Physics 2310-001 Exam 4

Name: Solutions (with correct m_{el})

SHOW ALL WORK

1) A certain special metal has a work function of 3.2 eV. What is the threshold wavelength of light that can eject an electron from this metal?

a) 352.4nm
b) 387.8nm
c) 472.2nm
d) 498.5nm
e) 515.8nm
f) 545.1nm
g) 587.3nm
h) 620.4nm
i) 647.7nm
j) 660.7nm

$KE_{max} = hf - \phi = 0$ at thresh.

$\therefore hf = \phi = \frac{hc}{\lambda} \Rightarrow$

$3.2eV = \frac{(4.136 \times 10^{-15} eV \cdot s)(3 \times 10^{8} m/s)}{\lambda} \Rightarrow$

$\boxed{\lambda = 387.8 nm}$

2) An electron is in a 1D infinite square well (a box) that is 3.0 nm wide. The electron makes a transition from the n = 3 to the n = 4 state, what is the wavelength of the absorbed photon?

a) 119nm
b) 331nm
c) 526nm
d) 734nm
e) 974nm
f) 1123nm
g) 2008nm
h) 3654nm
i) 4243nm
j) 5321nm

$E_n = \frac{n^2 h^2}{8mL^2}$

$\therefore (E_4 - E_3) = (16-9)\frac{h^2}{8mL^2}$

$= \frac{7(6.626 \times 10^{-34} J \cdot s)^2}{8(9.11 \times 10^{-31} kg)(3 \times 10^{-9} m)^2}$

$\frac{hc}{\lambda} = 4.685 \times 10^{-20} J$

$\therefore \boxed{\lambda = \frac{hc}{4.685 \times 10^{-20} J} = 4243 nm}$

$= 4615$ nm with $m_{el} = 9.91 \times 10^{-31}$ kg

3) How fast must a nonrelativistic electron move so its de Broglie wavelength is the same as the wavelength of a 4.2eV photon?

a) 3040 m/s
b) 1210 m/s
c) 2630 m/s
d) 2465 m/s
e) 2830 m/s
f) 1870 m/s
g) 1010 m/s
h) 1350 m/s
i) 2266 m/s
j) 3860 m/s

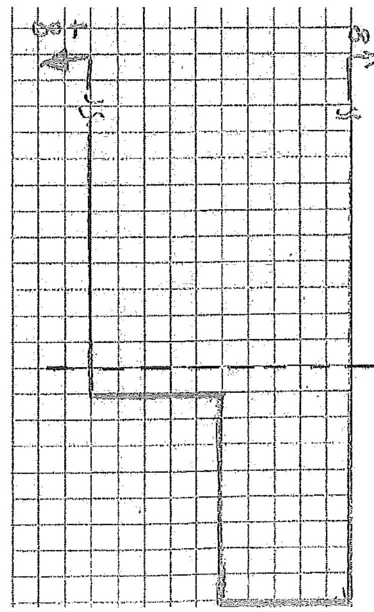
$$m_{el} v = p_{el} = \frac{h}{\lambda} \quad , \quad E_\gamma = hf = \frac{hc}{\lambda} \Rightarrow \lambda = 295\,\text{nm}$$

$$v = \frac{h}{m_{el}\lambda} = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{9.11\times10^{-31}\,\text{kg}\cdot 295\times10^{-9}\,\text{m}}$$

$$\boxed{v = 2465\ \text{m/s}}$$

2266 m/s with $m_{el} = 9.91\times10^{-31}$ kg

An infinitely deep potential energy well has a "shelf" in it on the left side, as shown. Suppose an electron is a stationary state of this potential with the energy shown as the dashed line.



4) Will the wavelength of the electron be shorter or longer in the deeper part (right side) of the well?

a) Shorter
b) Longer
c) Same in both parts
d) No way to tell

In the deeper side, $KE = E - U$ is higher than the shallow side. Since the K.E is given by the second derivative of the w.f., it must have a shorter wavelength in the deeper side.

5) Will the amplitude of the wavefunction be higher or lower in the deeper part (right side) of the well?

a) Higher
b) Lower
c) Same in both parts
d) No way to tell

Since it is going more slowly in shallow part (less K.E. there), it will spend more time there per unit distance. Therefore the w.f. will have a lower amplitude in the deeper side

6) Which is an allowed set of quantum numbers for an electron in an atom?

a) $n = 3$, $\ell = -1$, $m_\ell = -1$, $m_s = 1/2$
b) $n = 2$, $\ell = 1$, $m_\ell = 1$, $m_s = 0$
c) $n = 0$, $\ell = 0$, $m_\ell = 0$, $m_s = 1/2$
d) $n = 2$, $\ell = 1$, $m_\ell = 0$, $m_s = 1/2$
e) $n = 1$, $\ell = 1$, $m_\ell = -1$, $m_s = -1/2$
f) $n = 3$, $\ell = 2$, $m_\ell = 2$, $m_s = -1/2$
g) $n = 3$, $\ell = 0$, $m_\ell = 1$, $m_s = 1/2$
h) more than one set shown is allowed (circle the allowed sets)

$n \neq 0$ excludes c)
$\ell = 0, 1, \cdots n-1$ excludes a) and e)
$m_\ell = -\ell \cdots 0 \cdots \ell$ excludes g)
$m_s = \pm \frac{1}{2}$ excludes b)

7) Consider the wave function (for an unspecified potential) formed by adding two stationary states (with energies $E_1 = E_2$) together (with appropriate normalization):

$$\Psi(\vec{r},t) = \frac{1}{\sqrt{2}}\psi_1(\vec{r})e^{-iE_1t/\hbar} + \frac{1}{\sqrt{2}}\psi_2(\vec{r})e^{-iE_2t/\hbar}$$

What is true for this wave?

