Lecture 17
(Geometric Optics I
Plane and Spherical Optics)

Physics 2310-01 Spring 2020

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Optics -Wikipedia

- Optics is the branch of physics which involves the behavior and properties of light, including its interactions with matter and the construction of instruments that use or detect it. Optics usually describes the behavior of visible, ultraviolet, and infrared light. Because light is an electromagnetic wave, other forms of electromagnetic radiation such as X-rays, microwaves, and radio waves exhibit similar properties.

- Optics began with the development of lenses by the ancient Egyptians and Mesopotamians. The earliest known lenses, made from polished crystal, often quartz, date from as early as 700 BC for Assyrian lenses such as the Layard/Nimrud lens. The ancient Romans and Greeks filled glass spheres with water to make lenses. These practical developments were followed by the development of theories of light and vision by ancient Greek and Indian philosophers, and the development of geometrical optics in the Greco-Roman world. The word optics comes from the ancient Greek word ὀπτική, meaning "appearance, look".

How We Perceive

- Our perception of an object is based on the directionality of the light (rays) coming from that object.

- More on how the eye works later.

- Think of an object as a collection of point sources, each radiating light uniformly (towards the observer, anyway).

- By placing some medium between the *object* and our observation, we can manipulate the *image* that we see.

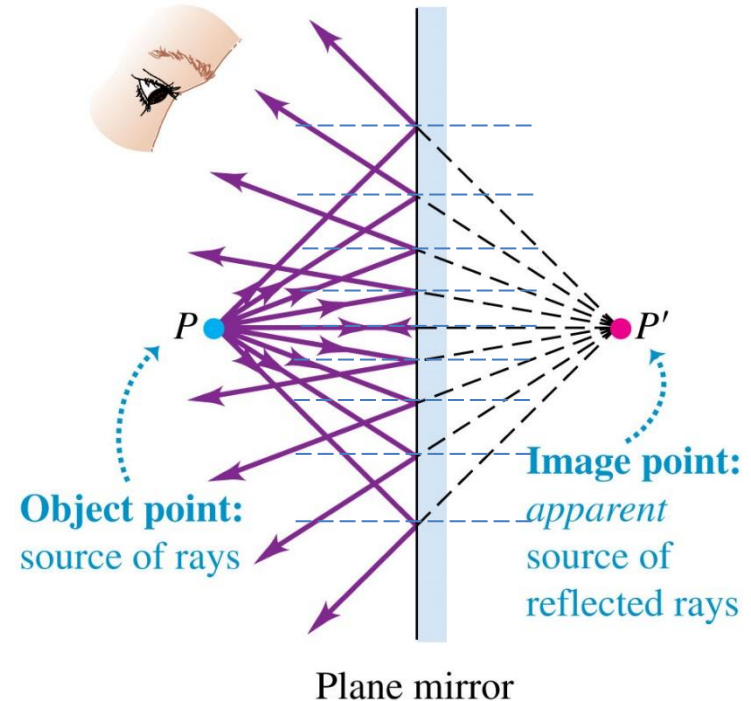- We only perceive an image by the rays that end up hitting our eyes!
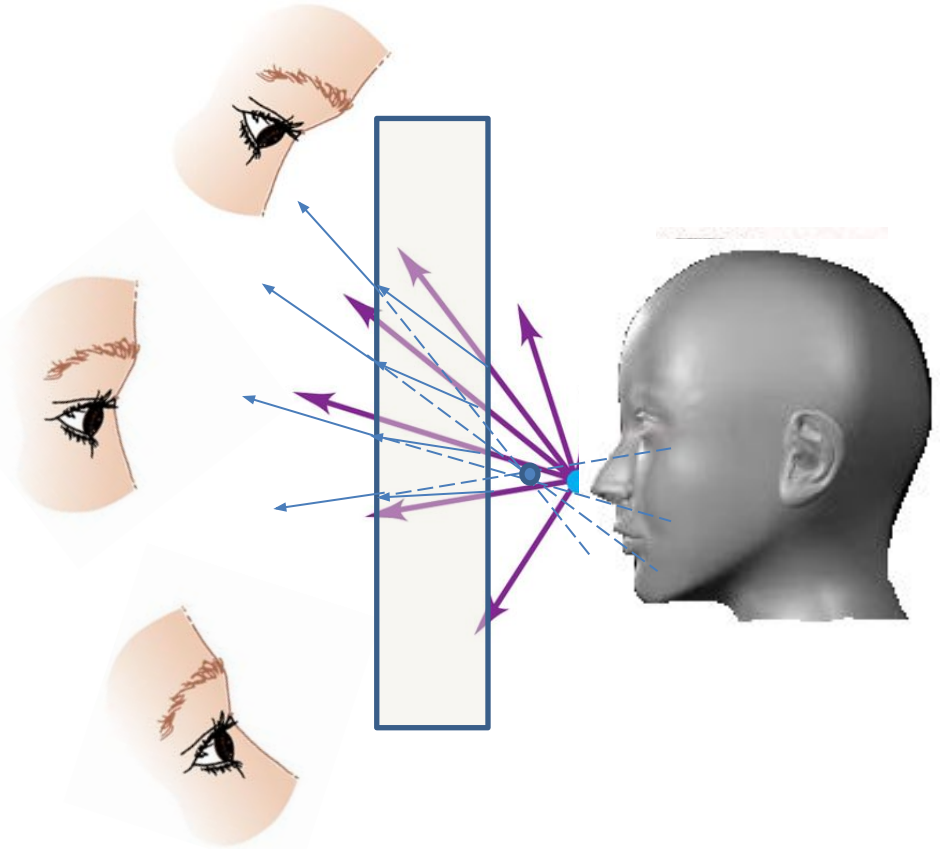
Virtual and Real Images

