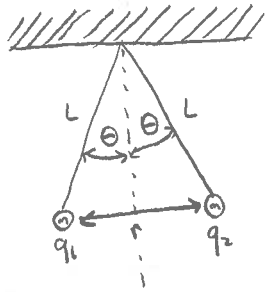
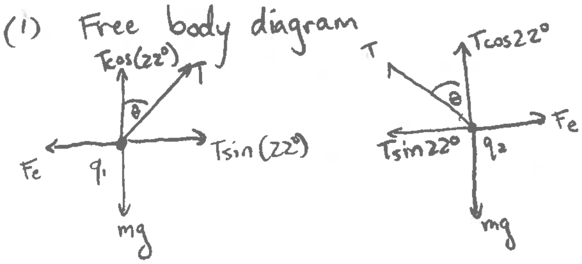
HW#5 Solutions

21.1



$L = 0.5\,m$
$m = 10\,g$
$\theta = 22°$

(1) Free body diagram



- where T is the tension of the string, F_e is the electric force, mg is the force due to gravity

(2) Let's start w/ mg

$$\boxed{mg = 0.098\ N}$$

$$mg = T\cos 22 \implies T = \frac{mg}{\cos 22} = \frac{0.098}{0.927} = \boxed{0.106\ N}$$

$$T\sin 22 = (.106)\sin 22 = \boxed{0.04\ N}$$

$$T\sin 22 = F_e = \frac{k q_1 q_2}{r_1^2}$$ where r is the distance between the spheres

we need to find r_1:

$$r_1 = 2L\sin 22 = \boxed{0.375\,m}$$

(3) from r so we can now compute $q_1 q_2$

$$F_e = \frac{k q_1 q_2}{r_1^2} \rightarrow q_1 q_2 = \frac{F_e r^2}{k} = \frac{(.04)(.375)^2}{9\times 10^9} = \boxed{6.25\times 10^{-13}\ C^2}$$

(4) Now we connect the wire between q_1 + q_2 $\implies$ $q_1 = q_2$ but $r_1 \rightarrow r_2$ because θ changes to 32

$$F_e = \frac{kQ^2}{r_2^2} \quad \text{where } Q = \frac{q_1 + q_2}{2} \qquad r_2 = 2(.5)\sin 32 = \underline{.55\,m}$$

$$F_e = T\sin 32 = \frac{mg}{\cos 32}\sin 32 = mg\tan 32 = \underline{0.065\ N}$$

$Q^2 = q_1 q_2 =$

$\implies$ over

$$Q = \frac{q_1 + q_2}{2} = \sqrt{\frac{F_e r^2}{k}} = \underline{1.47 \times 10^{-6}\ C}$$

from (3) $q_1 q_2 = 6.25 \times 10^{-13}\ C^2$

$$q_1 = 2.95 \times 10^{-6} - q_2 \longrightarrow (2.95 \times 10^{-6} - q_2) q_2 = 6.25 \times 10^{-13}$$

$$2.95 \times 10^{-6} q_2 - q_2^2 = 6.25 \times 10^{-13}$$

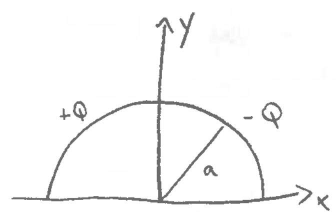
solving the quadratic gives

$$\boxed{q_2 = 2.7 \times 10^{-6}\ C}$$

plugging in above $\Longrightarrow$ $q_1 = 2.95 \times 10^{-6} - 2.7 \times 10^{-6}$

$$\boxed{= 2.5 \times 10^{-7}\ C}$$

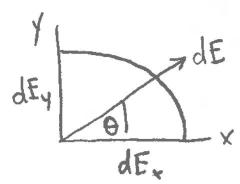
#21.2



We must find the x & y components of the electric field from each half.

$dE_x = dE \cos\theta$

$dE_y = dE \sin\theta$



$$dE = \frac{1}{4\pi\epsilon_0} \frac{|dQ|}{a^2}$$

but $dQ = \left(\frac{a\,d\theta}{\frac{1}{2}\pi a}\right) Q = \frac{2Q}{\pi} d\theta$

where $a\,d\theta$ is a small segment of charge Q

$$\Longrightarrow dE = \frac{1}{4\pi\epsilon_0 a^2} \frac{2Q}{\pi} d\theta = \frac{Q}{2\pi^2 a^2 \epsilon_0} d\theta$$

$$E_x = \int dE_x = \int dE \cos\theta = \frac{Q}{2\pi^2\epsilon_0 a^2} \int_a^b \cos\theta\, d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2} \sin\theta \Big|_a^b$$

