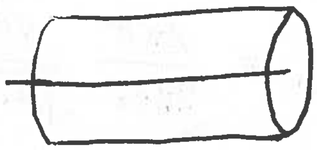
HW#7 Solutions

23.1



$R_c = 1.8\text{ cm} = 0.018\text{m}$

$r_w = 1.45\times10^{-6}\text{ m}$ $\qquad r = 0.012\text{ m}$

$E = 2\times10^4\text{ V/m}$

From Example 23.10 $\quad V = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_0}{r}\right)$ where r_0 is the distance when $V=0$

since we are probing at $r = 0.012\text{m}$ we are between the two "cylinders"

$$\Rightarrow V = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R_c}{r}\right) \qquad \text{when} \quad r = r_w \rightarrow V_w = \frac{\lambda}{2\pi\epsilon_0}\left(\frac{R_c}{r_w}\right)$$

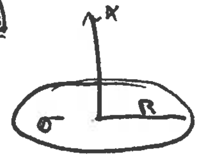
$$r = R_c \rightarrow V_c = 0$$

$$\text{so} \quad V_{wc} = V_w - V_c = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{R_c}{r_w}\right)$$

$$E = -\frac{\partial V}{\partial r} = -\frac{\lambda}{2\pi\epsilon_0}\frac{\partial}{\partial r}\ln\left(\frac{R_c}{r}\right) = +\frac{\lambda}{2\pi\epsilon_0}\left(\frac{\cancel{r}}{\cancel{R_c}}\right)\left(-\frac{R_c}{r\cancel{r}}\right) = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{r} = \underline{\frac{V_{wc}}{\ln(R_c/r_w)}\frac{1}{r}}$$

$$\Rightarrow V_{wc} = E\,r\ln\left(\frac{R_c}{r_w}\right) = (2\times10^4)(0.012)\ln\left(\frac{0.018}{1.45\times10^{-6}}\right) = \boxed{1157\text{ V}}$$

$\boxed{23.2}$



For a thin ring of radius y with width dy

$$A = 2\pi y\,dy \quad \& \quad dq = \sigma(2\pi y\,dy)$$

From Example 23.11:

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{\cancel{2\pi}\sigma}{\cancel{4\pi}_{2}\epsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}$$

$$= \frac{\sigma}{2\epsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}}$$

using u subst. $u = \sqrt{x^2+y^2}$

$$\frac{du}{dy} = \cancel{\tfrac{1}{2}}\left(x^2+y^2\right)^{-1/2}\cancel{2}y$$

$$du\sqrt{x^2+y^2} = y\,dy$$

$$= \frac{\sigma}{2\epsilon_0}\int_0^R du = \frac{\sigma}{2\epsilon_0}\,u\Big|_0^R$$

$$= \frac{\sigma}{2\epsilon_0}\sqrt{x^2+y^2}\Big|_0^R = \boxed{\frac{\sigma}{2\epsilon_0}\left(\sqrt{x^2+R^2} - x\right)}$$

$$E = -\frac{\partial V}{\partial x} = -\frac{\sigma}{2\epsilon_0}\frac{\partial}{\partial x}\left(\sqrt{x^2+R^2} - x\right) = -\frac{\sigma}{2\epsilon_0}\left(\frac{x}{\sqrt{x^2+R^2}} - 1\right)$$

$$= \boxed{\frac{\sigma x}{2\epsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)}$$

$\boxed{23.3}$

$$F = \frac{kq_1q_2}{r^2} \qquad F = -\frac{dU}{dr} \Rightarrow U = \frac{kq_1q_2}{r_i}$$

$r_i = \left(\frac{d_1}{2}\right) + \left(\frac{d_2}{2}\right) = 0.52\,m$

$m_1 = 0.053\,kg \qquad q_1 = -1.0\times10^{-5}\,C$

$m_2 = 0.16\,kg \qquad q_2 = -3.4\times10^{-5}\,C$

<u>finding max velocity</u>:

Using conservation of energy $\rightarrow$ $K_i + U_i = K_f + U_f$

at $t=0$ spheres are not moving $\rightarrow K_i = 0$

much later the spheres are far apart $(r=\infty) \rightarrow U_f = \frac{kq_1q_2}{r} = 0$

$$\Rightarrow \frac{kq_1q_2}{r_i} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

conservation of momentum $\Rightarrow$ $m_1v_1 = m_2v_2$

$(.053)v_1 = (.16)v_2 \Rightarrow v_1 = 3.02\,v_2$

$$\Rightarrow 5.88 = \frac{1}{2}(.053)v_1^2 + \frac{1}{2}(.16)v_2^2$$

$$= .027(3.02\,v_2)^2 + .08\,v_2^2$$

$$= .326\,v_2^2 \Rightarrow v_2 = \boxed{4.25\ m/s}$$

$$v_1 = -3.02(4.25) = \boxed{12.8\ m/s}$$

<u>Max accel</u>:

$$\frac{kq_1q_2}{r_i^2} = ma \qquad m_1a_1 = m_2a_2$$

$$a_1 = 3.02\,a_2$$

$\downarrow$

$$11.3 = m_1a_1 = .053\,a_1 \Rightarrow \boxed{a_1 = 213.5\ m/s^2}$$

$$a_1 = 3.02\,a_2$$

$$\Rightarrow \boxed{a_2 = 70.7\ m/s^2}$$