Physics 2310-001 Fall 2019 Exam #2

Name: Solutions **SHOW ALL WORK!**

There exists a plane electromagnetic wave in vacuum whose electric field is totally described by:
$\vec{E} = -(100\,V/m)\hat{\imath}\sin\left[ky + (3.040\times10^{15}s^{-1})t\right]$

1) What is the wavelength λ of the wave?

$\omega = 3.040\times10^{15}s^{-1}$ + $c = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \Rightarrow$

$$\lambda = \frac{2\pi c}{\omega} = \frac{2\cdot\pi\cdot3\times10^{8}\,m/s}{3.040\times10^{15}s^{-1}} = \boxed{620\,nm}$$

2) What is the direction of travel and the polarization direction of the wave?

Since the phase changes with ky, and also with $+\omega t$, then the wave direction is in the $-y$ direction.

The E-field is in the $\pm\hat{\imath}$ direction, so the polarization is along the x-axis.

3) What is the magnitude and direction of the magnetic field, $\vec{B}$, at the point (1mm, 0,0) at t=0?

We know that the amplitude of the B-field is related to the amplitude of the E-field as: $B_0 = \frac{E_0}{c} = \frac{100\,V/m}{3\times10^{8}\,m/s} = 3.33\times10^{-7}T$

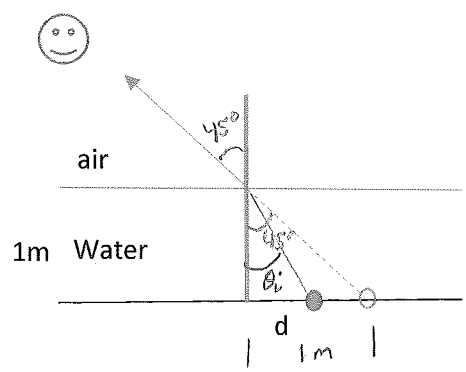
We also know that the phase of the B-field + E-field are the same for a plane wave, and in this case doesn't depend on x. So $\sin(k\cdot\emptyset + \omega\cdot\emptyset) = \emptyset$.

So the B-field is $\emptyset$ at that point in space and time.

4) What is the radiation pressure experienced by a **reflective** panel that is perpendicular to the Poynting vector of the electromagnetic wave from problem 1? [$\mu0 = 4\pi \times 10^{-7}$ H/m]

$$P_{rad} = \frac{2I}{c} = \frac{2E_0^2}{2\mu_0 c^2} = \frac{2\cdot\left(100\frac{V}{m}\right)^2}{2\cdot\left(4\pi\times10^{-7}\frac{H}{m}\right)\left(3\times10^8\frac{m}{s}\right)^2} = 8.84\times10^{-8}\frac{N}{m^2}$$

5) A child sees a coin at the bottom of a pool of water 1m deep. The coin appears to be 1m behind (away from him) a vertical pole sticking up out of the water. What is the actual distance d from the pole to the coin?



$n_{water} = 1.33$

The light from the coin is exiting the water at an angle of refraction $= 45°$, so

$$n_w \sin\theta_{in} = n_a \sin\theta_{ref}$$

$$1.33 \sin\theta_{in} = 1\cdot\sin 45°$$

$$\theta_{in} = \sin^{-1}\left(\frac{1}{1.33}\sin 45°\right)$$

$$\theta_{in} = 32.1°$$

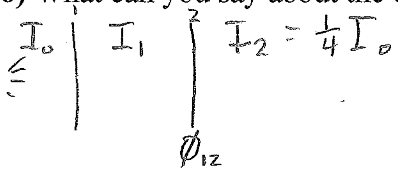
and

$$\tan\theta_{in} = \frac{d}{1m}$$

$$\therefore d = 0.63\ m$$

Unpolarized light with original intensity, I_0, is incident on a set of two ideal polarizers. After the second polarizer, the intensity is one fourth of the original.

6) What can you say about the orientation of the two polarizers?



