Lecture 19
(Interference I
Two-Source Interference)

Physics 2310-01 Spring 2020

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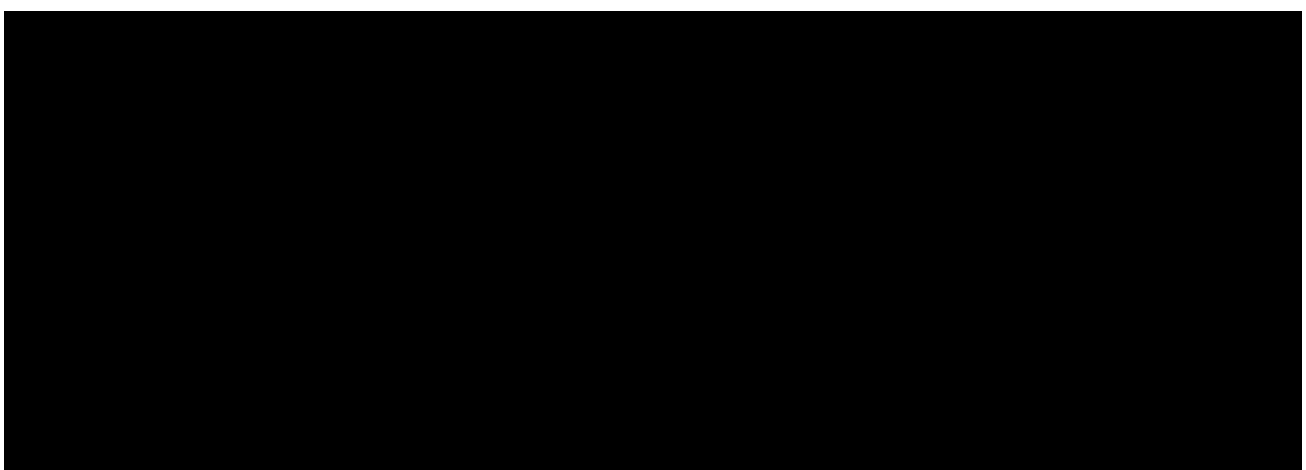
http://www.cabrillo.edu/~jmccullough/Applets/OSP/Oscillations_and_Waves/waves_interference.jar

https://www.youtube.com/watch?v=Iuv6hY6zsd0

Principle of Superposition- Wikipedia

- In physics and systems theory, the **superposition principle**,[1] also known as **superposition property**, states that, for all linear systems, the net response at a given place and time caused by two or more stimuli is the sum of the responses which would have been caused by each stimulus individually. So that if input *A* produces response *X* and input *B* produces response *Y* then input (*A* + *B*) produces response (*X* + *Y*).
- The superposition principle applies to *any* linear system, including algebraic equations, linear differential equations, and systems of equations of those forms. The stimuli and responses could be numbers, functions, vectors, vector fields, time-varying signals, or any other object which satisfies certain axioms. Note that when vectors or vector fields are involved, a superposition is interpreted as a vector sum.



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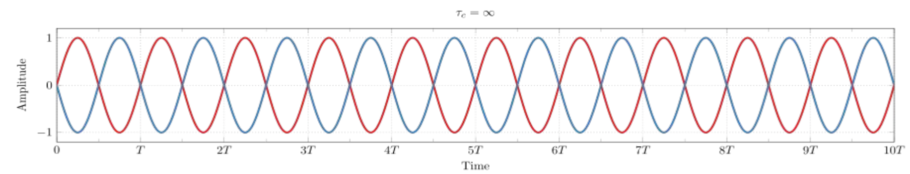
What’s the difference between superposition and interference?

- Nothing really.
- Generally speaking, when one talks about interference, the sources of light are ***coherent***.

Coherence - Wikipedia

- In [physics](), two wave sources are perfectly coherent if they have a constant [phase difference]() and the same frequency. It is an ideal property of [waves]() that enables stationary (i.e. temporally and spatially constant) [interference](). It contains several distinct concepts, which are limiting cases that never quite occur in reality but allow an understanding of the physics of waves, and has become a very important concept in quantum physics. More generally, **coherence** describes all properties of the [correlation]() between [physical quantities]() of a single wave, or between several waves or wave packets.
- Spatial coherence describes the correlation (or predictable relationship) between waves at different points in space, either lateral or longitudinal.[1]
- Temporal coherence describes the correlation between waves observed at different moments in time.



