https://phet.colorado.edu/sims/radiating-charge/radiating-charge_en.html

Lecture 11
(Electromagnetic Waves)

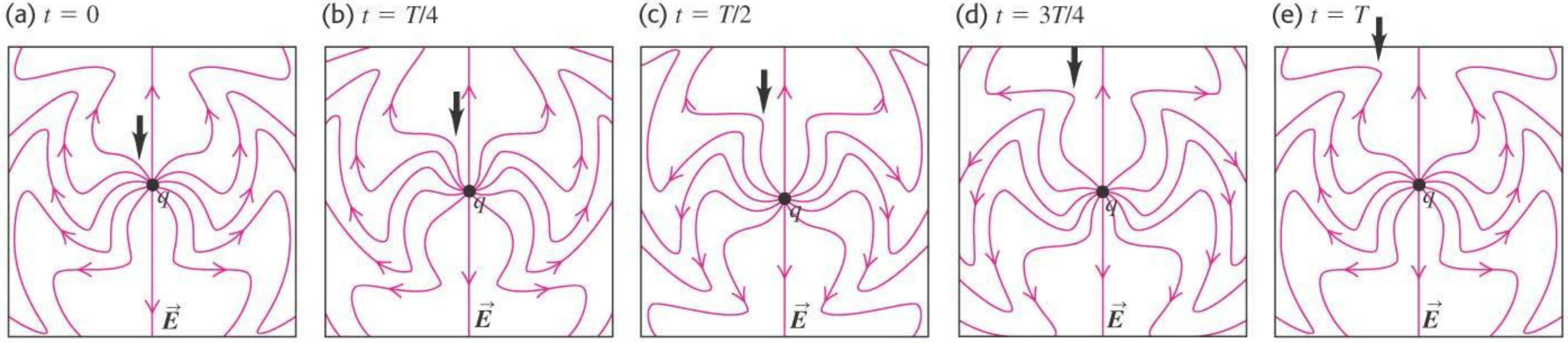
Physics 2310-01 Spring 2020
Douglas Fields

Qualities of the EM Wave

- The electric and magnetic fields are always perpendicular to the direction the wave is travelling – transverse waves.
- The electric field is always perpendicular to the magnetic field.
- The magnitudes of the electric and magnetic fields are governed by: $|\vec{E}| = c|\vec{B}|$
- The direction of the propagation of the wave is given by the right-hand-rule: $\vec{E} \times \vec{B}$
- The polarization direction (for linearly polarized waves) is defined by the direction of the electric field.

