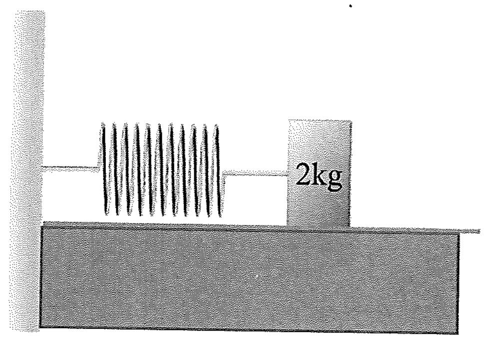
Physics 262-001 Final Exam

Name: Solutions

Mechanical waves:

1) A 2kg mass is mounted to a spring on a horizontal frictionless surface. The spring is compressed by 1cm when the mass is pushed to the left by a force of 10N. If it is then released at t=0, what is the correct equation of motion for the mass?



$F = kx \Rightarrow 10N = k \cdot 0.01m$

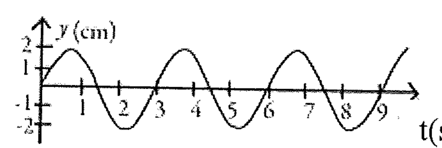
$\therefore k = 1000 \frac{N}{m}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1000 \frac{N}{m}}{2kg}} = \sqrt{500}\ s^{-1} = 22.36\ s^{-1}$

@ $t=0$, $x = -0.01m$

$\therefore x(t) = -0.01m \cos(22.36 s^{-1} \cdot t)$

2) A graph of the transverse displacement of a wave at ~~x~~z=0 is shown in the figure to the right. If the wave velocity is 3.0m/s $\hat{k}$, what is the **complete** equation for the wave?



$y(z,t) = A \overset{\sin \text{ or}}{\cos}(kz - \omega t)$, $A = 0.02m$

$f = \frac{1}{3} s^{-1}$ and $|V| = 3.0 \frac{m}{s}$

since $V = \lambda f$, $\lambda = 9m$

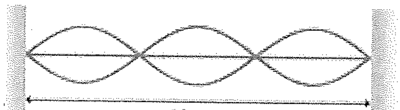
$\therefore k = \frac{2\pi}{\lambda} = 0.7 m^{-1}$

$\omega = 2\pi f = 2.09 s^{-1}$

since @ $z=0$, $t=0$ $y(0,0) = 0$, choose sin

$\therefore y(z,t) = 0.02m \sin(0.7m^{-1} \cdot z - 2.09 s^{-1} \cdot t)$

3) A standing wave is oscillating at 440 Hz on a string, as shown in the figure. The tension in the string is 100N. What is the mass of this string?



$\lambda = 40cm = 0.4m$

$f = 440 s^{-1}$

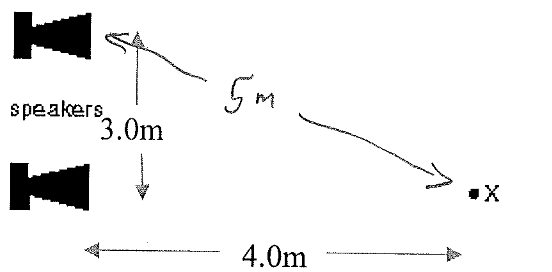
$\therefore v = \lambda f = 0.4m \cdot 440 s^{-1}$

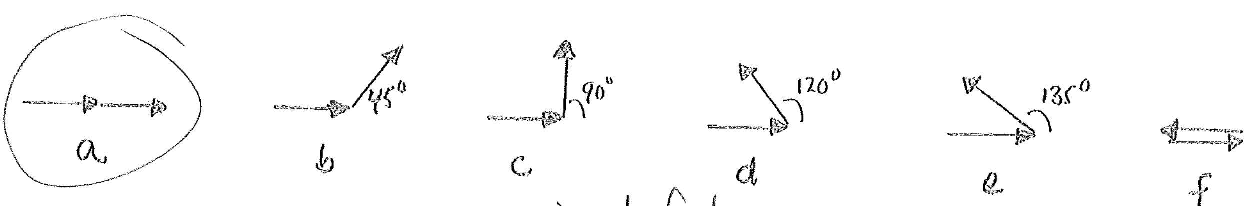
$= 176 \, m/s$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100N}{\frac{m}{L}}} \Rightarrow \frac{mv^2}{L} = 100N \Rightarrow m = \frac{100N \cdot L}{v^2} = \frac{100N \cdot .60m}{(176 \, m/s)^2}$