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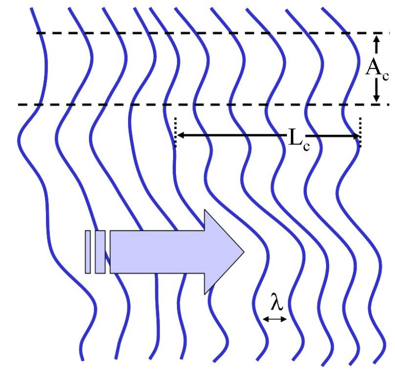

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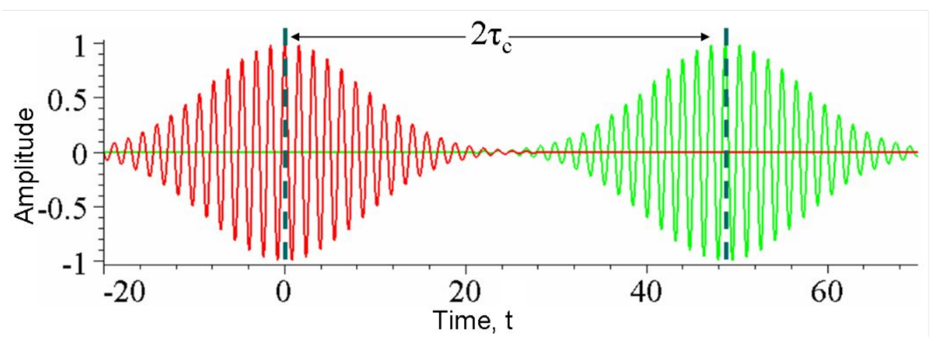

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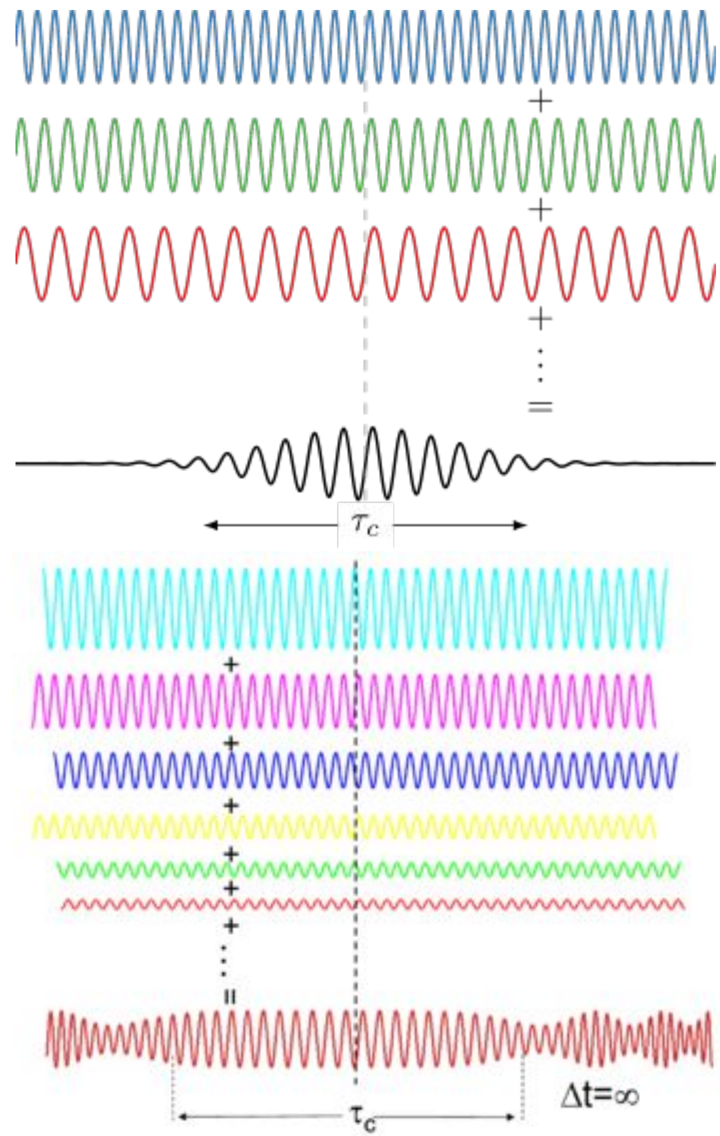
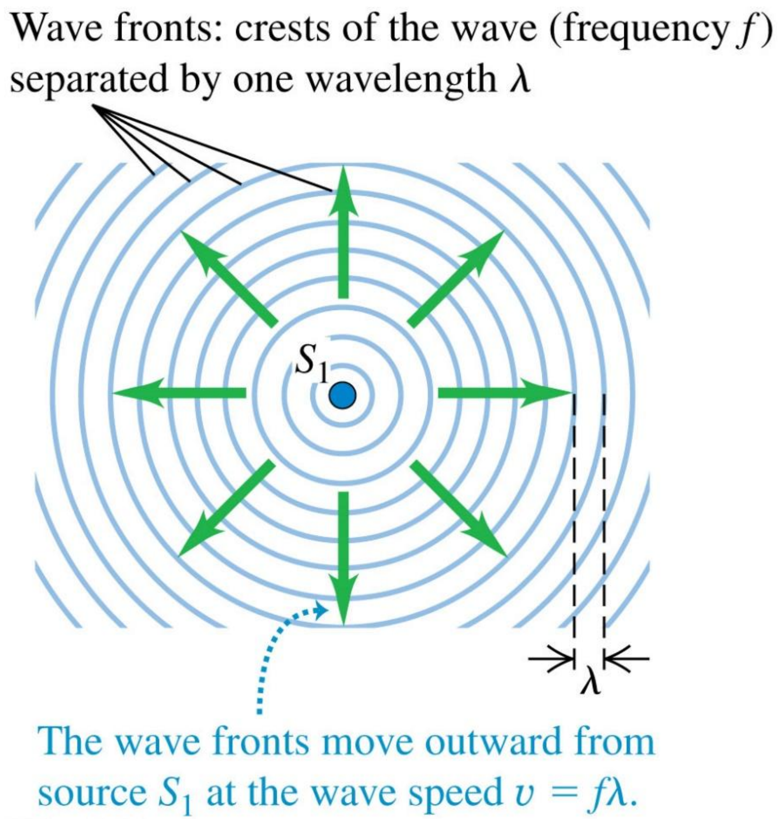
Coherence and Interference