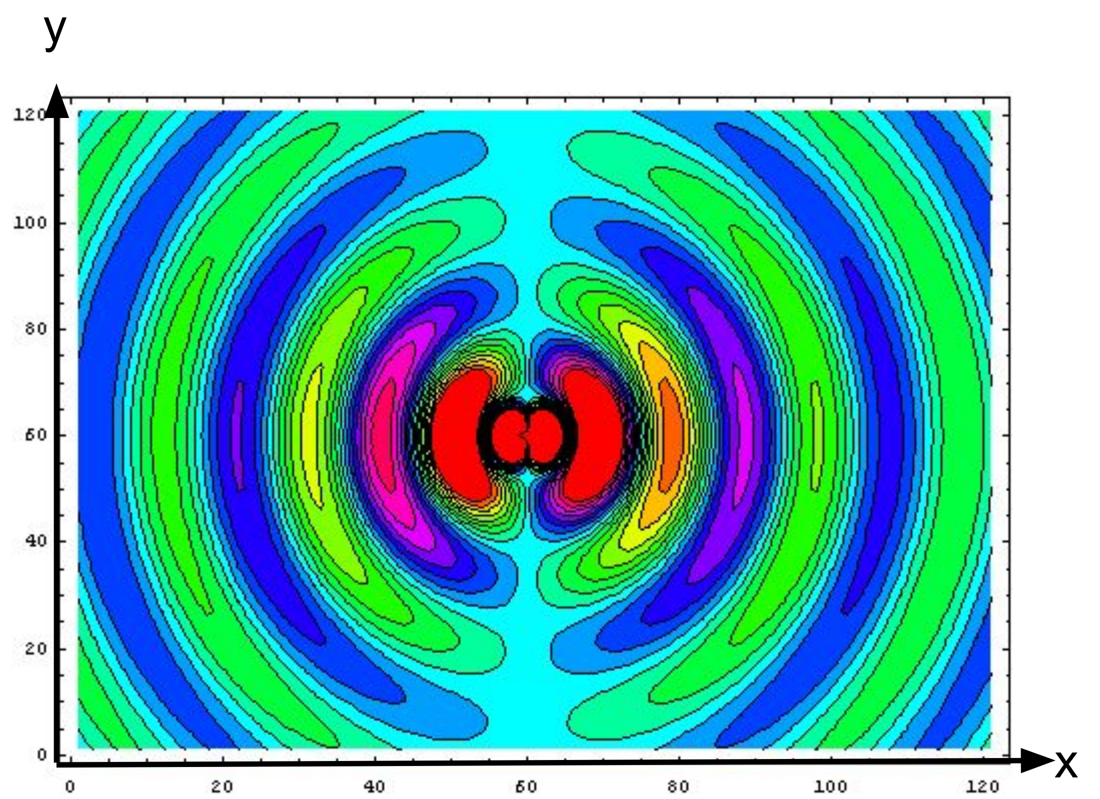
Sources of EM waves

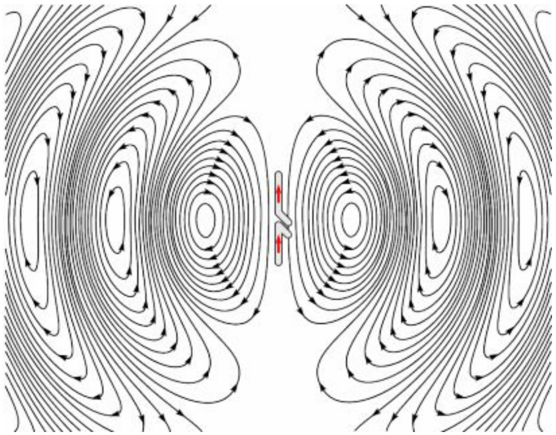
- In analogy to string waves being caused by moving one end of a string up and down (or in 3D, a hummingbird's flapping wings causing a sound wave), EM waves are caused by a moving (actually accelerating) charge (or charge distribution).
- Just like for waves on a string, EM waves don't have to be strictly sinusoidal, they can be a pulse or whatever.
- But we will restrict our discussions mostly to sinusoidal waves…



Dipole Antenna

- Dipole oriented in +/- y-direction.
- Contours are B-field strength in z-direction (green/yellow and blue/red in opposite directions).
- Which direction is E-field?





Electromagnetic Wave Equations (3D)

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{d^2 B}{dt^2} \qquad \nabla^2 E = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}$$

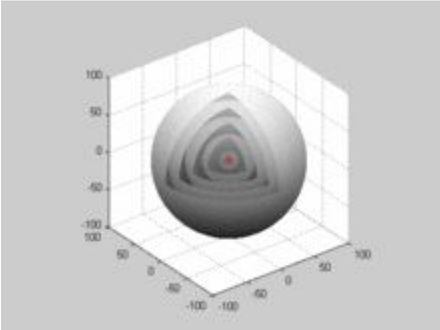
$$v = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}} = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\, H/m\right) \times \left(8.854 \times 10^{-12}\, F/m\right)}} = 2.998 \times 10^8\, m/s = c$$

- This is a four-dimensional differential equation. It's solutions depend on the boundary conditions (in both time and space).
- For free-space (vacuum everywhere), and no angular dependence (l=0),

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

- the solutions look like:

