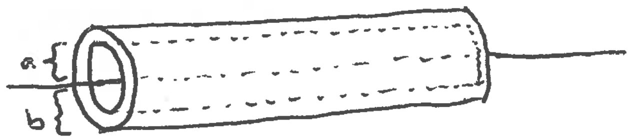
HW #6 Solutions

22.1



Gauss' law: $\oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$

for cylinder $\oint E \cdot dA = E\, 2\pi r L$

where L is the length of the Gaussian surface.

ⓐ <u>for $r<a$</u>: $E(2\pi r \ell) = \frac{Q_{enc}}{\epsilon_0}$ where $Q_{enc} = \lambda \ell$ $\quad \ell$ is length of cylinder

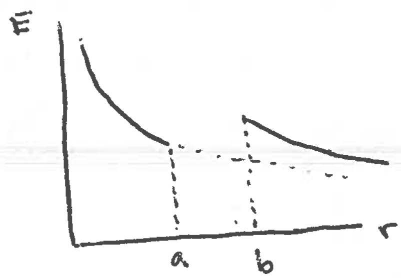
$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0} \rightarrow \boxed{E = \frac{\lambda}{2\pi r \epsilon_0}}$$

<u>for $a<r<b$</u>: $Q_{enc} = 0$ because surface is inside conductor $\Longrightarrow \boxed{E=0}$

<u>for $r>b$</u>: $Q_{enc} = 2\lambda \ell$ because line charge $= \lambda$ + cylinder $= \lambda$

$$\Longrightarrow \boxed{E = \frac{\lambda}{\pi r \epsilon_0}}$$

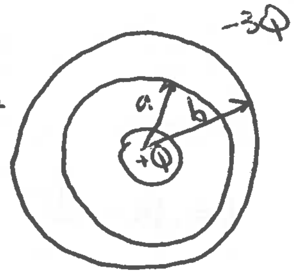
ⓑ



ⓒ <u>inner surface charge</u>: since the line carries charge λ and from $a<r<b$ the net charge is $\emptyset$, the inner surface must carry charge $\boxed{-\lambda}$

<u>outer surface</u>: since net charge on tube is λ & charge on inside is $-\lambda$ the outside must carry charge of $\boxed{2\lambda}$

22.2



Gauss' Law for sphere: $E = \frac{1}{4\pi\epsilon_0} \frac{Q_{enc}}{r^2}$

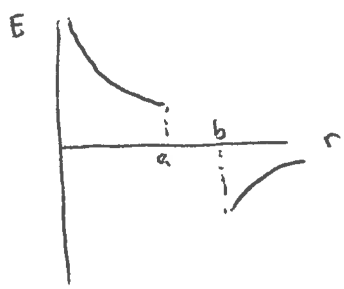
(a) for $r<a$: $\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}}$

for $a<r<b$: $\boxed{E=0}$ because $Q_{enc}=0$ ~~for~~ inside conducting material

for $r>b$: $Q_{enc} = Q - 3Q = -2Q$

$\Longrightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\left(\frac{-2Q}{r^2}\right)}$

(b)



(c) inside shell: for $a<r<b$ there must be 0 net charge so the charge on the inner shell is $-Q$.

$\Longrightarrow \boxed{\sigma = -\frac{Q}{4\pi a^2}}$

outside shell: for $r>b$ the charge is $-3Q$, but there is $-Q$ on the inner surface, so the outer surface must be $-2Q$.

$\Longrightarrow \boxed{\sigma = -\frac{2Q}{4\pi b^2}}$

$\boxed{22.3}$

$$\rho(r) = \begin{cases} \rho_0\left[1-\frac{4r}{3R}\right] & \text{for} \quad r \le R \\ 0 & \text{for} \quad r > R \end{cases}$$

(a) $Q = \int \rho(r)\,dV$ our dV is a sphere so $\rightarrow$ $dV = 4\pi r^2 dr$

$$\Rightarrow Q = 4\pi\int_0^\infty \rho(r)\,r^2 dr = 4\pi\rho_0\int_0^R\left(1-\frac{4r}{3R}\right)r^2 dr$$

$$= 4\pi\rho_0\left[\int_0^R r^2 dr - \int_0^R \frac{4r^3}{3R}dr\right]$$

$$= 4\pi\rho_0\left[\frac{r^3}{3}\Big|_0^R - \frac{4r^4}{12R}\Big|_0^R\right]$$

$$= 4\pi\rho_0\left[\frac{R^3}{3} - \frac{R^3}{3}\right] = \boxed{0}$$

(b) $\underline{r>R}$: $\oint E\cdot dA = \frac{Q_{enc}}{\epsilon_0} = 0$ because $Q_{enc} = 0$ so $\boxed{E=0}$

$\underline{r<R}$: $\oint E\cdot dA = E\cdot 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \frac{4\pi}{\epsilon_0}\int_0^r \rho(r')\,r'^2 dr'$

$$= \frac{4\pi\rho_0}{\epsilon_0}\left[\int_0^r r'^2 dr' - \frac{4}{3R}\int_0^r r'^3 dr'\right]$$

$$E\cdot \cancel{4\pi} r^{\cancel{2}} = \frac{4\cancel{\pi}\rho_0}{\epsilon_0}\left[\frac{r^{\cancel{3}}}{3} - \frac{r^{\cancel{4}2}}{3R}\right]$$

$$\Rightarrow \boxed{E = \frac{\rho_0}{3\epsilon_0}r\left[1-\frac{r}{R}\right]}$$

(c) @ E_{max} $\frac{dE}{dr} = 0 \rightarrow \frac{dE}{dr} = \frac{\rho_0}{3\epsilon_0} - \frac{2\rho_0 r}{3\epsilon_0 R} = 0 \Rightarrow \frac{\cancel{\rho_0}}{\cancel{3\epsilon_0}} = \frac{2\cancel{\rho_0} r}{\cancel{3\epsilon_0} R}$

$\Rightarrow \boxed{r = \frac{R}{2}}$

$$\Rightarrow E = \frac{\rho_0}{3\epsilon_0}\frac{R}{2}\underbrace{\left[1-\frac{\cancel{R}/2}{\cancel{R}}\right]}_{=1/2} = \boxed{\frac{\rho_0 R}{12\epsilon_0}}$$