- Coherence is important in the study and observation of interference because when coherent light interferes, it creates stable (or stationary) interference patterns.
- Incoherent light still interferes, but the resultant pattern changes with time (and on a time scale that is proportional to the level of incoherence), making the patterns difficult to detect (or just uninteresting).
- We keep with coherent light (**spatially, temporally, and in phase**) in order to best understand the nature of interference.
- But we will have to look at spatial coherence to understand some phenomena (like the two-slit with the sun).



Single Source

- Remember, what we “see” of a light source on a screen, is the average of the electric field squared over time, the intensity.
- At any instant, the electric field can be anywhere from $-E_{max}$ to E_{max}.
- When we deal with interference, we have to deal with the waves (and hence worry about phase).
- We will get to the intensity next lecture.



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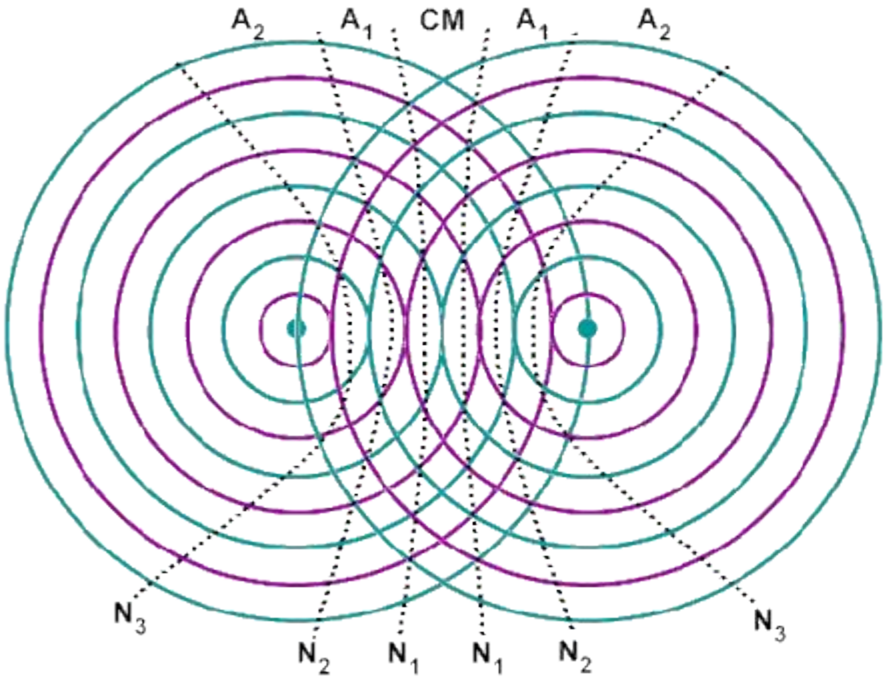
Two Sources