$m = 0.0019 \, kg = 1.9g$

4) Two small identical speakers are connected to the same source, **but completely out of phase.** The speakers are 3.0m apart. A microphone is located at X, in the same plane as the two speakers and 4.0m in front of one speaker as shown. Which of the phasor diagrams below best represent the situation at X if the wavelength of the sound played is 2.0m?





source is out of phase

$\delta\phi = k\delta d + \pi$

$= \frac{2\pi}{\lambda} \cdot \delta d + \pi$

$= \frac{2\pi}{2m} \cdot 1m + \pi$

$= \pi + \pi = 2\pi$

Electromagnetic waves:

5) There exists a plane electromagnetic wave in vacuum whose magnetic field is totally described by: $\vec{B} = \left(8.00 \times 10^{-8} T\right)\hat{i}\cos\left[\left(6.283 m^{-1}\right) y + \omega t\right]$

At t=0s, what is the magnitude and direction of the instantaneous Poynting vector at the point (1.0m, 0.5m, 0.0m)?

x y z

$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ direction is in direction of wave: $-\hat{j}$

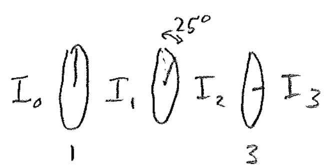
$|\vec{S}| = \frac{1}{\mu_0} c B^2 = \frac{c}{\mu_0}\left(8.0\times10^{-8}T\right)^2 \cos^2\left[2\pi m^{-1} \cdot y + \omega t\right]$

at $t = 0 \Rightarrow |\vec{S}| = \frac{3\times10^{8}\ m/s}{1.26\times10^{-6}\ \frac{T\cdot m}{A}}\left(8.0\times10^{-8}T\right)^2 \cancel{\cos^2}\left[2\pi \cancel{m}^{-1}\cdot 0.5\cancel{m} + 0\right]$

and $y = 0.5m$

$|\vec{S}| = 1.52\ \frac{T\cdot A}{s} = 1.52\ W/m^2$

6) Unpolarized light with original intensity, I_0, is incident on a set of two ideal polarizers. After the second polarizer, no light emerges. If a third polarizer is placed between these two at an angle of 25 degrees to the first polarizer, how much of the original intensity, I_0, exits from the final polarizer?



$I_1 = \frac{1}{2} I_0$

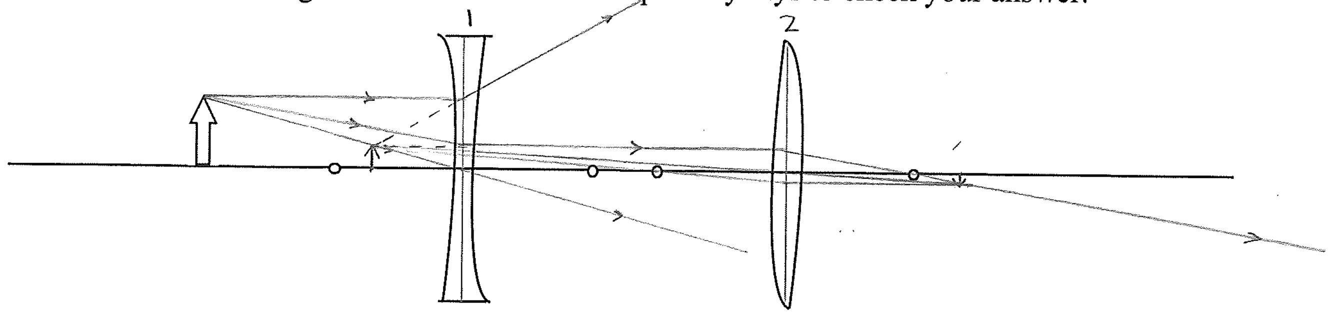
$I_2 = I_1 \cos^2\phi_{12} = \frac{1}{2} I_0 \cos^2(25°)$