- Notice that the rays that we see form the image don't actually pass through the image.
- In this case, the solid blue lines are the real rays, and the dotted blue lines are just the direction of the rays as we perceive them.
- This image is then a *virtual* image.
- Without the block of glass, the rays we see are actually tracing back to the *image* that we perceive.
- This image is then a *real* image.
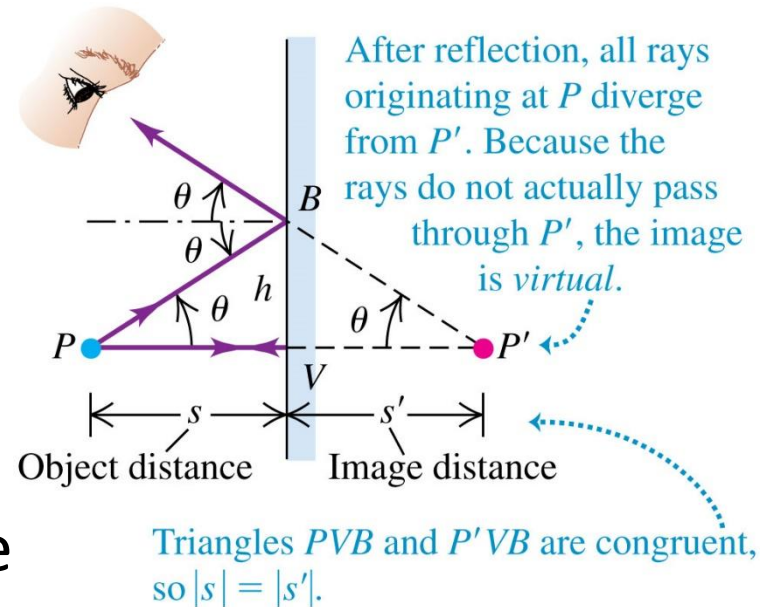- Practice will make this clearer…
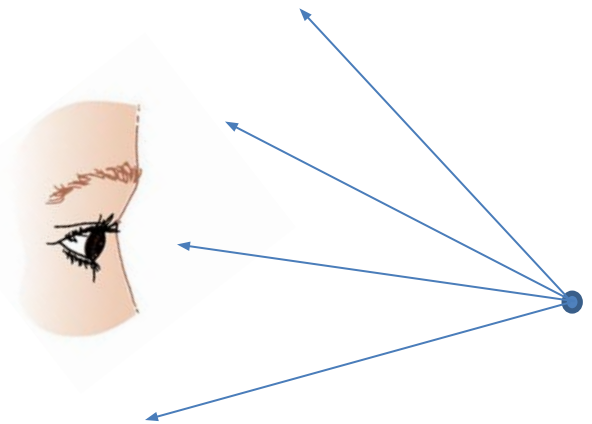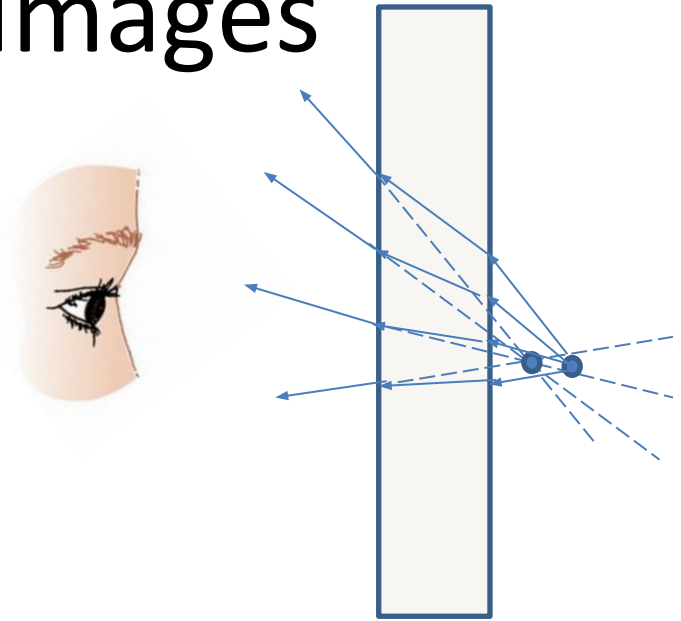
Plane Mirror

