Lecture 14 (Total Internal Reflection and Dispersion)

Physics 2310-01 Fall 2019

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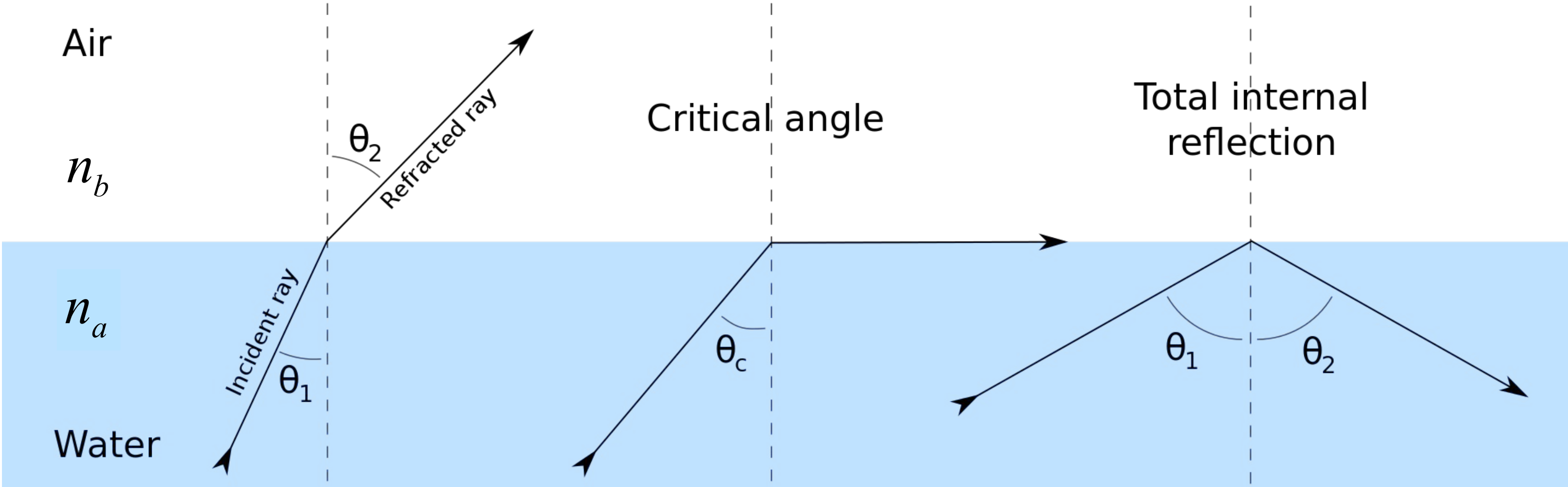
https://phet.colorado.edu/sims/html/bending-light/latest/bending-light_en.html

Total Internal Reflection

- An interesting thing happens when light goes from a higher to a lower index of refraction…
- As the angle of incidence is increased, the refracted light is bent more and more parallel to the surface.
- At an angle (called the critical angle), the light is refracted completely parallel to the interface.
- At this point then,

$$n_a \sin\theta_a = n_b \cancel{\sin\theta_b}^{\,1} \Rightarrow$$

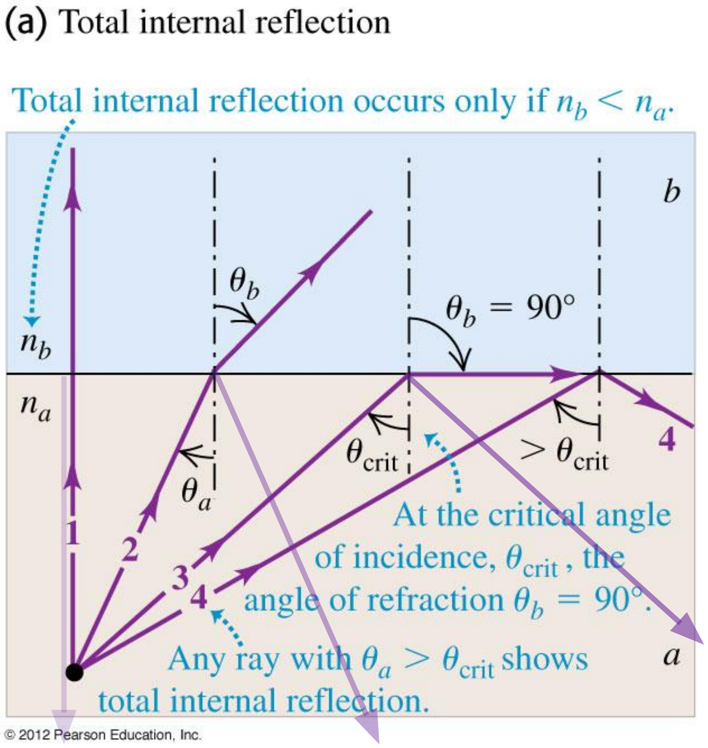
$$\sin\theta_{critical} = \frac{n_b}{n_a}$$



