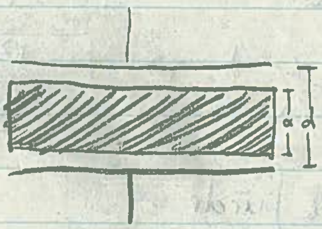
# HW # 8 Solutions

**24.1**




This is like having 2 capacitors ($C_1$) in series, with separation $\frac{1}{2}(d-a)$

a.) $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_1} = \frac{2}{C_1} \Longrightarrow C = \frac{1}{2}C_1$$

$$= \frac{1}{2}\frac{\epsilon_0 A}{(d-a)/2} = \boxed{\frac{\epsilon_0 A}{d-a}}$$

b.) $$C = \frac{\epsilon_0 A}{d-a} = \underbrace{\frac{\epsilon_0 A}{d}}_{C_0} \frac{d}{d-a} = \boxed{C_0 \frac{d}{d-a}}$$

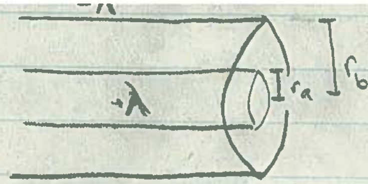
c.) when $a \to 0$: $C = C_0 \frac{d}{d-\cancel{a}\,0} = C_0$

so the metal slab has no effect

when $a \to d$: $C = C_0 \frac{d}{d-a}$ $\quad d-a \to 0$

$$\Longrightarrow C \to \infty$$

24.2




for $r < r_a$ $E = 0$

$r_a < r < r_b$ $E = \frac{\lambda}{2\pi\epsilon_0 r}$

$r > r_b$ $E = 0$

a.) $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \left(\frac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \boxed{\frac{\lambda^2}{8\pi^2\epsilon_0 r^2}}$

b.) $U = \int u\,dV = 2\pi L \int u r\,dr$

$$= 2\pi L \int_{r_a}^{r_b} \frac{\lambda^2}{8\pi^2\epsilon_0 r^2} r\,dr$$

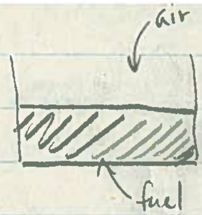
$$= \frac{L\lambda^2}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{1}{r}\,dr = \frac{L\lambda^2}{4\pi\epsilon_0}\ln\left(\frac{r_b}{r_a}\right)$$

$$\Longrightarrow \boxed{\frac{U}{L} = \frac{\lambda^2}{4\pi\epsilon_0}\ln\left(\frac{r_b}{r_a}\right)}$$

c.) $U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi\epsilon_0 L}\ln\left(\frac{r_b}{r_a}\right) = \frac{\lambda^2 L}{4\pi\epsilon_0}\ln\left(\frac{r_b}{r_a}\right)$

$$\Longrightarrow \boxed{\frac{U}{L} = \frac{\lambda^2}{4\pi\epsilon_0}\ln\left(\frac{r_b}{r_a}\right)}$$

24.3




This system is like two capacitors in series: $C_1$: with plate separation $d$, plate area $w(L-h)$, air

$C_2$: plate separation $d$, area $wh$, $K$

$$C_{eq} = \frac{K_{eff}\,\epsilon_0 A}{d} \qquad A = wL$$

$$C_{eq} = C_1 + C_2$$

$$C_1 = \frac{\epsilon_0 w(L-h)}{d}$$

$$C_2 = \frac{\epsilon_0 w h K}{d}$$

$$C_{eq} = \frac{\epsilon_0 w(L-h)}{d} + \frac{\epsilon_0 w h K}{d}$$

$$= \frac{\epsilon_0 wL}{d}\underbrace{\left[1 - \frac{h}{L} + \frac{hK}{L}\right]}_{K_{eff}}$$

$$\Longrightarrow \boxed{K_{eff} = 1 + \frac{Kh}{L} - \frac{h}{L}}$$