- Let's begin with a plane mirror, and a point source of light.
- Rays emanate from the point source out in all directions, but we only concern ourselves with those heading in the direction of the mirror.
- The law of reflection says that each ray striking the mirror reflects with the same angle of reflection as its angle of incidence. $\theta_r = \theta_a$
- The rays are then perceived to come from a point on the other side of the mirror.
- Is this a real or virtual image?

Object point: source of rays

P

Image point: *apparent* source of reflected rays

P'

Plane mirror

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Plane Mirror

- Let's use some geometry to analyze the apparent distance to the image.
- We don't need all the rays, just choose two that are simple to analyze.
- The two triangles (PVB and P'VB) are congruent, so therefore the distance from the mirror to the image is the same as the distance from the mirror to the object, or

$$|s| = |s'|$$

After reflection, all rays originating at P diverge from P'. Because the rays do not actually pass through P', the image is *virtual*.

Object distance   Image distance

Triangles *PVB* and *P'VB* are congruent, so $|s| = |s'|$.

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Sign Conventions

- Now, we will want to develop a set of equations that relate things like distances to images, focal points and magnification.

- In order to do this in a consistent way, we have to set up some conventions for how we assign signs (+/-) to distances.

- These don't really have anything to do with vectors, just relations between rays and positions:

  - Object distance is positive if it is on the same side (of interface) as the incoming rays, else it is negative.