Total Internal Reflection

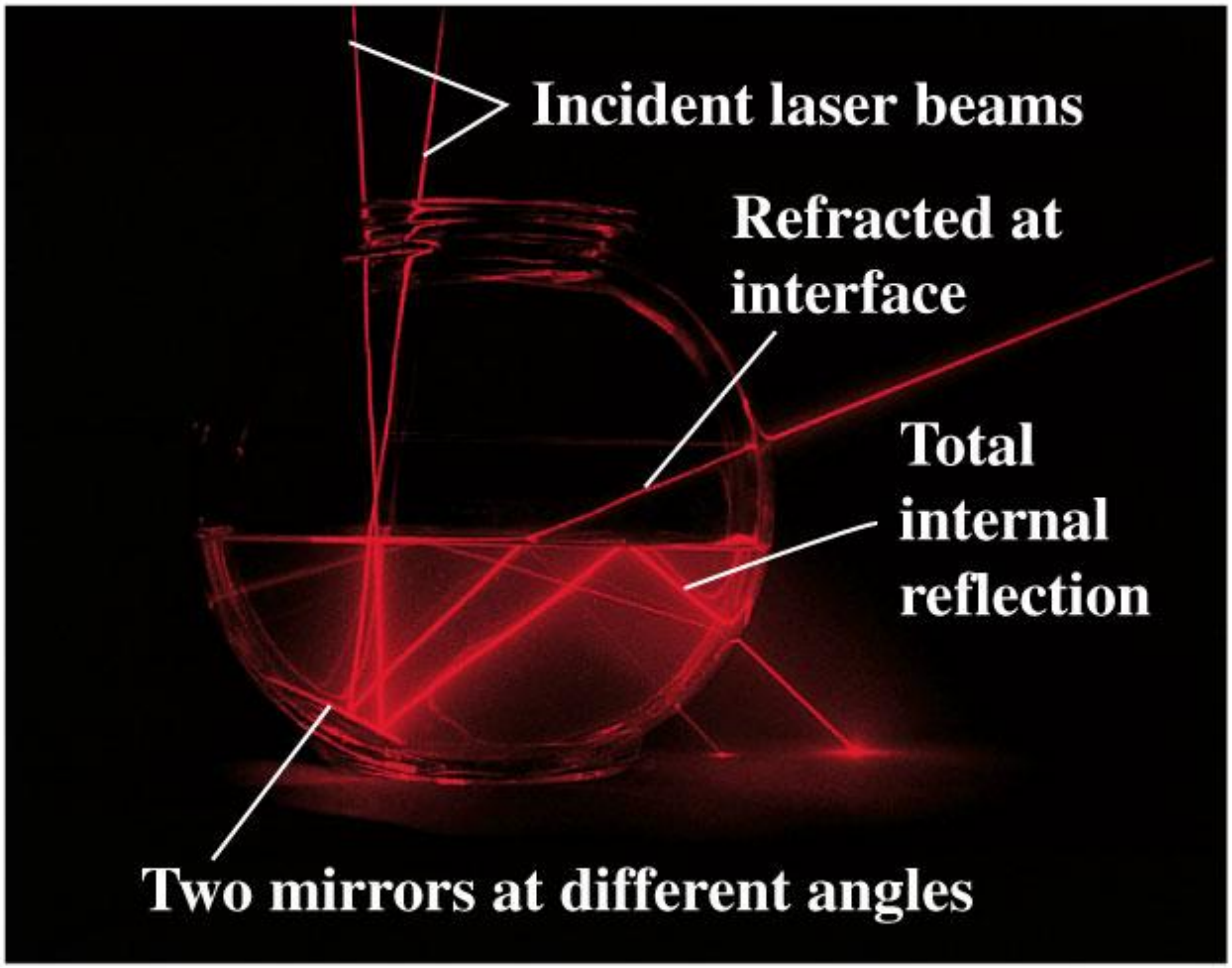
- Notice that this can only happen when $n_a > n_b$
- What the previous picture didn't show is that there is also reflection at the other angles!
- The fraction that is reflected increases up to the critical angle, and then is 100%.