- Some things to remember:
  - With point sources, there is radiation out in 3D.
  - We will usually define a plane through both points at random.
  - We will start by confining ourselves to purely coherent and monochromatic sinusoidal sources.
  - We will begin by looking at sources with no phase difference (will relax this later).



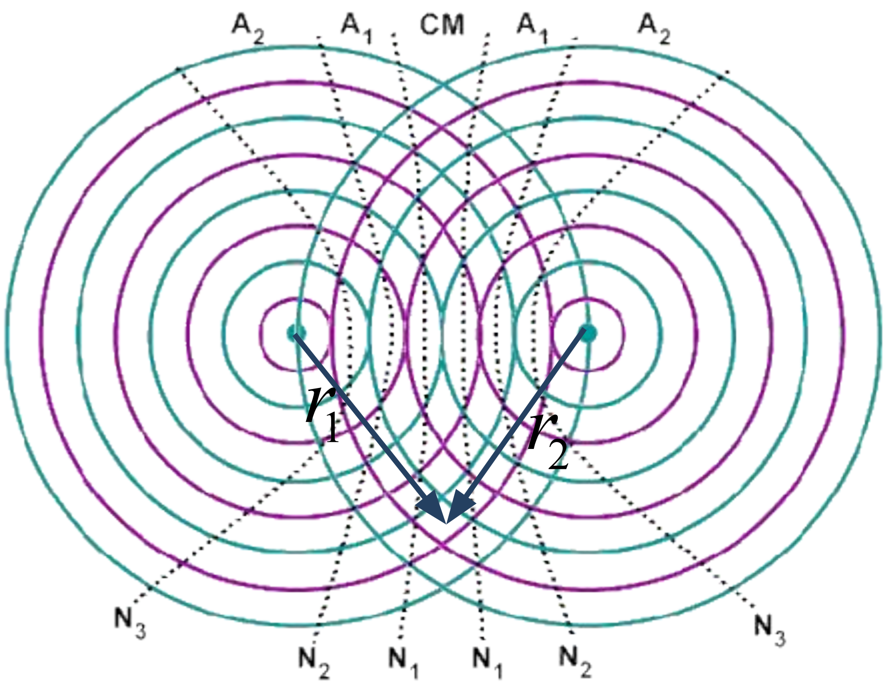
Phase Difference

- As the waves from the sources move out, they superimpose.
- If we draw rays out from the center to a particular point, we can determine the relative phases of the waves by looking at the distances they travelled from their source (since they started in phase):

$$E_1(r_1,t) = E(r_1)\cos(kr_1 - \omega t)$$
$$E_2(r_2,t) = E(r_2)\cos(kr_2 - \omega t)$$

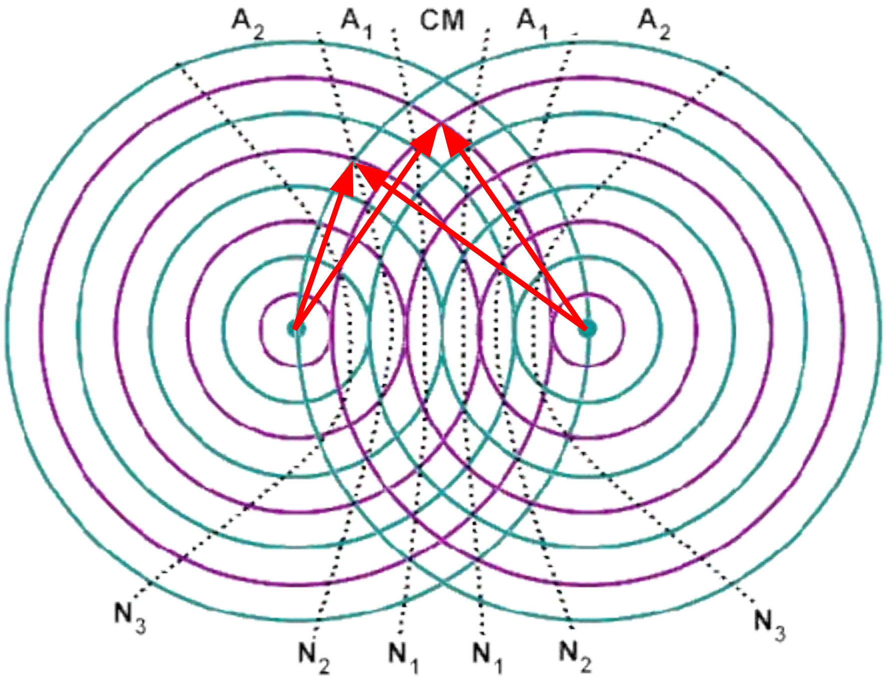
- At a particular point in space and time (same for both waves), the phase difference is just:

$$\delta\phi = kr_1 - kr_2 = \frac{2\pi}{\lambda}(r_1 - r_2)$$



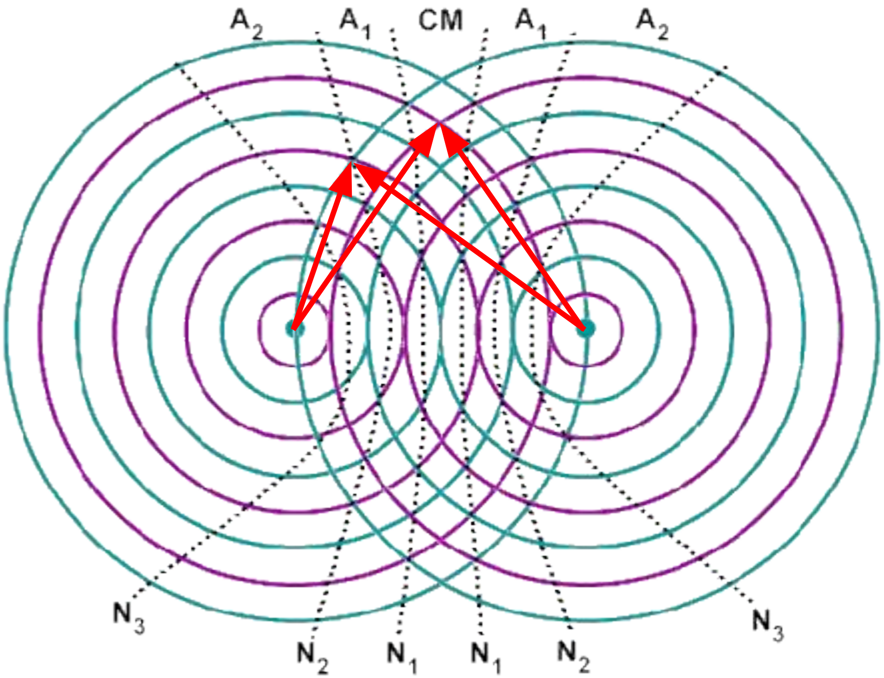
Constructive and Destructive Interference

- Now, if the phase difference between the two waves at a certain point (and all times) is π (or 3π, or 5π, etc.), then at that point, and for all times, they are out of phase (completely) and destructively interfere.
- If, on the other hand, the phase difference is 0 (or 2π, or 4π, etc.) then at that point and for all times they add constructively.
- Of course, all other possibilities also exist, where the interference is neither completely destructive or constructive.



$$\delta\phi = kr_1 - kr_2 = \frac{2\pi}{\lambda}(r_1 - r_2)$$

Constructive and Destructive Interference



- So, for destructive interference:

$$\delta\phi = \frac{2\pi}{\lambda}(r_1 - r_2) = -5\pi, -3\pi, -\pi, \pi, 3\pi, 5\pi \ldots \Rightarrow$$

$$(r_1 - r_2) = \frac{-5\lambda}{2}, \frac{-3\lambda}{2}, \frac{-\lambda}{2}, \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots$$

$$(r_1 - r_2) = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2 \ldots$$

- And for constructive interference:

$$\delta\phi = \frac{2\pi}{\lambda}(r_1 - r_2) = -4\pi, -2\pi, 0, 2\pi, 4\pi \ldots \Rightarrow$$

$$(r_1 - r_2) = -2\lambda, -\lambda, 0, \lambda, 2\lambda \ldots = m\lambda, \quad m = 0, \pm 1, \pm 2 \ldots$$