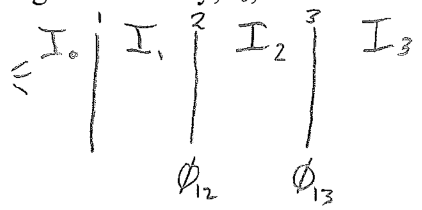
$$I_1 = \frac{1}{2} I_0$$

and $$I_2 = I_1 \cos^2 \phi_{12}$$

$$\frac{1}{4} \cancel{I_0} = \frac{1}{2} \cancel{I_0} \cos^2 \phi_{12}$$

$$\therefore \cos \phi_{12} = \frac{1}{\sqrt{2}} \Rightarrow \phi_{12} = 45°$$

7) Now, the two polarizers are set up so that none of the light emerges, and a third polarizer is placed between these two at an angle of 45 degrees to the first polarizer. How much of the original intensity, I_0, exits from the final polarizer?

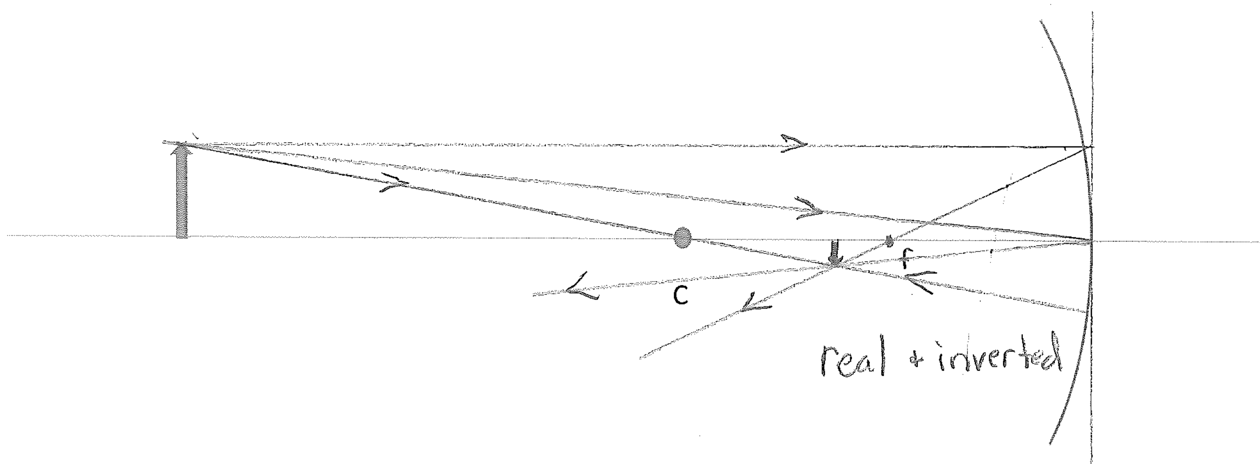


$$I_1 = \frac{1}{2} I_0, \quad \phi_{13} = 90°, \quad \phi_{12} = 45° \therefore \phi_{23} = 45°$$

$$I_2 = I_1 \cos^2 \phi_{12} = \frac{1}{2} I_0 \cos^2 45° = \frac{1}{4} I_0$$

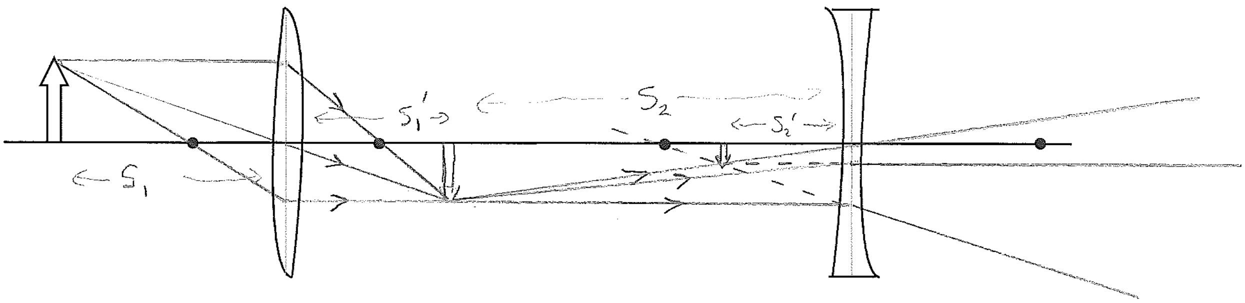
$$I_3 = I_2 \cos^2 \phi_{23} = \frac{1}{4} I_0 \cos^2 45° = \frac{1}{8} I_0$$

8) An object sits in front of a convex spherical mirror as shown below (with its center of curvature marked C). Draw three principle rays, locate the image, and indicate if it is real or virtual, upright or inverted.