Since $E_1 = E_2$, time dependence in the cross products of the probability distribution will go away.

a) it is also a stationary state
b) it is not a solution to the full Schödinger equation
c) it is a solution to the full Schödinger equation, but has an imaginary probability density
d) it is a solution to the full Schödinger equation, but the probability density now changes with time
e) it is a solution, but it doesn't make any sense because it has an undetermined energy

8) Given an electron put into a 1D Gaussian wavepacket of width 8nm, what is the uncertainty in the velocity of this particle?

a) $7.23 \times 10^3\ m/s$
b) $7.13 \times 10^2\ m/s$
c) $1.33 \times 10^4\ m/s$
d) $2.66 \times 10^4\ m/s$
e) $8.36 \times 10^4\ m/s$
f) $4.21 \times 10^{-3}\ m/s$
g) $2.27 \times 10^{-8}\ m/s$
h) $9.22 \times 10^{-12}\ m/s$
i) $4.19 \times 10^{-12}\ m/s$
j) $3.34 \times 10^{-12}\ m/s$

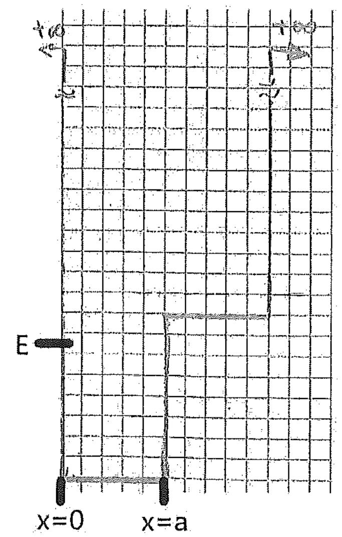
$\Delta x \Delta p = \frac{\hbar}{2} = \frac{(6.626 \times 10^{-34}\ J \cdot s)}{4\pi}$

$\therefore \Delta p = m \Delta V = \frac{6.626 \times 10^{-34}\ J \cdot s}{4 \cdot \pi \cdot (8 \times 10^{-9}\ m)}$

$\boxed{\Delta V = 7{,}235\ m/s}$

$6.65 \times 10^3\ m/s$ with $m_{el} = 9.11 \times 10^{-31}\ kg$

9) For a particle with energy E, in the potential shown to the right, what can be said about the particle's wavefunction at the points x=0 and x=a?



a) The wavefunction must be zero at x=0 and at x=a.
b) The wavefunction can be non-zero at x=0, but must be zero at x=a.
c) The wavefunction can be non-zero at both x=0 and x=a.
d) The wavefunction must be zero at x=0, but is non-zero at x=a.

10) A particle is in a 3D box with side lengths (in x,y,z) of L, L, 2L. What is the degeneracy of the third excited state of this particle?

a) 0
b) 1
c) 2
d) 3
e) 4

Energies must be enumerated:

$$E = \frac{n_x^2 \pi^2 \hbar^2}{2mL^2} + \frac{n_y^2 \pi^2 \hbar^2}{2mL^2} + \frac{n_z^2 \pi^2 \hbar^2}{2m(2L^2)}$$

$$= \left(n_x^2 + n_y^2 + \frac{n_z^2}{4}\right)\frac{\pi^2 \hbar^2}{2mL^2}$$

$$E_G = \left(1^2 + 1^2 + \frac{1^2}{4}\right)\underbrace{\frac{\pi^2\hbar^2}{2mL^2}}_{E_0} = 2\tfrac{1}{4} E_0 \quad \text{deg. } 1$$

$$E_1 = \left(1^2 + 1^2 + \frac{2^2}{4}\right)E_0 = 3 E_0 \quad \text{deg. } 1$$

$$E_2 = \left(1^2 + 1^2 + \frac{3^2}{4}\right)E_0 = 4\tfrac{1}{4} E_0 \quad \text{deg. } 1$$

$$E_3 = \left(2^2 + 1^2 + \frac{1^2}{4}\right)E_0 = 5\tfrac{1}{4} E_0$$
$$\text{or } \left(1^2 + 2^2 + \frac{1^2}{4}\right)E_0$$
deg. 2

$$E_4 = \left(2^2 + 1^2 + \frac{2^2}{4}\right)E_0 = 6 E_0$$
$$\text{or } \left(1^2 + 2^2 + \frac{2^2}{4}\right)E_0$$
$$\text{or } \left(1^2 + 1^2 + \frac{4^2}{4}\right)E_0$$
deg. 3