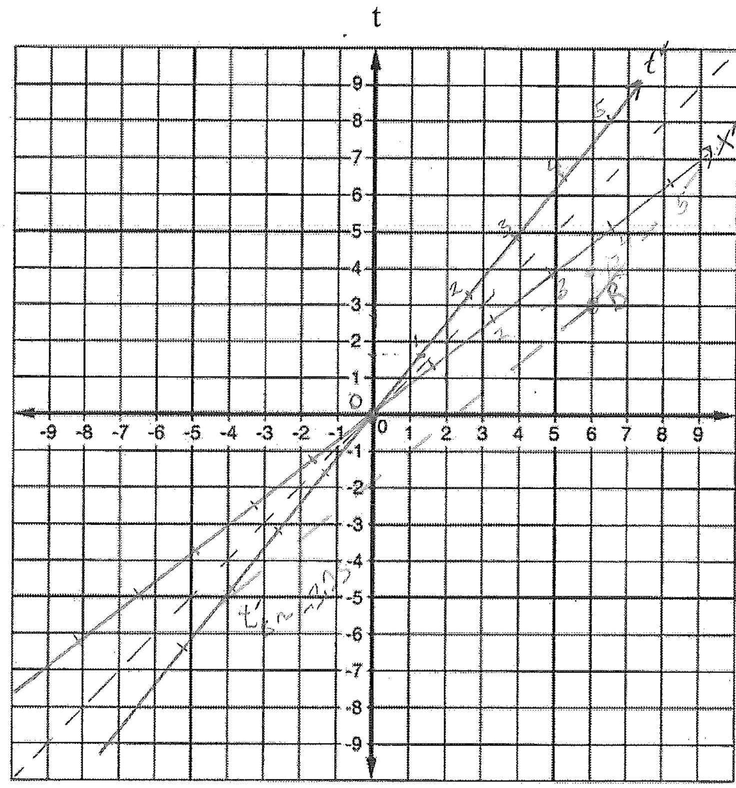
Physics 262-001 Exam #3

Name: Solutions

SHOW ALL WORK!

1) On the graph below, draw a two-frame space time diagram with the moving frame (relative to the home frame) having having a speed of 4/5c. Given that each grid on the graph corresponds to one nanosecond in the home frame, mark the appropriate scale also on the moving frame (Other Frame) axes, and label all axes.

$\beta = \frac{4}{8}$ so $\gamma = \frac{1}{\sqrt{1-(\frac{4}{5})^2}} = 1.67$



$x'_B \sim 6\,s$

2) Lightning strikes at t=x=0 in the home frame (call that Event O), and then 6ns away in the positive direction and 3ns later in time (according to an observer in the home frame) a bomb explodes (Event B). What are locations and times for Event O and Event B according to an observer in the Other Frame? Show graphically on your two-frame graph above and then use the appropriate transforms to confirm your findings.

$x'_B = \gamma(x_B - \beta t_B) = 6.0\,s$ ✓

$t'_B = \gamma(t_B - \beta x_B) = -3.0\,s$ ✓

3) Could Event B have been caused by Event O in problem 2? Is there a frame in which it would appear that Event B was caused by Event O? Why or why not?

No, the spacetime interval is less than zero since the time between O + B is less than the distance.

4) In one inertial frame, two events are measured to occur 8 s apart in time and 4 s apart in space. In another reference frame, these events are measured to occur 6 s apart in space. What is the time separation of these events in that frame?

$$\Delta s^2 = \Delta t^2 - \Delta d^2 = 64 s^2 - 16 s^2 = 48 s^2$$

$$\Delta s'^2 = \Delta t'^2 - \Delta d'^2 = \Delta t'^2 - 36 s^2$$

$$\therefore 48 s^2 = \Delta t'^2 - 36 s^2 \Rightarrow$$

$$\Delta t'^2 = 84 s^2$$

$$\Delta t' = \sqrt{84}\ s$$

5) What is the velocity of the frame (relative to the original frame) where the events from problem 4 take place at the same location?

The two events would both have to be on a [illegible] parallel to the moving frame axis, so

$$\beta = \frac{\Delta x}{\Delta t} = \frac{4 s}{8 s} = 0.5$$

6) Planet Fields was recently discovered roaming near our solar system just 3 light-days from earth (as measured by NASA). We decided to send our newest spaceship, capable of constant speed 0.5, to explore it. According to earth, how long does the spaceship take to reach the planet?

$$\Delta t = \frac{\Delta d}{v} = \frac{3\text{ days}}{0.5} = 6\text{ days}$$

7) According to the spaceship, at what speed is the planet coming towards him?

$$\beta = -0.5$$

8) According to the spaceship, what is the distance to the planet at time t=t'= 0?

$$x' = \frac{x}{\gamma} \quad \text{where } \gamma = \frac{1}{\sqrt{1-(.5)^2}} = 1.15$$

$$\therefore x' = \frac{3\text{ days}}{1.15} = 2.60\text{ days}$$

9) According to the spaceship, how long did it take to reach the planet?

$$\Delta t' = \frac{\Delta d'}{v} = \frac{2.6\text{ days}}{0.5} = 5.2\text{ days}$$

10) An observer on earth, E, sees two alien spacecraft, Y and Z, approaching from opposite sides of the galaxy, both at a distance of one light-year and with speeds 0.75. What is the velocity which Y sees Z approaching?

$$V_x' = \frac{V_x - \beta}{1 - \beta V_x}$$ where the two frames are the earth and spaceship Y,

so $\beta = 0.75$, $V_x = -0.75$

$$\therefore V_x' = \frac{-0.75 - 0.75}{1 + 0.75^2} = \boxed{-0.96}$$

11) A particle of mass m at rest decays into two identical particles, each of mass 1/3 m. Conservation of spatial momentum means that the product particles must move off in opposite directions with the same speed. What is the relativistic kinetic energy and velocity of each particle?

Before: $\overset{m}{o}$

After: $\overset{1/3 m}{\leftarrow o}$ $\overset{1/3 m}{o \rightarrow}$

Energy cons. gives: $\cancel{m} = \frac{1}{3}\gamma \cancel{m} + \frac{1}{3}\gamma \cancel{m} \Rightarrow \gamma = \frac{3}{2}$

$\therefore E_T = \gamma m_p = \frac{3}{2}\left(\frac{1}{3}m\right) = \frac{1}{2}m$

and $KE = (\gamma - 1)m_p = \left(\frac{3}{2} - 1\right)\frac{1}{3}m = \boxed{\frac{1}{6}m = KE}$

and $\gamma = \frac{3}{2} = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \frac{9}{4} = \frac{1}{1-\beta^2} \Rightarrow 1 - \beta^2 = \frac{4}{9} \Rightarrow \beta^2 = \frac{5}{9}$

$$\boxed{\beta = \frac{\sqrt{5}}{3}}$$

12) A particle moving at velocity 0.95c relative to the lab has a lifetime of 3.0×10^{-8}s. How far (in meters) will the particle travel in the lab before it decays?

$\beta = 0.95 \longrightarrow \gamma = \frac{1}{\sqrt{1-\beta^2}} = 3.2$

and $t' = 3.0 \times 10^{-8}s$

so $t = \gamma t' = 9.6 \times 10^{-8}s$

$d = vt = (.95) \times (9.6 \times 10^{-8}s) = 9.12 \times 10^{-8}s$

in meters,

$d = 9.12 \times 10^{-8}s \times 3 \times 10^{8}\frac{m}{s} = 27.36\ m$