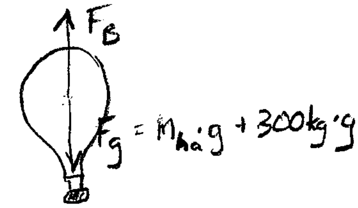
18.1 Recall that the buoyant force is equal to the weight of the fluid displaced:



$$F_B = (\rho_{air} V) \cdot g$$

Now, from a Newton's Law analysis;

$$F_B - m_{hot\,air} \cdot g - (300kg) \cdot g = 0$$

$$\rho_{air} \cdot V \cdot \cancel{g} - \rho_{ha} \cdot V \cdot \cancel{g} - 300kg \cdot \cancel{g} = 0 \Rightarrow$$

$$\rho_{ha} = \rho_{air} - \frac{300\,kg}{V}$$

$$= 1.2\,kg/m^3 - 0.5\,kg/m^3$$

$$= 0.7\,kg/m^3$$

Now, use the ideal gas law to find the temperature that air must have to have that density:

$$p_1 V_1 = n_1 R T_1 \quad \text{at } 20°C = 293\,K$$

But remember that the hot air is still at atmospheric pressure! (The balloon is open at the bottom) So,

$$pV = n_1 R T_1 = n_2 R T_2$$

The density can be written as:

$$\rho_{air} = \frac{n_1 M_{air}}{V} = \frac{pM}{RT_{air}} \Rightarrow$$

$$\rho_{air} \cdot T_{air} = \rho_{ha} \cdot T_{ha} \Rightarrow$$

$$T_{ha} = (293\,K)\left(\frac{1.2\,kg/m^3}{0.7\,kg/m^3}\right) = \boxed{502\,K}$$

18.2

Just out of curiosity, let's calculate how many grams of ethane are originally in the flask:

$$pV = nRT \Rightarrow n = \frac{pV}{RT} = \frac{(1.013\times10^5\ Pa)(3.1\times10^{-3}\ m^3)}{(8.31\ J/mol\cdot K)(305\ K)}$$

$$n = 0.124\ mol \Rightarrow m = nM = (0.124\ mol)(30.1\ g/mol) = 3.73\ g$$

Now, with the stop cock open, the pressure and volume stay the same, but the number of moles (<u>inside</u> the flask!) changes:

$$n_2 = \frac{pV}{RT_2} = \frac{(1.013\times10^5\ Pa)(3.1\times10^{-3}\ m^3)}{(8.31\ J/mol\cdot k)(395\ k)}$$

$$= 0.096\ mol$$

$$\therefore\ m = nM = (0.096\ mol)(30.1\ g/mol) = \boxed{2.89\ g}$$

We can now calculate the new pressure after cooling:

$$p = \frac{nRT}{V} = \frac{(0.096\ mol)(8.31\ J/mol\cdot k)(305\ K)}{3.1\times10^{-3}\ m^3}$$

$$\boxed{p = 0.78\times10^5\ Pa}$$

18.3

For each degree of freedom (d.o.f) each molecule has $\frac{1}{2}kT$ of energy:

$$E = \frac{1}{2}kT/\text{d.o.f.}/\text{molecule}$$

For rotation, there are 2 d.o.f, and there are N_A molecules/mol.

$$\therefore E_{tot.rot.} = 2 \cdot \frac{1}{2}kT \cdot N_A \cdot n$$
$$= n \cdot (k \cdot N_A) \cdot T$$
$$= 1\,\text{mol} \cdot 8.31\,\text{J/mol}\cdot\text{K} \cdot 300\,\text{K}$$
$$= 2{,}500\,\text{J}$$

Now, the moment of inertia, $I = \Sigma m r^2$

$$= m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2$$
$$= 2m\frac{d^2}{4} = \frac{1}{2}md^2$$

for oxygen,

$$m_O = \frac{M}{N_A} = \frac{16\,\text{g/mol}}{6.02\times10^{23}\,\frac{\text{atom}}{\text{mol}}} = 2.66\times10^{-26}\,\text{kg}$$

$$\therefore I = \frac{1}{2}(2.66\times10^{-26}\,\text{kg})(1.21\times10^{-10}\,\text{m})^2 = 1.94\times10^{-46}\,\text{kg}\,\text{m}^2$$

The rotational kinetic energy is given by:

$$K_{rot} = \frac{1}{2}I\omega^2 = \frac{2500\,\text{J}}{6.02\times10^{23}\,\text{atoms}}$$

$$\therefore \omega^2 = \frac{2 \cdot 2500\,\text{J}}{(6.02\times10^{23}\,\text{atoms})(1.94\times10^{-46}\,\text{kg}\,\text{m}^2)}$$

$$= 4.26\times10^{25}\,\text{s}^{-2}$$

$$\text{so } \omega_{rms} = \sqrt{\omega^2} = \boxed{6.53\times10^{12}\,\text{s}^{-1}}$$