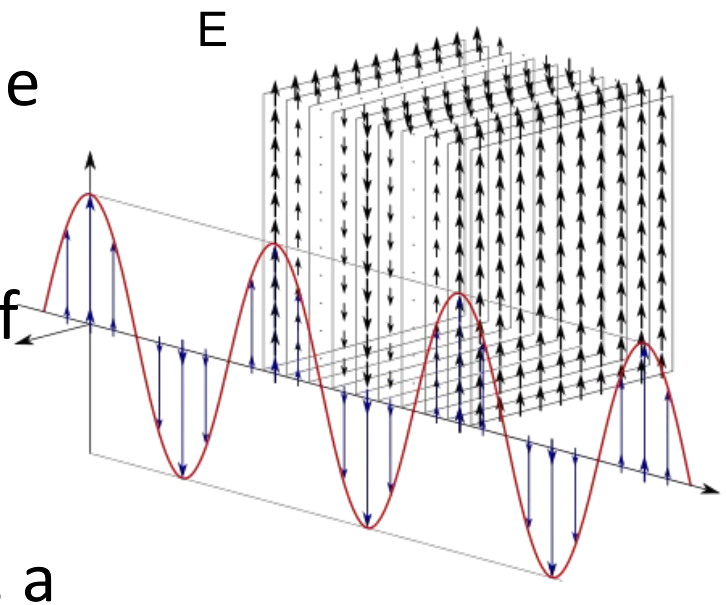
$$\left|\vec{B}(r,t)\right| = \frac{B_0}{r}\sin(kr - \omega t)$$



Can you confirm that this solution satisfies the wave equation?

Plane Waves

- Initially, we will limit our discussions to plane waves.
- Plane waves have the same value of the fields at all locations in space within a plane perpendicular to the direction of motion.
- How can that be?
- Think of the surface of a sphere, a large distance from the center compared to the distance on the surface you are sampling.



Electromagnetic Wave Equations (1D)

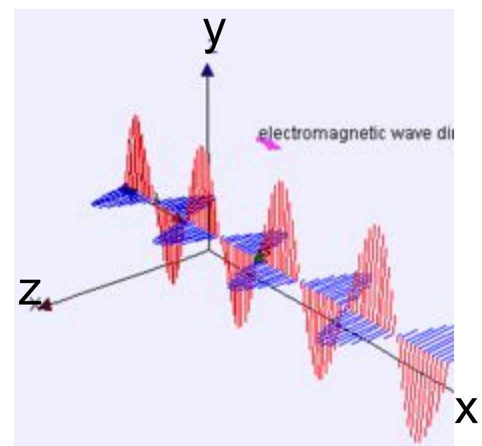
$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{d^2 B}{dt^2} \qquad \nabla^2 E = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}$$

$$v = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}} = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\, H/m\right) \times \left(8.854 \times 10^{-12}\, F/m\right)}} = 2.998 \times 10^8\, m/s = c$$

- In the case of plane waves, we can restrict the solutions to the one-dimensional case.
- That is, ***moving*** in one dimension:

$$\vec{E}(x,t) = \hat{j} E_{\max} \sin(kx - \omega t + \phi_E)$$
$$\vec{B}(x,t) = \hat{k} B_{\max} \sin(kx - \omega t + \phi_B)$$

$$c = \frac{\omega}{k}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \; \phi_E = \phi_B$$



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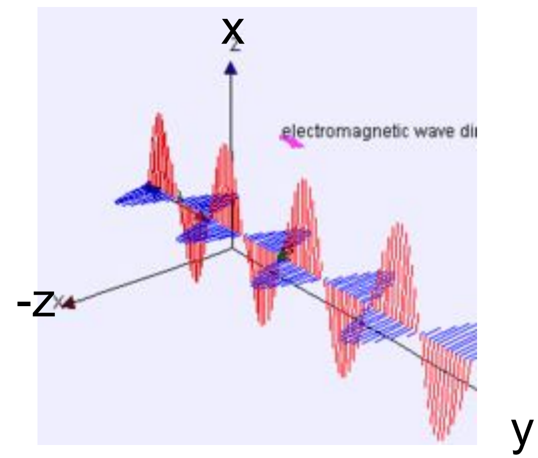
- In the case of plane waves, we can restrict the solutions to the one-dimensional case.
- That is, ***moving*** in one dimension:

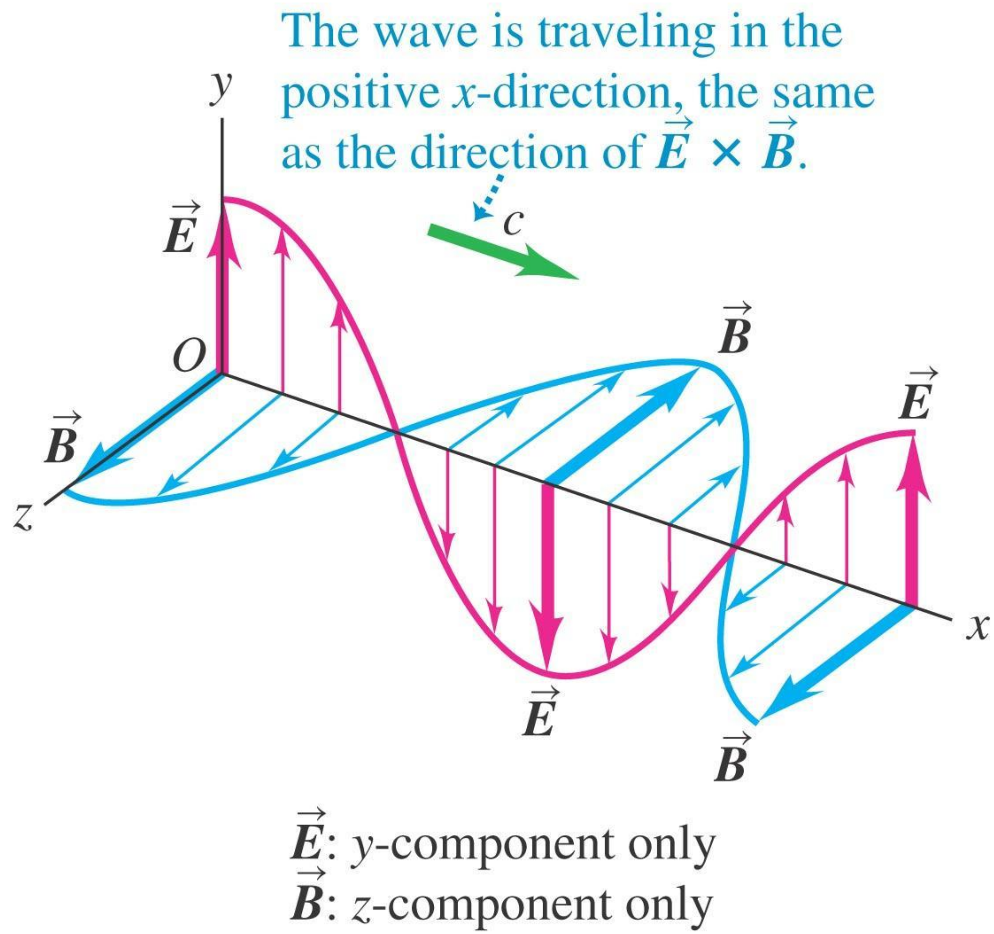
$$\vec{E}(x,t) = \hat{i} E_{\max} \sin\left(ky - \omega t + \phi_E\right)$$
$$\vec{B}(x,t) = -\hat{k} B_{\max} \sin\left(ky - \omega t + \phi_B\right)$$

$$c = \frac{\omega}{k}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f, \; \phi_E = \phi_B$$