  - Image distance is positive if it is on the same side (of interface) as the outgoing rays, else it is negative.
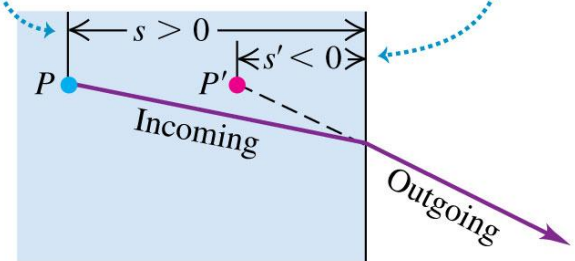
(a) Plane mirror

Outgoing

Incoming

P P'

$\longleftarrow s > 0 \longrightarrow$ $\longleftarrow s' < 0 \longrightarrow$
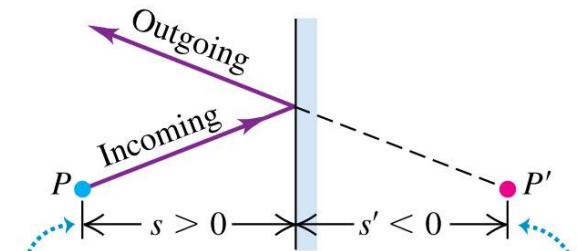
In both of these specific cases:

Object distance s is *positive* because the *object is on the same side as the incoming light.*

Image distance s' is *negative* because the image is NOT on the same side as the outgoing light.

(b) Plane refracting interface

$\longleftarrow s > 0 \longrightarrow$ $\longleftarrow s' < 0 \longrightarrow$

P P'

Incoming

Outgoing

Extended Objects and Magnification

- One can just think of an extended object as a collection of point sources, but to tell what the image is going to look like, you generally don't need to trace rays from every point.

- For a plane mirror, for instance, one need to only trace two rays from the tip of the object…

- The (lateral) magnification of an object is defined as the ratio of the object's lateral size to the images lateral size:

$$m = \frac{y'}{y}$$

- For a plane mirror the magnification is 1.
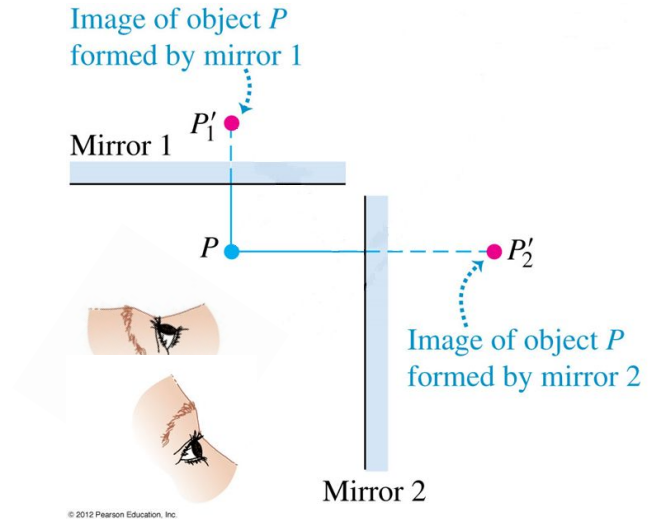
- Notice, that since s = -s':

$$m = \frac{-s'}{s}$$

For a plane mirror, PQV and $P'Q'V$ are congruent, so $y = y'$ and the object and image are the same size (the lateral magnification is 1).
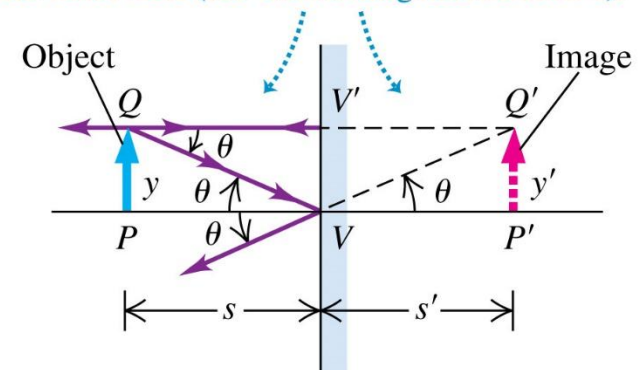
Object   Image

