HW#11 Solutions

27.1

a.) $\vec{F} = q\vec{v} \times \vec{B}$

$\vec{v} = -v\hat{j} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{F} = q\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -v & 0 \\ B_x & B_y & B_z \end{vmatrix} = (-qvB_z - 0)\hat{i} - (0-0)\hat{j} + (0 + qvB_x)\hat{k}$$

$$= \boxed{qvB_x\hat{k} - qvB_z\hat{i}}$$

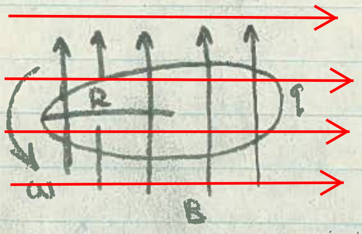
b.) $q<0 \qquad B_x = B_y = B_z > 0$

$$\vec{F} = |q|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & v & 0 \\ B_x & B_x & B_x \end{vmatrix} = \boxed{|q|vB_x\hat{i} - |q|vB_x\hat{k}}$$

$$|\vec{F}| = \sqrt{(|q|vB_x)^2 + (|q|vB_x)^2} = \sqrt{2(|q|vB_x)^2}$$

$$= \boxed{\sqrt{2}\,|q|vB_x} \quad =$$

27.2



$$I = \frac{q}{T} = qf = \frac{q\omega}{2\pi}$$

$\tau = \mu B \sin\theta$ $\quad \theta = 90$ so $\sin\theta = 1$

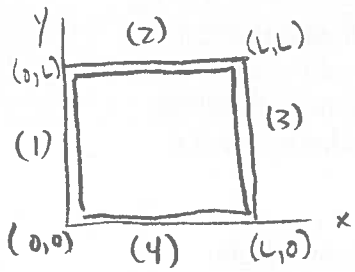
$\mu = IA$

$\Longrightarrow \tau = IAB$

$$= \left(\frac{q\omega}{2\pi}\right)\pi r^2 B$$

$$\boxed{= \frac{q\omega}{2} r^2 B}$$

27.3



$$\vec{B} = \frac{B_0 x}{L}\hat{i} + \frac{B_0 y}{L}\hat{j}$$

a) $F = \int I\,d\vec{l} \times \vec{B}$

(1) $F_1 = \frac{IB_0}{L}\int_0^L dy\,\hat{j} \times (\cancel{x}^{\,0}\hat{i} + y\hat{j}) = 0$

(2) $F_2 = \frac{IB_0}{L}\int_0^L dx\,\hat{i} \times (x\hat{i} + \cancel{y}^{\,L}\hat{j}) = \frac{IB_0}{L} L\, x\Big|_0^L \hat{k} = IB_0 L\hat{k}$

(3) $F_3 = \frac{IB_0}{L}\int_L^0 -dy\,\hat{j} \times (\cancel{x}^{\,L}\hat{i} + y\hat{j}) = -\frac{IB_0}{L} L\, y\Big|_0^L \hat{k} = -IB_0 L\hat{k}$

(4) $F_4 = \frac{IB_0}{L}\int_L^0 -dx\,\hat{i} \times (x\hat{i} + \cancel{y}^{\,0}\hat{j}) = 0$

b) $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}_1 = \vec{\tau}_4 = 0$

$\vec{\tau}_2 = L\hat{j} \times IB_0 L\hat{k} = IB_0 L^2\hat{i}$

$\vec{\tau}_3 = \int_L^0 r \times dF = -\int_0^L y\hat{j} \times -IB_0\,dy\,\hat{k} = +\frac{IB_0}{2} y^2\Big|_0^L \hat{i} = +\frac{1}{2} IB_0 L^2 \hat{i}$

$\vec{\tau}_{net} = IB_0 L^2\hat{i} + \frac{1}{2} IB_0 L^2\hat{i} = \boxed{\frac{3}{2} IB_0 L^2\hat{i}}$

c.) $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}_1 = \vec{\tau}_4 = 0$

$\vec{\tau}_2 = \int r \times dF = \int_0^L x\hat{i} \times IB_0\,dx\,\hat{k} = -\frac{1}{2} IB_0 x^2\Big|_0^L \hat{j} = -\frac{1}{2} IB_0 L^2\hat{j}$

$\vec{\tau}_3 = L\hat{i} \times -IB_0 L\hat{k} = IB_0 L^2\hat{j}$

$\vec{\tau}_{net} = -\frac{1}{2} IB_0 L^2\hat{j} + IB_0 L^2\hat{j} = \boxed{\frac{1}{2} IB_0 L^2\hat{j}}$