$= 0.411\, I_1$

$I_3 = I_2 \cos^2\phi_{23} = 0.411 I_0 \cos^2(90° - 25°)$

$= 0.073\, I_0$

7) Two lenses are set up as shown below. The focal lengths of the first (diverging) and second (converging) lenses are both 10cm, and they are 25cm apart. The object shown is 20cm to the left of the first lens. Use the image-object equation and the magnification equation and determine the **overall** magnification and draw three primary rays to check your answer.



$$\frac{1}{o_1} + \frac{1}{i_1} = \frac{1}{f_1}$$

$$\frac{1}{20cm} + \frac{1}{i_1} = \frac{1}{-10cm}$$

$$\therefore i_1 = -6.6\overline{6}cm$$

$$m_1 = \frac{-i_1}{o_1} = \frac{6.6\overline{6}cm}{20cm} = 0.3\overline{3}$$

$$\frac{1}{o_2} + \frac{1}{i_2} = \frac{1}{f_2}$$

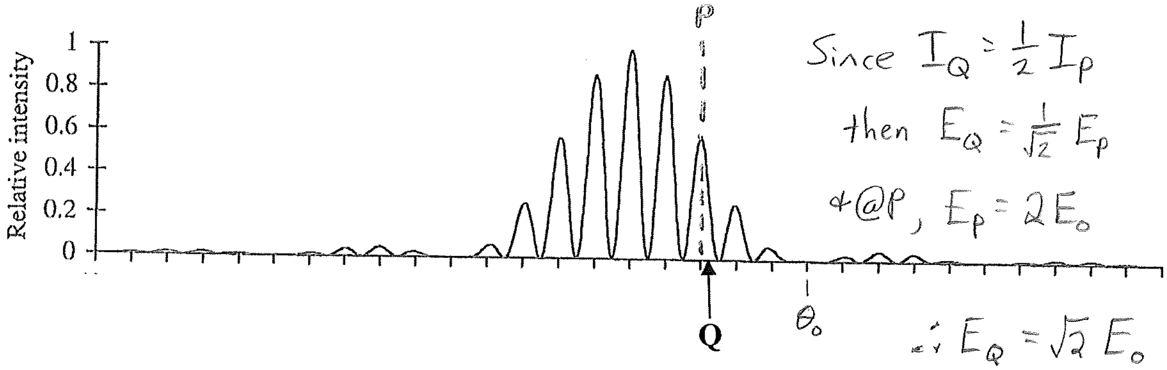
$$\frac{1}{6.6\overline{6}cm + 25cm} + \frac{1}{i_2} = \frac{1}{10cm}$$

$$\therefore i_2 = 14.6cm$$

$$m_2 = \frac{-i_2}{o_2} = \frac{-14.6cm}{31.6\overline{6}cm} = -0.46$$

$$\therefore m_T = m_1 \times m_2 = -0.154$$

Two slits are separated by a distance ***d***, with a slit width ***a***; they are illuminated by a plane wave and thus are emitting waves in phase. Crests are shown.



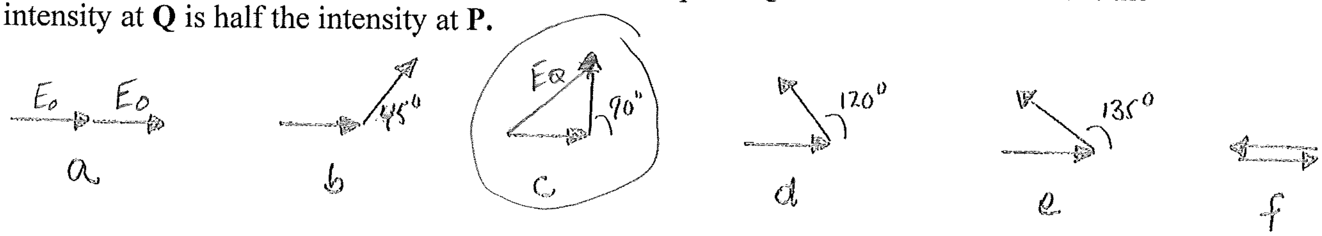
Since $I_Q = \frac{1}{2} I_P$

then $E_Q = \frac{1}{\sqrt{2}} E_P$

& @P, $E_P = 2E_0$

$\therefore E_Q = \sqrt{2} E_0$

8) What phasor diagram best represents the fields at point **Q**? Additional information: the intensity at **Q** is half the intensity at **P.**



9) The two-slit interference/diffraction pattern above was made with light with wavelength 680 nm. What is the ratio of the slit width ***a*** to the slit separation ***d***?