$$\sin\theta_{critical} = \frac{n_b}{n_a}$$



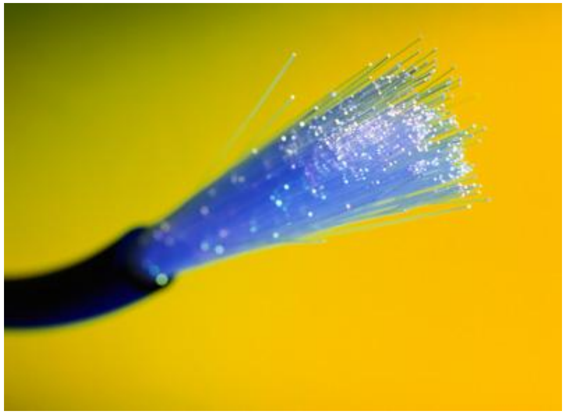
See?

(b) Total internal reflection demonstrated with a laser, mirrors, and water in a fishbowl

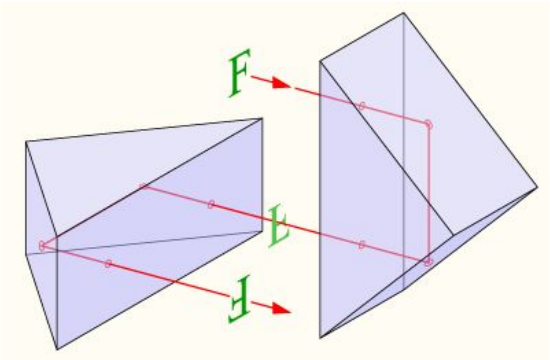


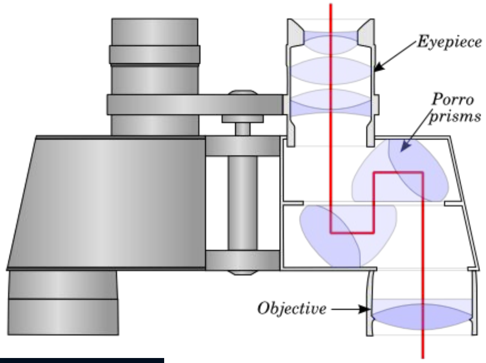
Uses

- Fiber Optics



- Porro Prisms

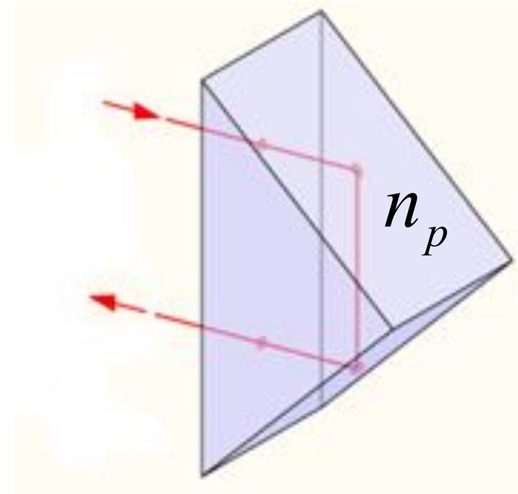




- Endoscopes

- Diamonds…

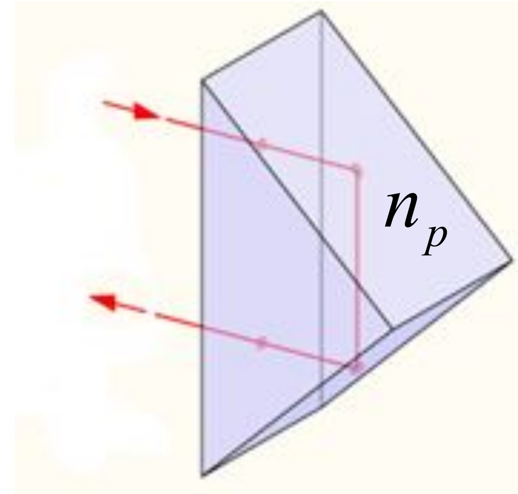
Exercise



$n_{H_2O} = 1.333$

- A porro prism made of a material with index of refraction n_p **just barely** gives *total* internal reflection when immersed in water. What is the index of refraction n_p ?