Two lenses are set up as shown below. The focal length of the converging lens is 10cm, that of the diverging lens is 20cm, and they are 60cm apart. The object shown is 25cm to the left of the converging lens.

9) Draw as many primary rays as you need to find the position of the image as viewed from the right.



10) Confirm with the image-object equation and the magnification equation your approximate findings above.

$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1}$$

$$\frac{1}{25cm} + \frac{1}{s_1'} = \frac{1}{10cm}$$

$$s_1' = 16.67cm \checkmark$$

$$m_1 = -\frac{s_1'}{s_1} = \frac{-16.67cm}{25cm} = \frac{-2}{3}$$

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2}$$

$$\frac{1}{(60cm - 16.67cm)} - \frac{1}{-20cm} = -\frac{1}{s_2'}$$

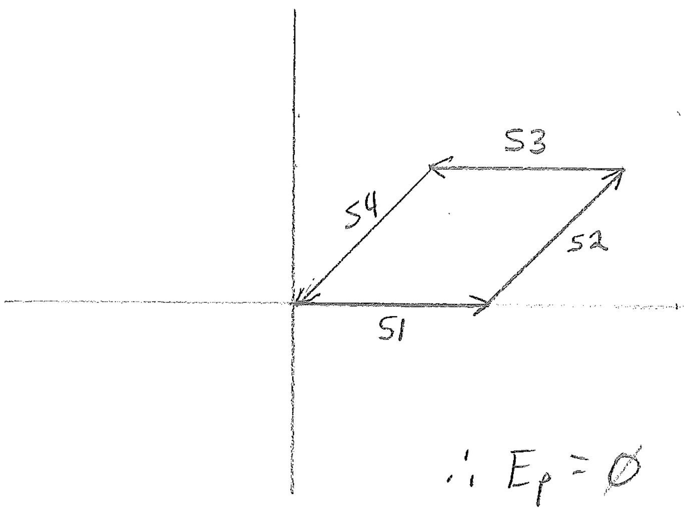
$$s_2' = -13.68cm \checkmark$$

$$m_2 = \frac{-s_2'}{s_2} = \frac{+13.68cm}{43.333cm} = 0.316$$

$$m_T = m_1 \times m_2 = 0.21 \checkmark$$

11) Four coherent sources of the same wavelength and electric field strength, E_0, are in phase at their points of origin. The table below describes what happens to the light between the sources and a point on a screen. Draw a phasor diagram for the light from these sources at that point, and determine the approximate value of the net electric field

S1	**Travels a distance D to point P in air.**
S2	**Travels the same total distance, but passes through a one-wavelength thick piece of material with index of refraction n=1.125.**
S3	**Travels the same total distance in air, but undergoes a reflection on the way.**
S4	**Travels a distance D+0.625λ to point P in air.**



For S2, In the material the wavelength is shorter by $\lambda_2 = \frac{\lambda_0}{n}$, so it goes through $1.125 = 1\frac{1}{8}$ complete phase changes (2π) instead of 1 ($1\lambda_0$ thick), so it is advanced in phase by $\frac{1}{8} \cdot 2\pi = \frac{\pi}{4}$

S3 undergoes a reflection, so it undergoes a π phase change

S4 is advanced by $0.625 \times 2\pi = \frac{5}{8} \times 2\pi$
$= \pi + \frac{\pi}{4}$

$\therefore E_P = 0$

12) Two stars are separated by $1\text{x}10^{-5}$ degrees in the sky. What size aperture (diameter) is needed to resolve them as separate stars assuming you are using a filter that only lets light of 500nm into your telescope?

$$\theta \sim \sin\theta = 1.22 \frac{\lambda}{D}$$

$$\frac{1 \times 10^{-5} \times 2\pi}{360} = 1.22 \frac{500 \times 10^{-9}\,m}{D} \Rightarrow$$

$$D = 3.5\ m$$