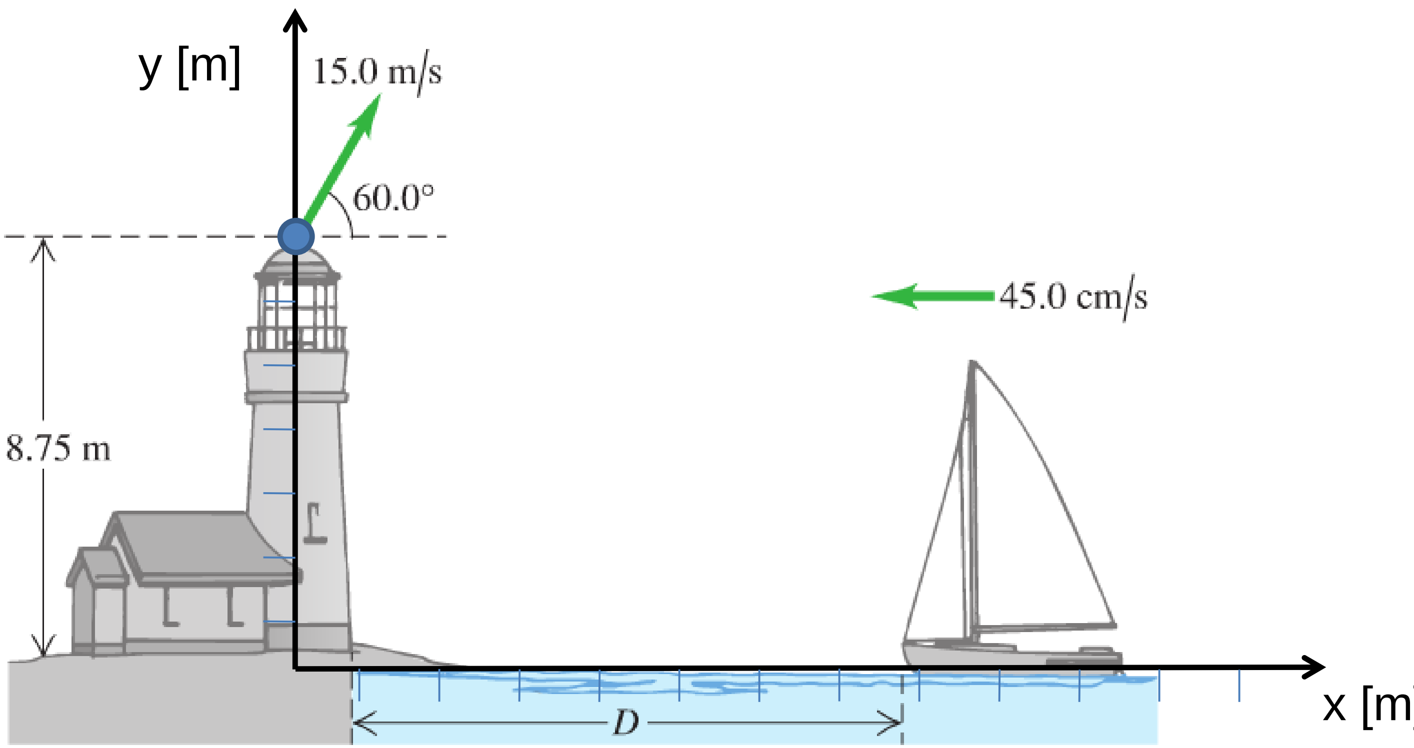
3.52 ••• A movie stuntwoman drops from a helicopter that is 30.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 15.0 m/s horizontal and toward the south. You can ignore air resistance. (a) Where on the ground (relative to the position of the helicopter when she drops) should the stuntwoman have placed the foam mats that break her fall? (b) Draw x-t, y-t, v_x-t, and v_y-t graphs of her motion.



Problem 3.56

3.56 ••• As a ship is approaching the dock at 45.0 cm/s, an important piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck (Fig. P3.56). For this equipment to land at the front of the ship, at what distance *D* from the dock should the ship be when the equipment is thrown? Air resistance can be neglected.