Exercise



$$n_{H_2O} = 1.333$$

- A porro prism made of a material with index of refraction n_p just barely gives *total* internal reflection when immersed in water. What is the index of refraction n_p ?

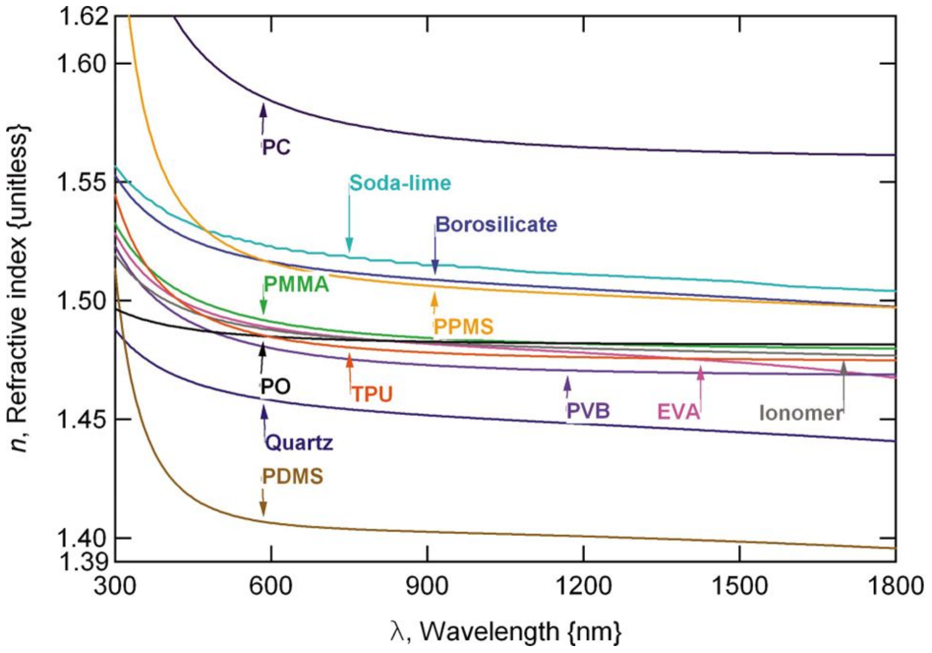
$$\sin\theta_c = \frac{n_b}{n_a}$$

$$\sin 45° = \frac{1.333}{n_a} \Rightarrow$$

$$n_a = \frac{1.333}{\sin 45°} = \frac{1.333}{0.707} = 1.886$$

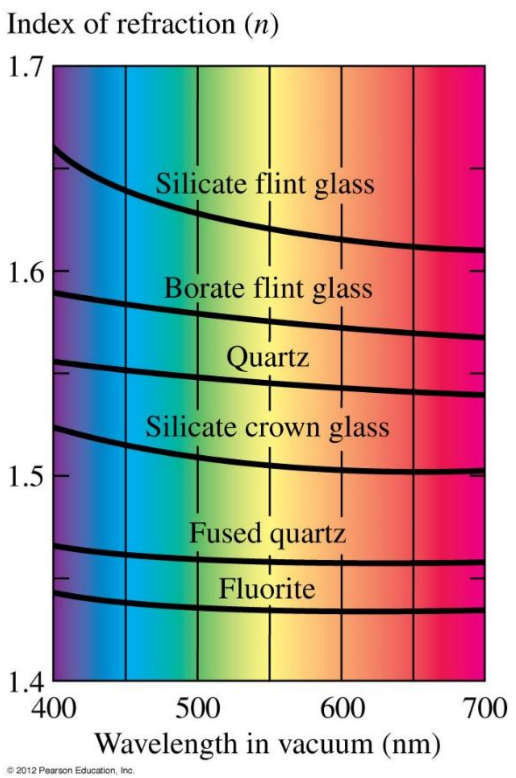
Dispersion

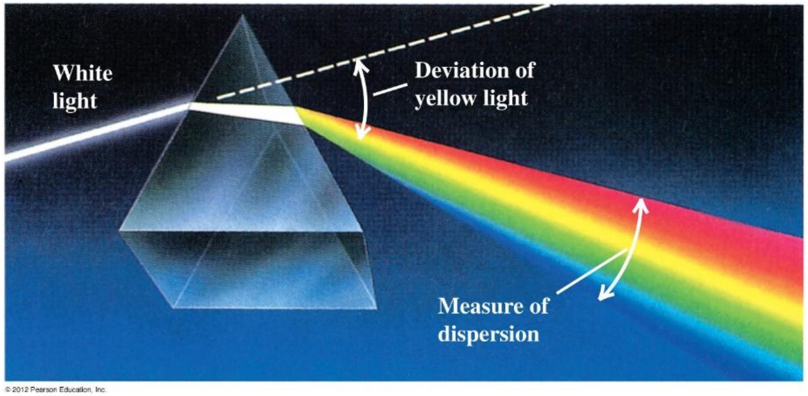
- Until now, we have considered that the index of refraction of a material is independent of the wavelength of the light.