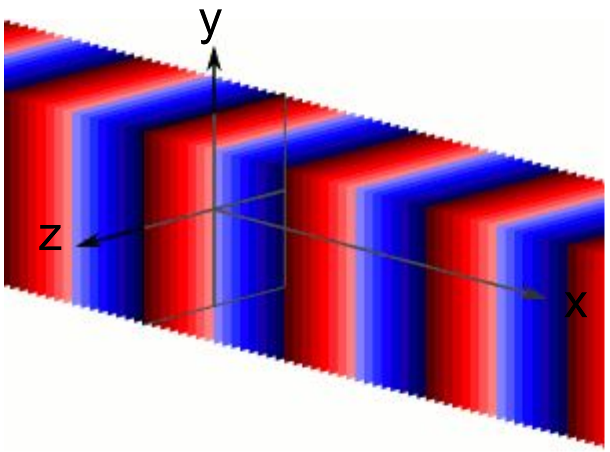
Confusion…

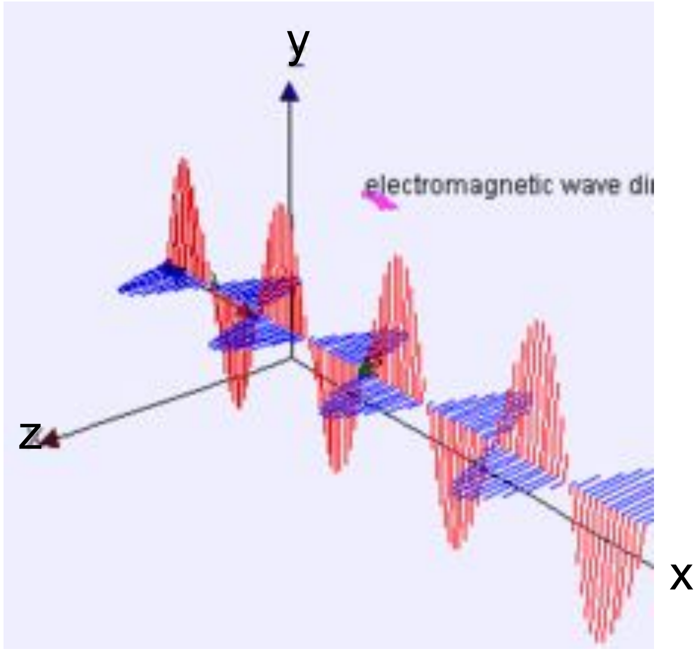
- Just like the electric field lines, the depiction of plane waves as in this figure can create conceptual confusion.



Plane Waves

- Don't forget that these planes move, so that at a specified point, the value of the field still oscillates.





Phase

- We can find the phase constant (same for E and B for plane waves) by identifying the position (x_1) of a maximum value of E along the x-axis at time t=0:

$$\vec{E}(x_1, 0) = \hat{j}E_{\max}\sin(kx_1 + \phi_E) = \hat{j}E_{\max}$$

$$or$$

$$\sin(kx_1 + \phi_E) = 1 \Rightarrow$$

$$kx_1 + \phi_E = \frac{\pi}{2}$$

Wavelength & Frequency

- Remember, in vacuum, the wave speed is fixed (speed of light = c), so that the wavelength and frequency both uniquely define the type of EM radiation.

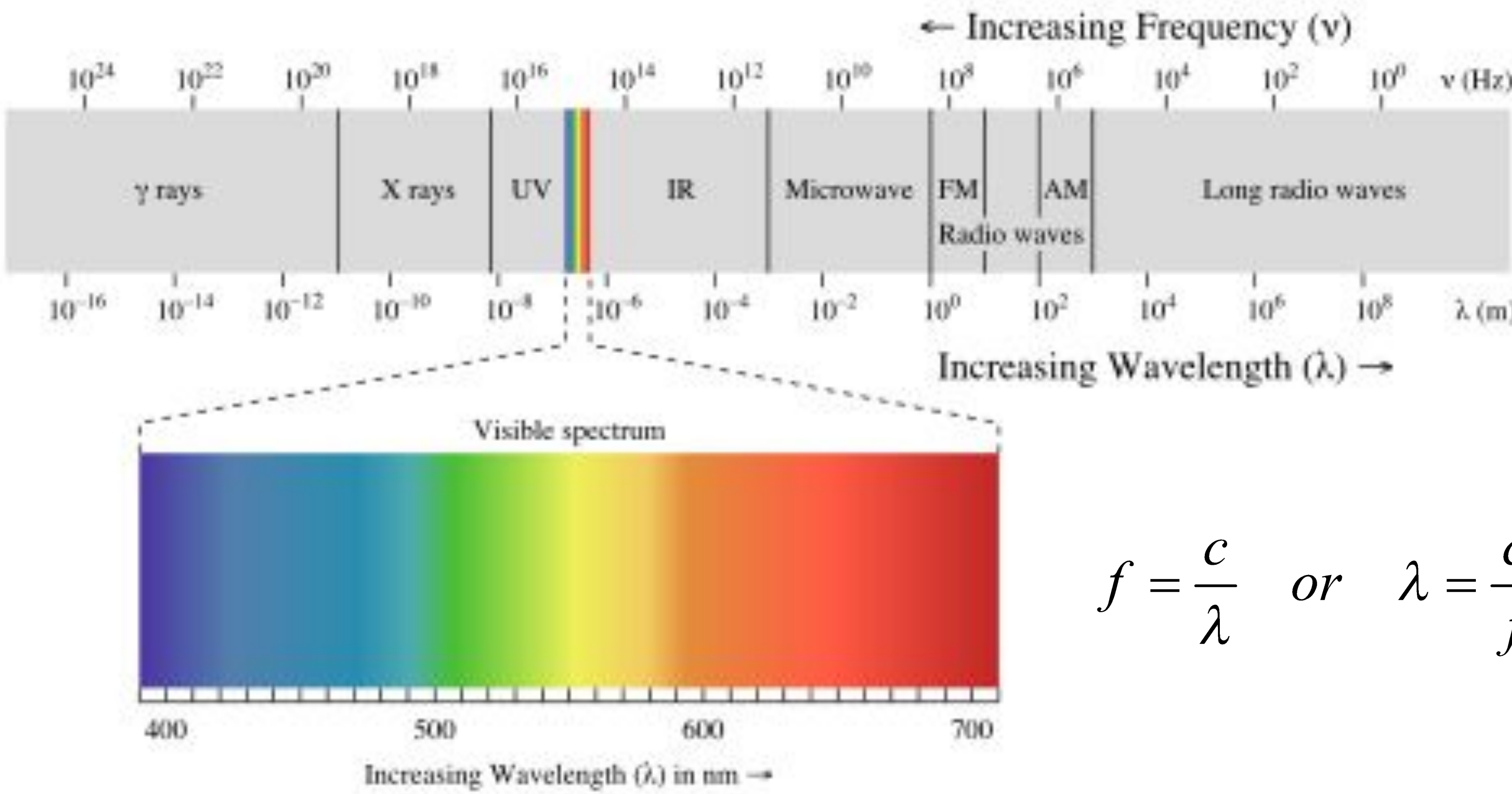
$$c = \frac{\omega}{k}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f \Rightarrow$$