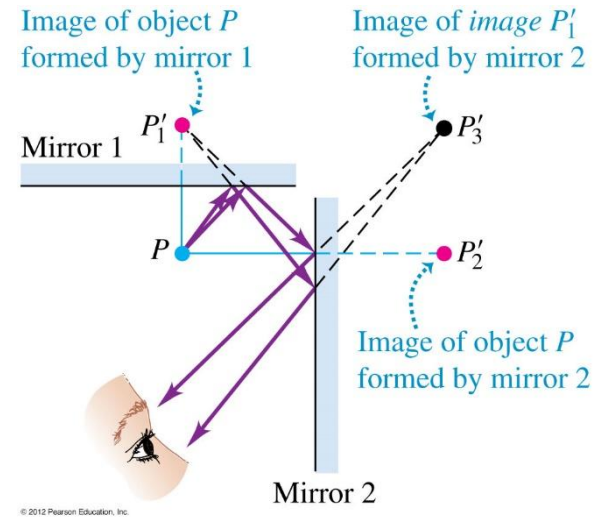© 2012 Pearson Education, Inc.

Images as Objects

- Are there any other images in the mirrors?

- Both P_1' and P_2' are virtual images of P.

- But there is another…

Image of object *P* formed by mirror 1

Mirror 1

P_1'

P

P_2'

Image of object *P* formed by mirror 2

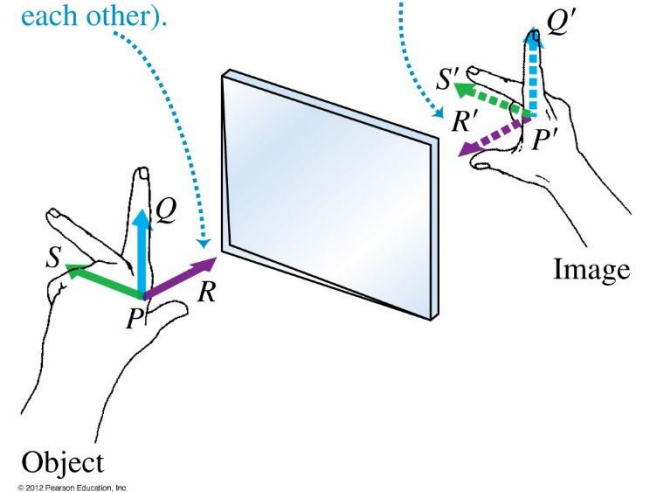Mirror 2

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Images as Objects

- An image can also play the role of an object.

- You can have an image of an image (of an image…).

- We will see more of this when we get to thin lenses.

Image of object P formed by mirror 1

Image of *image* P'_1 formed by mirror 2

Mirror 1

P'_1

P'_3

P

P'_2

Image of object P formed by mirror 2

Mirror 2

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Mirror Symmetry

An image made by a plane mirror is reversed back to front: the image thumb $P'R'$ and object thumb PR point in opposite directions (toward each other).

- A mirror does one important thing to the image of an object. It reverses it **back to front**.

- Notice that any arrow pointing in a direction in a plane parallel to the mirror has an image that points in the same direction.

- Whereas an arrow that points either towards or away from the mirror will have an image that points in its opposite direction.

- The image and object together create a bilateral symmetry (or mirror symmetry).

"Ophrys apifera (flower)" by © Hans Hillewaert /. Licensed under CC BY-SA 3.0 via Commons - https://commons.wikimedia.org/wiki/File:Ophrys_apifera_(flower).jpg #/media/File:Ophrys_apifera_(flower).jpg