- The different speeds of light in materials is caused by moving charges in the material absorbing and reemitting light at the same frequency but with a slight change in phase.
- The change in phase can also be wavelength dependent, causing dispersion.



Prisms

- Non-monochromatic light (e.g., white light) can be separated into its components through the use of a prism, which relies on dispersion.
- Shorter wavelengths see a higher index of refraction and are therefore refracted more than longer wavelengths.
- One can use this to analyze the color spectrum of a source, but there are better methods to do that which we will discuss later.



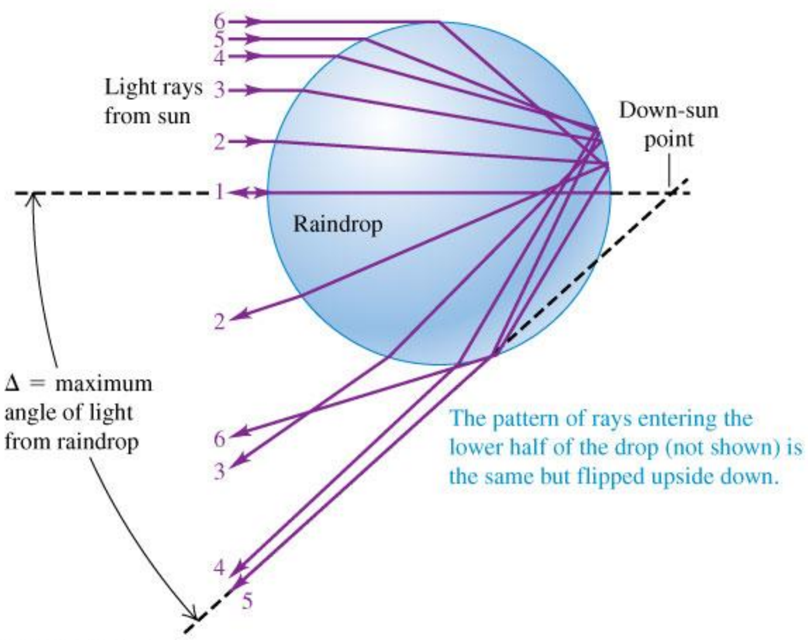


Got to love NM!