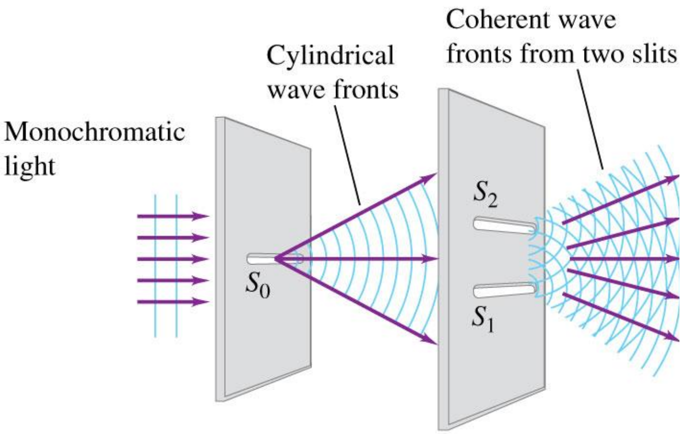
Adding Variables

- Now, other than just looking at the pretty picture and seeing where wave fronts overlap, this is so far not very useful.
- What is the quantitative effects of:
  - Moving the sources relative to each other?
  - Changing the phases of the sources?
  - Changing the wavelengths of the sources?

Young's Two-Slit Experiment

- In order to study the effect of changing the distance between the sources, we will go back to a seminal experiment performed in 1800 by Thomas Young.
- They didn't have coherent sources then, so Young created his own by splitting a single source into two.
- Because the two sources S_1 and S_2, are taken from the same wave, and from two places near each other, they are coherent.

(a) Interference of light waves passing through two slits



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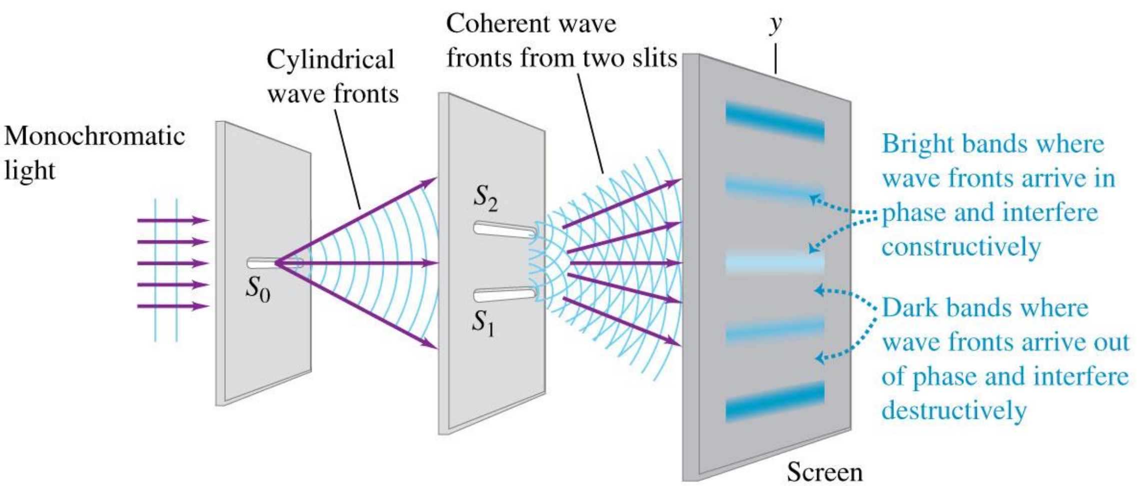
Young's Two-Slit Experiment

- If you then let the light waves from these two sources strike a screen some distance away, they will create a diffraction pattern, as we have seen before.
- But now, we have ways to play around with the geometry.