Figure **P3.56**

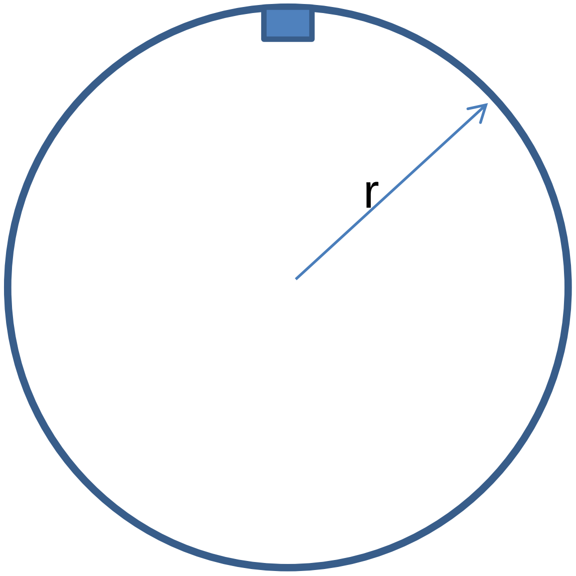


Problem 3.77

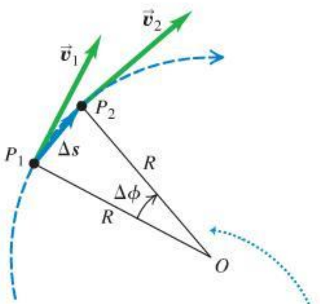
3.77 ••• In an action-adventure film, the hero is supposed to throw a grenade from his car, which is going 90.0 km/h, to his enemy's car, which is going 110 km/h. The enemy's car is 15.8 m in front of the hero's when he lets go of the grenade. If the hero throws the grenade so its initial velocity relative to him is at an angle of 45° above the horizontal, what should the magnitude of the initial velocity be? The cars are both traveling in the same direction on a level road. You can ignore air resistance. Find the magnitude of the velocity both relative to the hero and relative to the earth.

Uniform Circular Motion

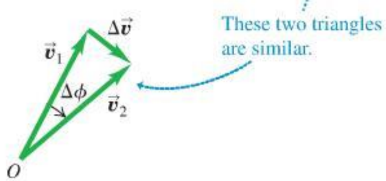
- Period, circumference, velocity…



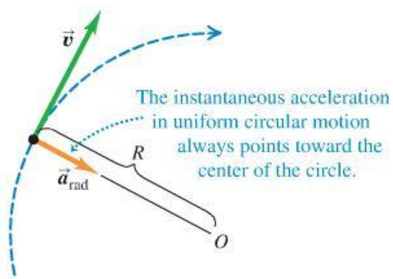
Uniform Circular Motion



(b) The corresponding change in velocity and average acceleration



(c) The instantaneous acceleration



The angles labeled $\Delta\phi$ in Figs. 3.28a and 3.28b are the same because $\vec{v}_1$ is perpendicular to the line OP_1 and $\vec{v}_2$ is perpendicular to the line OP_2. Hence the triangles in Figs. 3.28a and 3.28b are *similar*. The ratios of corresponding sides of similar triangles are equal, so

$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \qquad \text{or} \qquad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$

The magnitude a_{av} of the average acceleration during Δt is therefore

$$a_{\text{av}} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

The magnitude a of the *instantaneous* acceleration $\vec{a}$ at point P_1 is the limit of this expression as we take point P_2 closer and closer to point P_1:

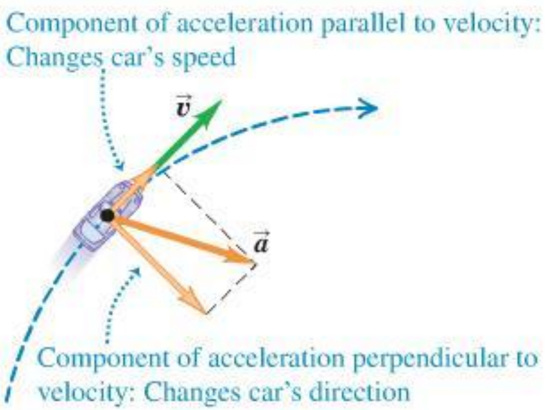
$$a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

But the limit of $\Delta s/\Delta t$ is the speed v_1 at point P_1. Also, P_1 can be any point on the path, so we can drop the subscript and let v represent the speed at any point. Then

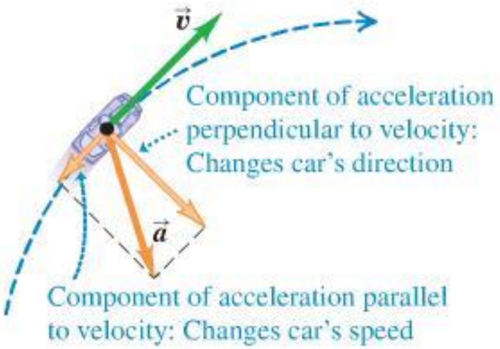
$$a_{\text{rad}} = \frac{v^2}{R} \qquad \text{(uniform circular motion)} \qquad (3.28)$$

Circular Motion

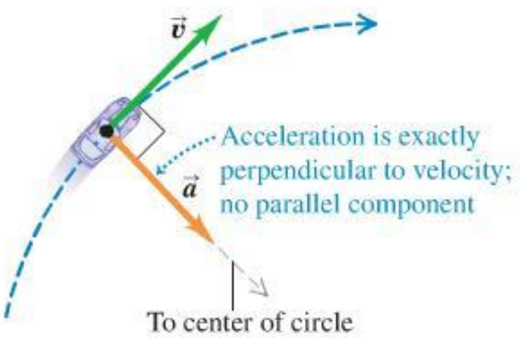
Car speeding up along a circular path



Car slowing down along a circular path



Uniform circular motion: Constant speed along a circular path



CPS Question 9-1

- **A race car going around a circular track is, at some moment slowing down. At that moment, his acceleration vector points**

  A) In his direction of motion.

  B) In the opposite direction of his motion.

  C) Towards the center of the track.

  D) Both towards the center of the track and opposite of his direction of motion.

  E) Not enough information.