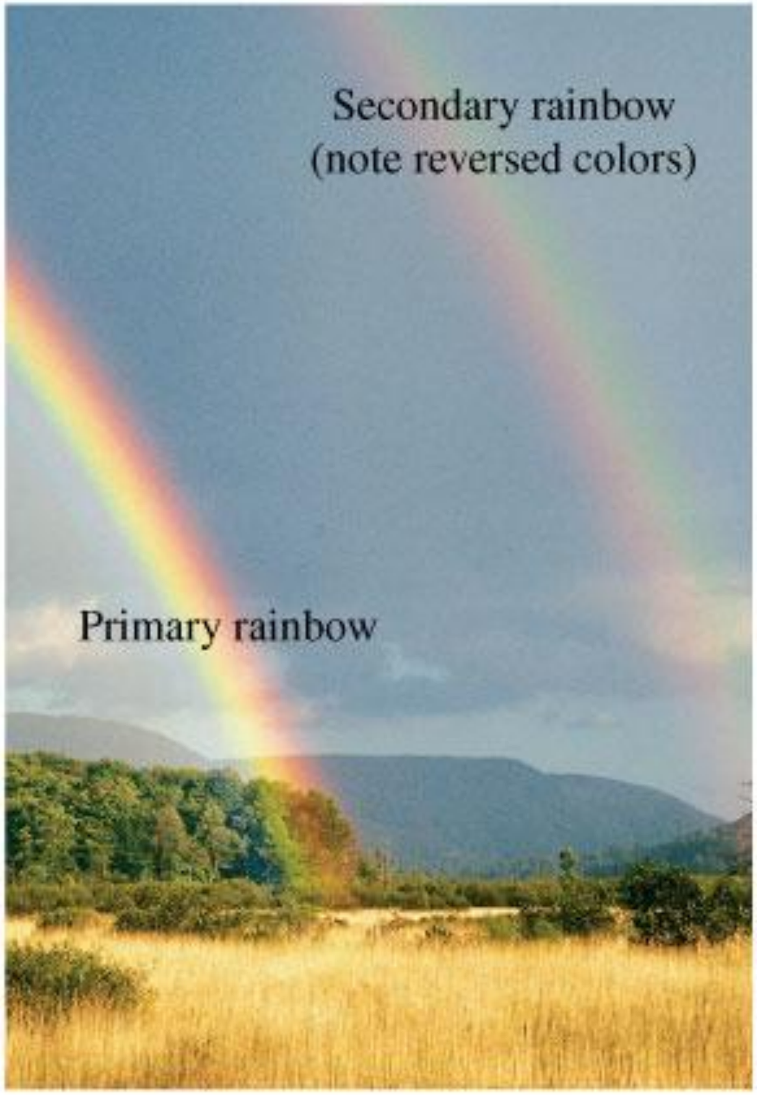
Rainbows

- Rainbows are another excellent example of dispersion.
- Without dispersion:

(b) The paths of light rays entering the upper half of a raindrop



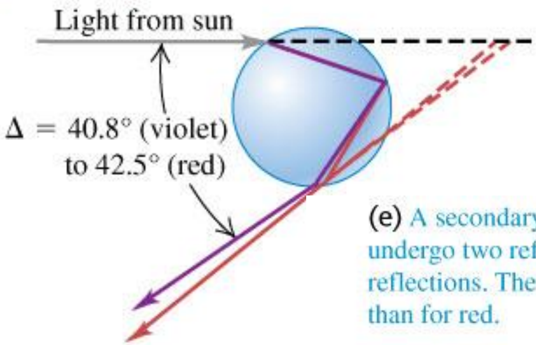
(a) A double rainbow



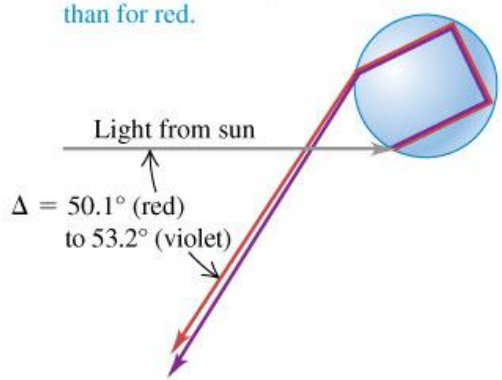
Rainbows

- Rainbows are another excellent example of dispersion.