Spherical Mirrors

- Rays striking a concave spherical mirror converge at a point creating a real image.
- We will limit our discussion of spherical mirrors to rays that make small angles, α, relative to the central axis of the mirror.
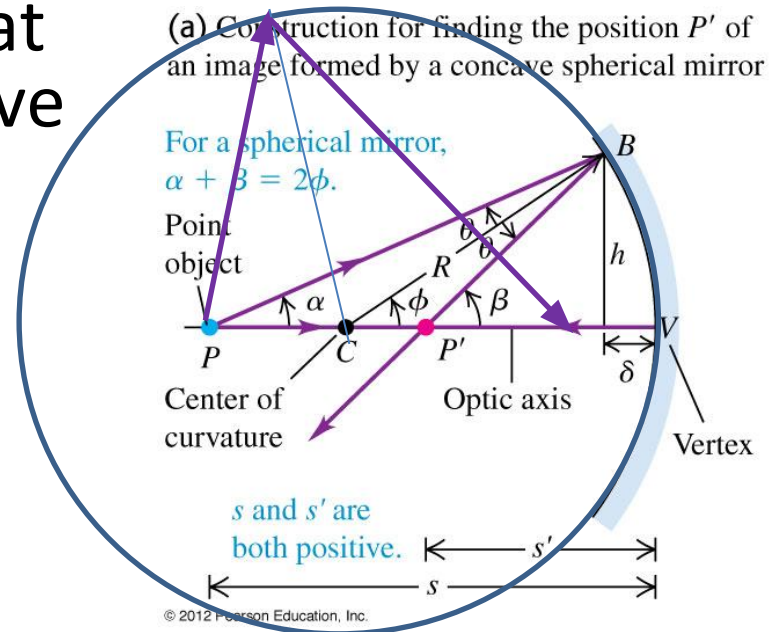- Here's why: Spherical aberration.

(b) The paraxial approximation, which holds for rays with small α
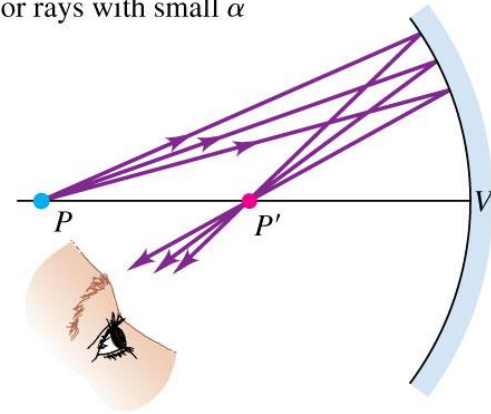
All rays from P that have a small angle α pass through P', forming a real image.

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(a) Construction for finding the position P' of an image formed by a concave spherical mirror

For a spherical mirror, $\alpha + \beta = 2\phi$.

Point object

Center of curvature

Optic axis

Vertex

s and s' are both positive.
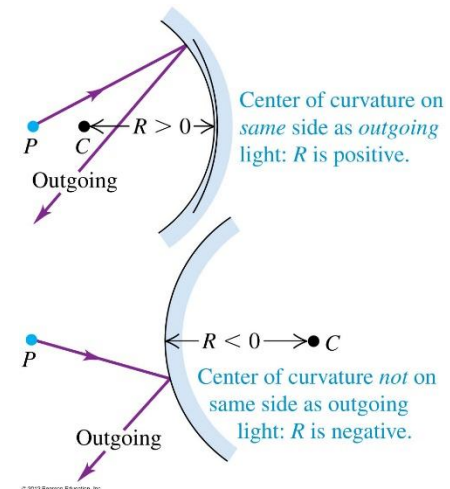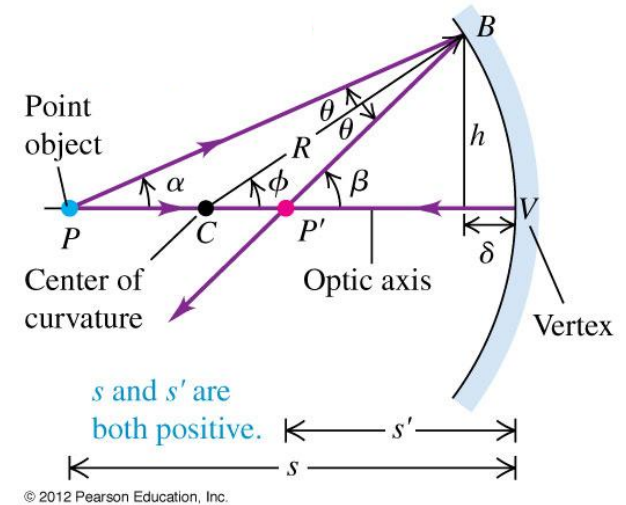
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Spherical Mirrors

- Keeping with that ***small angle assumption***, one can relate the object distance to the image distance and radius by:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

- Again, by convention, R is considered negative if the center of curvature is not on the same side as the outgoing light.