(a) Interference of light waves passing through two slits



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Young's Two-Slit Experiment

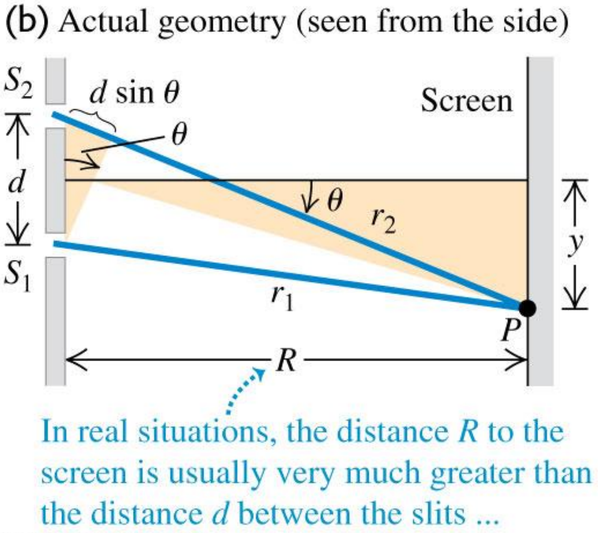
- Let's define the relevant geometric variables:
  - The distance between S_1 and S_2 we will call ***d***.
  - The distance to the screen is ***R***.
  - The wavelength is ***λ***.
  - The distance from the bisector of the two slits to a particular position on the screen is ***y***.
  - r_1 and r_2 are the distance that the waves have to travel to the screen.
- To see if there is constructive or destructive interference at the screen we apply our equations:

Constructive

$$(r_1 - r_2) = m\lambda, \quad m = 0, \pm 1, \pm 2 \ldots$$

Destructive

$$(r_1 - r_2) = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2 \ldots$$



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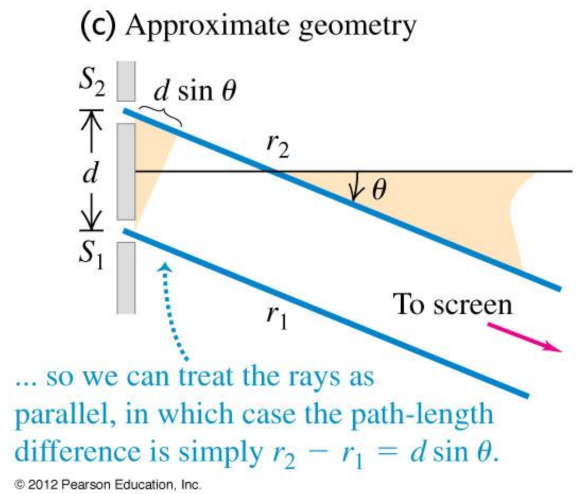
Young's Two-Slit Experiment

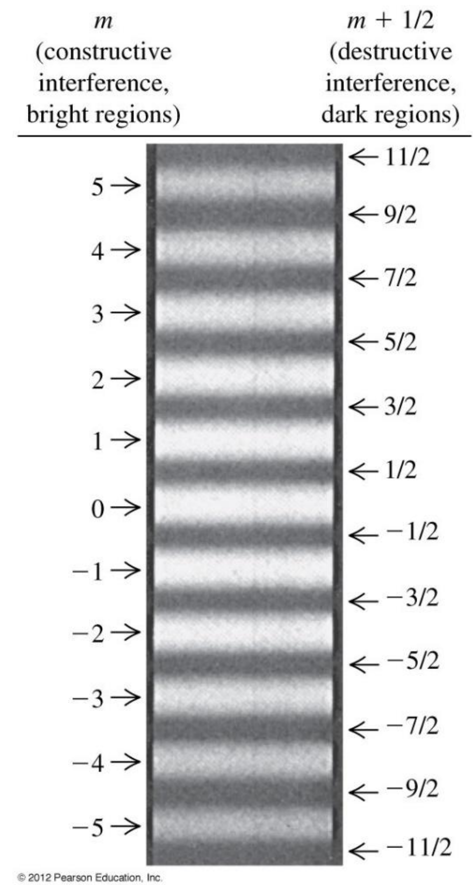
- If ***d*** is small compared to ***R*** (usually the case), then,

$$(r_1 - r_2) = d \sin\theta$$

and

$$\tan\theta \approx \sin\theta = \frac{y}{R}$$





Constructive

$$\frac{d}{R} y = m\lambda, \quad m = 0, \pm 1, \pm 2 \ldots \Rightarrow$$

$$y_m = R\frac{m\lambda}{d}, \quad m = 0, \pm 1, \pm 2 \ldots$$

Destructive

$$\frac{d}{R} y = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2 \ldots$$

$$y_m = R\frac{\left(m + \frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm 1, \pm 2 \ldots$$