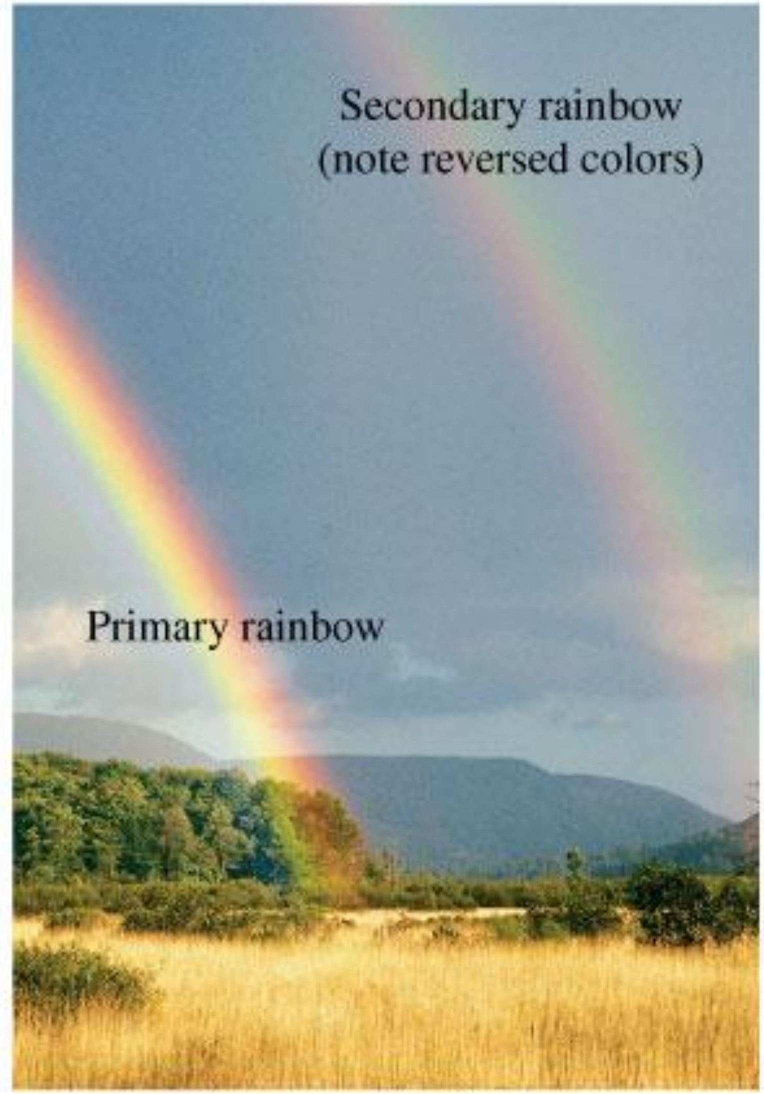
(d) A primary rainbow is formed by rays that undergo two refractions and one internal reflection. The angle Δ is larger for red light than for violet.