(a) Construction for finding the position P' of an image formed by a concave spherical mirror

s and s' are both positive.

Center of curvature on *same side* as *outgoing* light: R is positive.

$R > 0$

Center of curvature *not* on same side as outgoing light: R is negative.

$R < 0$

Focal Points

- If we let the object distance go to infinity, then we get plane waves (parallel rays), and the spherical mirror equations tells us that the *focal point* of the mirror is:
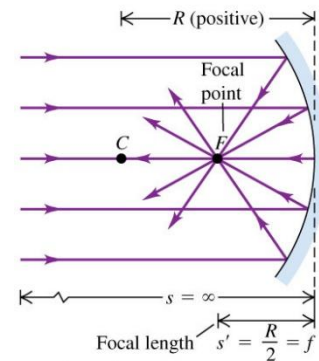
$$\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R}$$

$$f = \frac{R}{2}$$

- So, we can rewrite the equation as:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

- Focal point, f, is positive for concave, negative for convex.
- Notice that if you can turn the rays around…

(a) All parallel rays incident on a spherical mirror reflect through the focal point.

R (positive)

Focal point

C F

$s = \infty$

Focal length $s' = \frac{R}{2} = f$

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(a) Paraxial rays incident on a convex spherical mirror diverge from a virtual focal point.

R (negative)

F C

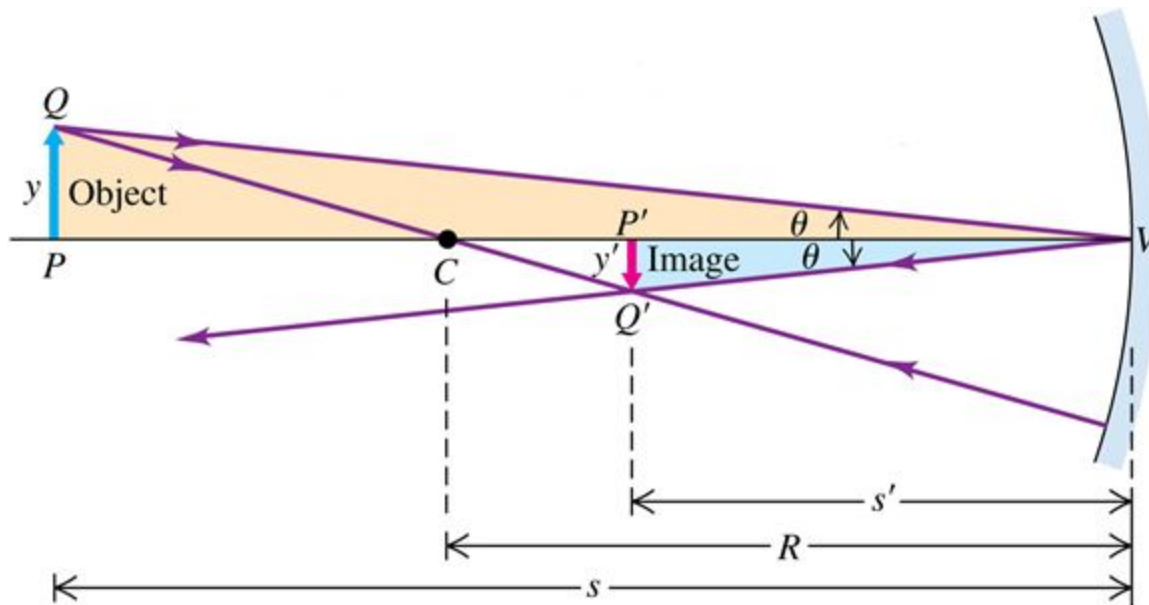Virtual focal point

$s' = \frac{R}{2} = f$

$s = \infty$
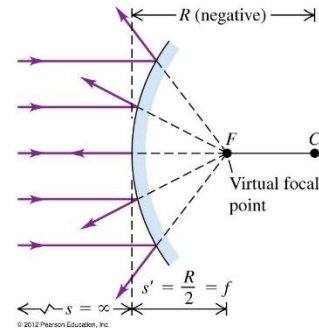
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Magnification