Since for two slit diffraction, $d\sin\theta = m_2\lambda$ for const. inter.

and for single slit diffraction $a\sin\theta = m_1\lambda$ for destruc. inter.

& we see that the $m_1 = 1$ minimum falls at the same position (angle) as the $m_2 = 5$ maximum:

$$d\sin\theta_0 = 5\lambda$$

$$\& \quad a\sin\theta_0 = 1\lambda$$

so dividing these two equations gives

$$\frac{a\sin\theta_0}{d\sin\theta_0} = \frac{a}{d} = \frac{1\lambda}{5\lambda} = \frac{1}{5} \Rightarrow$$

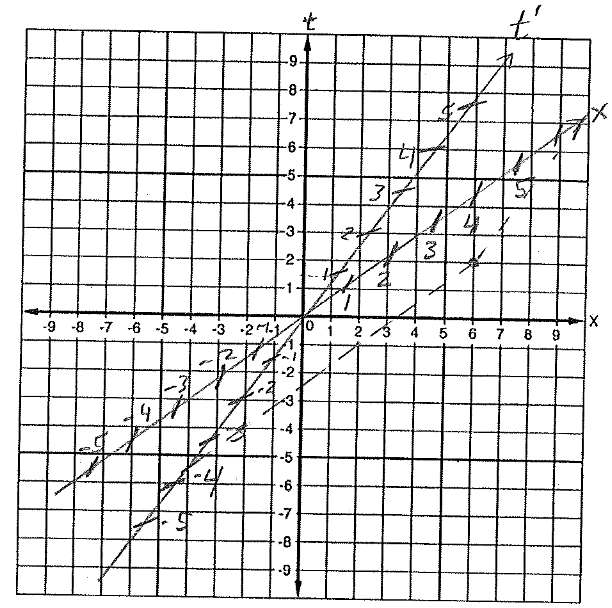
$$\frac{a}{d} = \frac{1}{5}$$

Relativity:

10) On the space-time diagram below with the home frame shown and scaled, draw and scale the other frame when β = 0.75.

for scaling, $t = \gamma t'$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.51$$



11) What is the space and time of an event (with t = 2s and x = 6s as measured by the home frame) as measured in the other frame? Show graphically and calculate exactly.

$\to x' \approx 6.5s$
$t' \approx -3.7s$

$$x' = \gamma(x - \beta t)$$
$$= 1.51(6s - 0.75 \times 2s)$$
$$= 1.51(4.5s)$$
$$= 6.8s$$

$$t' = \gamma(t - \beta x)$$
$$= \gamma(2s - .75 \times 6s)$$
$$= 1.51 \times (-2.5)$$
$$= -3.8s$$

12) Show that the spacetime interval between this event and the origin is the same in both frames.

$$\Delta s^2 = \Delta t^2 - \Delta x^2$$
$$= 4s^2 - 36s^2$$
$$= -32s^2$$

$$\Delta s'^2 = \Delta t'^2 - \Delta x'^2$$
$$= (-3.8s)^2 - (6.8s)^2$$
$$= -32s^2 \checkmark$$

13) A particle with mass = 2.0 kg has total energy = 6.0kg as measured in the laboratory, and has a lifetime of 1 s. How far (in meters) will the particle travel in the lab before it decays?

$$E = 6\,kg \Rightarrow E^2 = p^2 + m^2 \Rightarrow p = \sqrt{E^2 - m^2} = \sqrt{36kg^2 - 4kg^2} = 5.66\,kg$$

$$v = \frac{p}{E} = \frac{5.66\,kg}{6.0\,kg} = 0.94$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.94)^2}} = 3$$