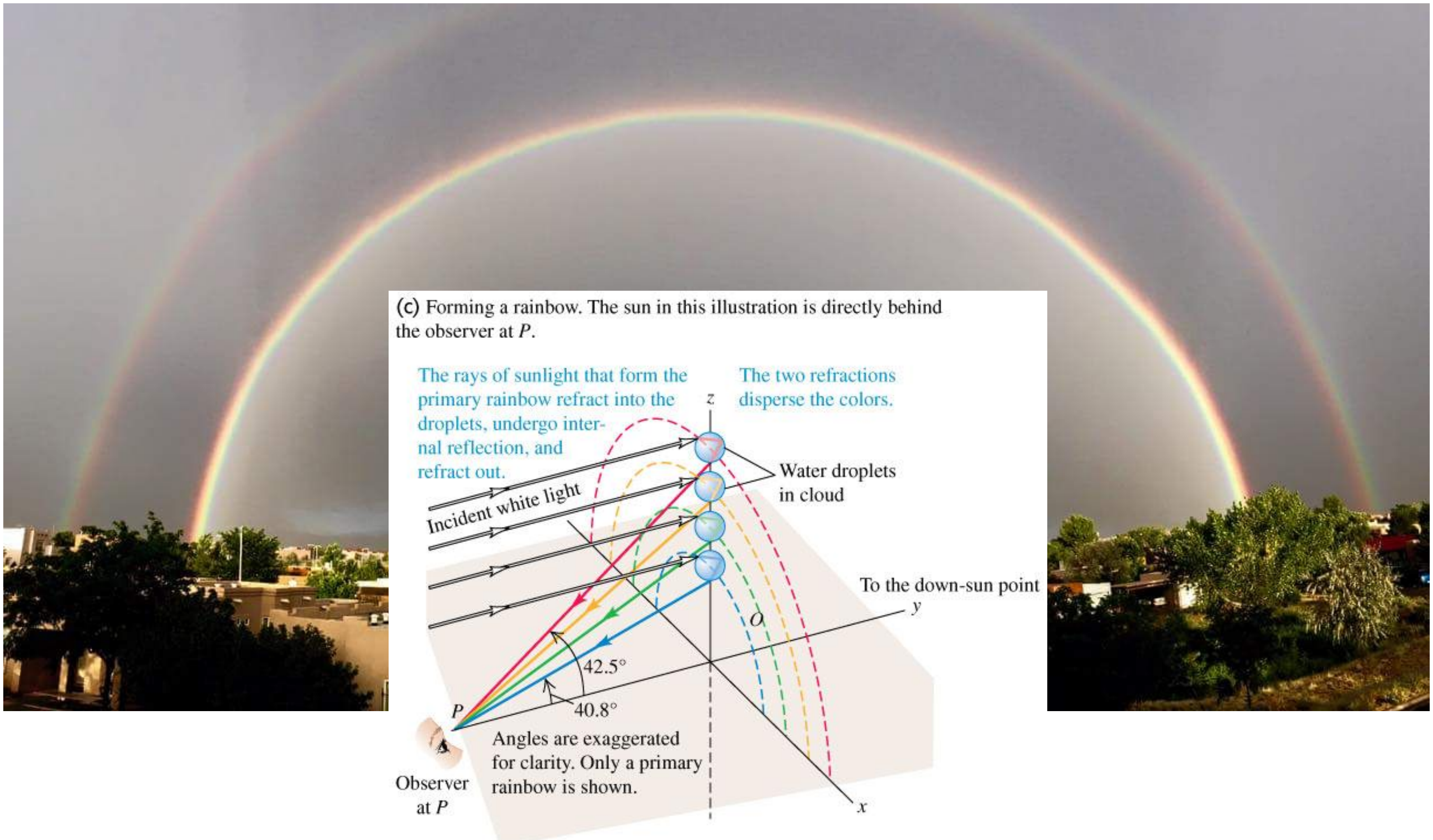


(e) A secondary rainbow is formed by rays that undergo two refractions and *two* internal reflections. The angle Δ is larger for violet light than for red.



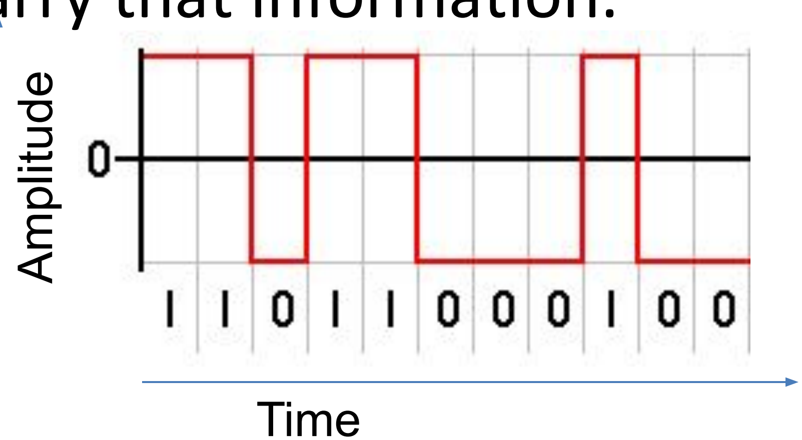
(a) A double rainbow




(c) Forming a rainbow. The sun in this illustration is directly behind the observer at P.
The rays of sunlight that form the primary rainbow refract into the droplets, undergo internal reflection, and refract out.
The two refractions disperse the colors.
z
Water droplets in cloud
Incident white light
To the down-sun point
y
O
42.5°
40.8°
P
Angles are exaggerated for clarity. Only a primary rainbow is shown.
Observer at P
x

Dispersion in Fiber Optics

- Okay, now let's think about dispersion in fiber optics.
- How do you send a signal down a fiber optic cable?
- A: a laser. But that is monochromatic, so no dispersion, right?
- But you have to send information, so the light beam must be modulated (amplitude must change with time) to carry that information.



Dispersion in Fiber Optics

- Once you modulate the amplitude, you no longer have a sinusoidal wave.
- That wave (let's take a simple square-wave function) is now a combination of frequencies, which can be determined by a Fourier transform of the square-wave.
- Each of these frequencies travel down the fiber optic at different velocities, determined by the dispersive properties of the glass.
- This limits the distance that signals can travel without re-modulation.

$$\phi(t) = \sum_{n=1}^{\infty} c_n \varphi_n(t)$$

$$\phi_{\text{square}}(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin\left(2\pi(2n-1)ft\right)}{2n-1}.$$

Colors