- We can use the geometry of a couple of rays to determine the lateral magnification for spherical mirrors.
- Note that the two triangles are similar, so the ratio of the image to object size must be the same as the ratio of the distance to the image and the distance to the object:
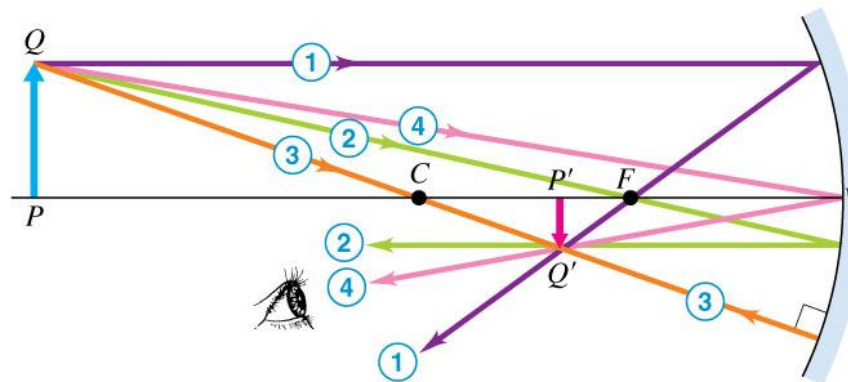
$$\cos\theta = \left|\frac{y}{s}\right| = \left|\frac{y'}{s'}\right| \Rightarrow \qquad m = \frac{y'}{y} = \frac{-s'}{s}$$

Principle Rays

- You can get a good feel for what a piece of optics can do by tracing rays.
- Some random ray can be difficult to properly draw, but…
- There is a set of rays that are relatively straightforward.
- These are called principle rays:

(a) Principal rays for concave mirror

1. Ray parallel to axis reflects through focal point.
2. Ray through focal point reflects parallel to axis.
3. Ray through center of curvature intersects the surface normally and reflects along its original path.
4. Ray to vertex reflects symmetrically around optic axis.

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Principle Rays

- You can get a good feel for what a piece of optics can do by tracing rays.
- Some random ray can be difficult to properly draw, but…
- There is a set of rays that are relatively straightforward.
- These are called principle rays:

(b) Principal rays for convex mirror
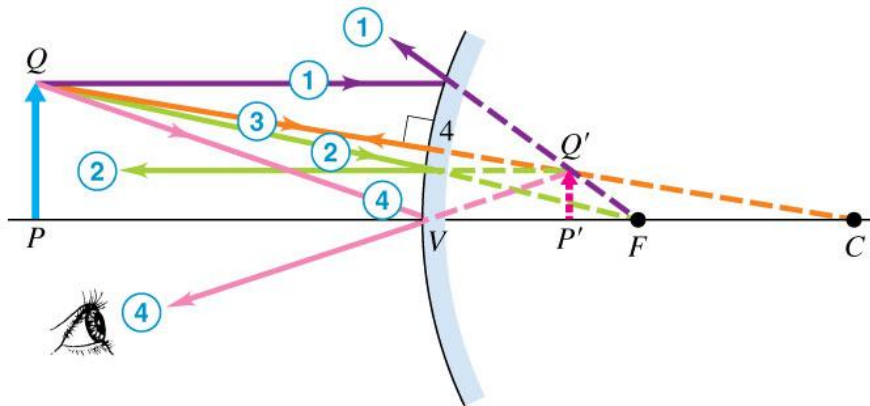
1. Reflected parallel ray appears to come from focal point.
2. Ray toward focal point reflects parallel to axis.
3. As with concave mirror: Ray radial to center of curvature intersects the surface normally and reflects along its original path.
4. As with concave mirror: Ray to vertex reflects symmetrically around optic axis.

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