(where b + a are angles of θ)

$$E_y = \int dE_y = \int dE \sin\theta = \frac{Q}{2\pi^2\epsilon_0 a^2} \int_a^b \sin\theta\, d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2} (-\cos\theta) \Big|_a^b$$

for the +Q side: $E_x = \frac{Q}{2\pi^2\epsilon_0 a^2} \sin\theta \Big|_{\pi/2}^{\pi} = \frac{-Q}{2\pi^2\epsilon_0 a^2}$

$$\frac{Q}{2\pi^2\epsilon_0 a^2} \underbrace{\sin(\pi)}_{0} - \frac{Q}{2\pi^2\epsilon_0 a^2} \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} =$$

$E_y = \frac{Q}{2\pi^2\epsilon_0 a^2} \cos\theta \Big|_{\pi/2}^{\pi} = -\frac{Q}{2\pi^2\epsilon_0 a^2}$

$\cos(\pi) - \cos\left(\frac{\pi}{2}\right)$
$-1 \quad - 0$

$\Longrightarrow$ (more)

for the $-Q$ side:

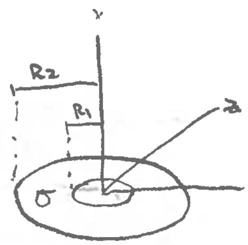
$$E_x = \frac{-Q}{2\pi^2\epsilon_0 a^2}\sin\theta\Big|_0^{\pi/2} = \frac{-Q}{2\pi^2\epsilon_0 a^2}$$

$$E_y = \frac{-Q}{2\pi^2\epsilon_0 a^2}\cos\theta\Big|_0^{\pi/2} = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$\sum E_x = -\frac{Q}{2\pi^2\epsilon_0 a^2} - \frac{Q}{2\pi^2\epsilon_0 a^2} = \frac{-\cancel{2}Q}{\cancel{2}\pi^2\epsilon_0 a^2} = \boxed{\frac{-Q}{\pi^2\epsilon_0 a^2}}$$

$$\sum E_y = -\frac{Q}{2\pi^2\epsilon_0 a^2} + \frac{Q}{2\pi^2\epsilon_0 a^2} = 0$$

#21.3



(1) total charge Q is:

$$Q = A\sigma$$

where σ is the charge density of the disk.

A is the area of the annulus which is

$$A = \pi r^2 = \pi(R_2^2 - R_1^2)$$

$$\Longrightarrow \boxed{Q = \pi\sigma(R_2^2 - R_1^2)}$$

(2) according to Eq. 21.11:

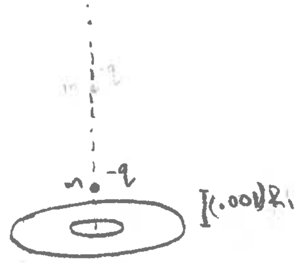
$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{\left(\frac{R}{x}\right)^2+1}}\right] \quad \text{for a disk}$$

$$\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left[\left(1 - \frac{1}{\sqrt{\left(\frac{R_2}{x}\right)^2+1}}\right) - \left(1 - \frac{1}{\sqrt{\left(\frac{R_1}{x}\right)^2+1}}\right)\right]\frac{|x|}{x}\hat{i}$$

$$= \frac{\sigma}{2\epsilon_0}\left[\frac{1}{\sqrt{\left(\frac{R_1}{x}\right)^2+1}} - \frac{1}{\sqrt{\left(\frac{R_2}{x}\right)^2+1}}\right]\frac{|x|}{x}\hat{i}$$

which means the E-field is in the $\hat{x}$ direction above and below the disk. ~~but we just want points above the annulus so~~

(3)



first we note that

$$\frac{1}{\sqrt{\left(\frac{R_1}{x}\right)^2+1}} = \frac{1}{\frac{R_1}{|x|}\sqrt{1+\left(\frac{x}{R_1}\right)^2}}$$ by pulling $\left(\frac{R_1}{x}\right)^2$ out of the radicle

$$\Rightarrow \frac{|x|}{R_1}\left[1+\left(\frac{x}{R_1}\right)^2\right]^{-1/2}$$ when $|x| \ll R_1$ then $\frac{x}{R_1} \ll 1$

so $$\frac{|x|}{R_1}\left[1+\left(\frac{x}{R_1}\right)^2\right]^{-1/2} \to \frac{|x|}{R_1}$$

so $$\vec{E}(x) = \frac{\sigma}{2\epsilon_0}\left[\frac{1}{\sqrt{\left(\frac{R_1}{x}\right)^2+1}} - \frac{1}{\sqrt{\left(\frac{R_2}{x}\right)^2+1}}\right]$$

$$= \frac{\sigma}{2\epsilon_0}\left[\frac{|x|}{R_1} - \frac{|x|}{R_2}\right]\frac{|x|}{x}\hat{i} = \frac{\sigma}{2\epsilon_0}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\frac{|x|^2}{x}\hat{i} = \frac{\sigma x}{2\epsilon_0}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\hat{i}$$

the force due to $\vec{E}(x)$ in the $\hat{x}$ direction is

$$F_x = -qE_x = -\underbrace{\frac{q\sigma}{2\epsilon_0}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]}_{\text{call this } k} x = -kx$$

the frequency of a spring is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and substituting for k above

so $$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\epsilon_0 m}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}}$$