$$2\pi f = \frac{2\pi}{\lambda} c \Rightarrow$$

$$f = \frac{c}{\lambda} \quad or \quad \lambda = \frac{c}{f}$$

Electromagnetic Spectrum

- All are electromagnetic waves.



$$f = \frac{c}{\lambda} \quad or \quad \lambda = \frac{c}{f}$$

However…

- Remember from where the wave equations originate – Maxwell's equations.
- In matter (not vacuum, but no free charges), Maxwell's equations are modified:

$$\oint_{\partial V} \vec{E} \cdot d\vec{S} = 0$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{S} = 0$$

$$\oint_{\partial S} \vec{B} \cdot d\vec{r} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint_{\partial S} \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t}\Phi_B$$

⇒

$$\oint_{\partial V} \vec{E} \cdot d\vec{S} = 0$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{S} = 0$$

Relative permeability

$$\oint_{\partial S} \vec{B} \cdot d\vec{r} = (\kappa_M \mu_0)(\kappa \epsilon_0) \frac{d\Phi_E}{dt}$$

Dielectric constant

$$\oint_{\partial S} \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t}\Phi_B$$

Speed of Light in Materials

- Then, because the only thing that has changed are these constants, we again get a wave equations from Maxwell's equations, but with a different wave speed:

$$\nabla^2 B = \mu\varepsilon \frac{d^2 B}{dt^2} \qquad \nabla^2 E = \mu\varepsilon \frac{d^2 E}{dt^2}$$

$$v = \frac{1}{\sqrt{(\mu\varepsilon)}} = \frac{1}{\sqrt{(\kappa\kappa_M)(\mu_0\varepsilon_0)}} = \frac{1}{\sqrt{(\kappa\kappa_M)}}c$$

Index of Refraction

- The index of refraction of a material is defined as:

$$v = \frac{1}{\sqrt{(\mu\varepsilon)}} = \frac{1}{\sqrt{(\kappa\kappa_M)(\mu_0\varepsilon_0)}} = \frac{1}{\sqrt{(\kappa\kappa_M)}}c \equiv \frac{c}{n} \Rightarrow$$

$$n = \sqrt{(\kappa\kappa_M)}$$

- So, the index of refraction (always > 1) has the effect of slowing the propagation of light in a material. This will have many consequences, as we will see later.