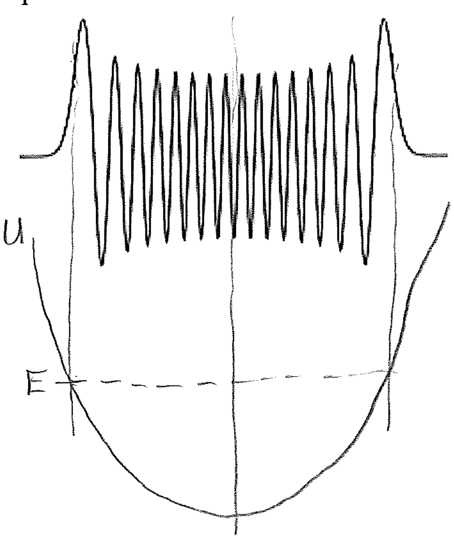
$$t = \gamma t' = 3 \cdot 1s = 3s$$

$$d = vt = 0.94 \times 3 = 2.83s$$

$$d = 2.83s \times 3\times10^8 \frac{m}{s} = 8.48\times10^8 m$$

__Quantum Mechanics:__

14) A particle is trapped in a 1D symmetric potential. It is in an excited state with its wavefunction drawn below. By examining the wavefunction, sketch the potential associated with it indicating the approximate energy of the particle on the same sketch. What is the quantum number?



Amplitude and wave length gets smaller towards the center, getting larger more quickly towards the edges. Also, wavefunction doesn't go to zero like a sinusoid, but more like an exponetial, so... It's actually a wavefunction for a harmonic oscillator potential. It has 30 nodes, so it has quantum # $n = 31$.

15) Consider the wave function for an infinite square well potential formed by adding two stationary states (with different energies E_1 and E_2) together (with appropriate normalization):

$$\psi_t(x,t) = \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}e^{-iE_2 t/\hbar}$$

Explicitly show the time dependence of the probability ~~dependence~~ density.

The probability density is given by:

$$\Psi^*\Psi = \left(\frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}e^{iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}e^{iE_2 t/\hbar}\right) \times \left(\frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{\pi x}{L}e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}e^{-iE_2 t/\hbar}\right)$$

$$= \frac{1}{L}\sin^2\frac{\pi x}{L} + \frac{1}{L}\sin^2\frac{2\pi x}{L} + \frac{1}{L}\sin\frac{2\pi x}{L}\sin\frac{\pi x}{L}\left[e^{i(E_2-E_1)t/\hbar} + e^{-i(E_2-E_1)t/\hbar}\right]$$

$$= \frac{1}{L}\left[\left(\sin^2\frac{\pi x}{L} + \sin^2\frac{2\pi x}{L}\right) + \sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}\cos\left[(E_2-E_1)t/\hbar\right]\right]$$

16) List all eigenstates of an electron in a hydrogen atom with the primary quantum number n = 3.

$n =$		3	
$\ell =$	0	1	2
$m_\ell =$	0	$-1, 0, 1$	$-2, -1, 0, 1, 2$
$m_s =$	$-\frac{1}{2}, +\frac{1}{2}$	$-\frac{1}{2}, +\frac{1}{2}$ $-\frac{1}{2}, +\frac{1}{2}$ $-\frac{1}{2}, +\frac{1}{2}$	. . . . .

so,

$3, 0, 0, \frac{1}{2}$
$3, 0, 0, -\frac{1}{2}$
$3, 1, -1, -\frac{1}{2}$
$3, 1, -1, +\frac{1}{2}$
$3, 1, 0, -\frac{1}{2}$
$3, 1, 0, +\frac{1}{2}$
$3, 1, 1, -\frac{1}{2}$
$3, 1, 1, +\frac{1}{2}$

$3, 2, -2, +\frac{1}{2}$
$3, 2, -2, -\frac{1}{2}$
$3, 2, -1, -\frac{1}{2}$
$3, 2, -1, +\frac{1}{2}$
$3, 2, 0, -\frac{1}{2}$
$3, 2, 0, +\frac{1}{2}$
$3, 2, 1, -\frac{1}{2}$
$3, 2, 1, +\frac{1}{2}$
$3, 2, 2, -\frac{1}{2}$
$3, 2, 2, +\frac{1}{2}$

18 states