# Lecture 20
# (Interference II
# Phasors and Interference Intensity)

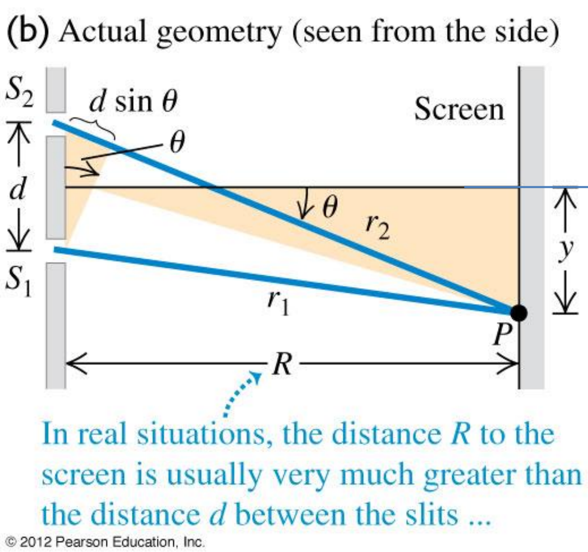
Physics 2310-01 Spring 2020

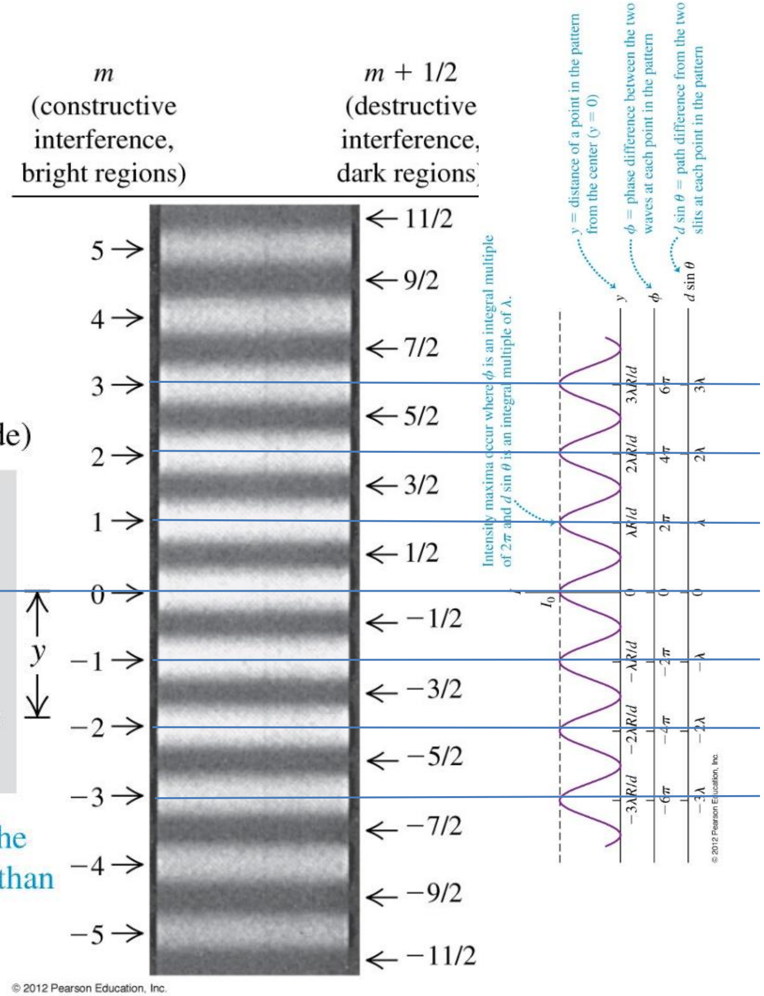
Douglas Fields

# Review

$$\delta\phi = kr_1 - kr_2 + (\phi_1 - \phi_2)$$
$$= \frac{2\pi}{\lambda}(r_1 - r_2) + (\phi_1 - \phi_2)$$

$$(r_1 - r_2) = d\sin\theta$$




$d\sin\theta = 3\lambda$

$d\sin\theta = 2\lambda$

$d\sin\theta = 1\lambda$

$d\sin\theta = 0\lambda$

For small angles: $\sin\theta \approx \tan\theta = \frac{y}{R}$

$$y_m = R\frac{m\lambda}{d}, \quad m = 0, \pm1, \pm2\ldots \quad \text{Constructive}$$

$$y_m = R\frac{\left(m + \frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm1, \pm2\ldots \quad \text{Destructive}$$

# Young's Two-Slit Experiment

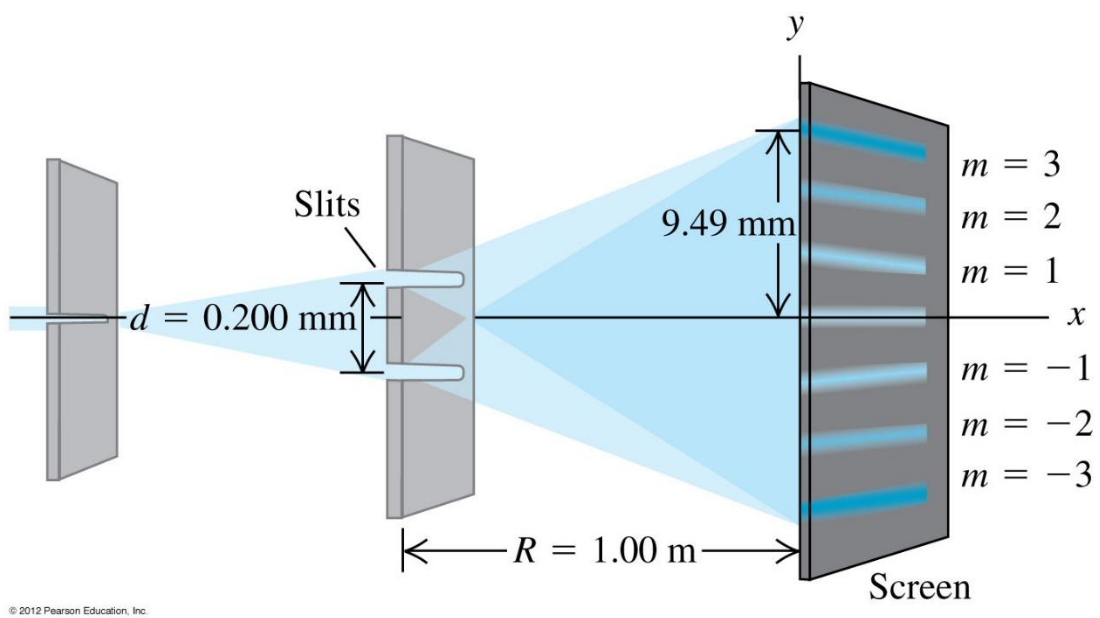
- What is the wavelength of the light in this example?

$$\frac{d}{R}y = m\lambda, \quad m = 0, \pm1, \pm2\ldots \Rightarrow$$

$$y_m = R\frac{m\lambda}{d}, \quad m = 0, \pm1, \pm2\ldots$$

$$\frac{d}{R}y = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm1, \pm2\ldots$$

$$y_m = R\frac{\left(m + \frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm1, \pm2\ldots$$

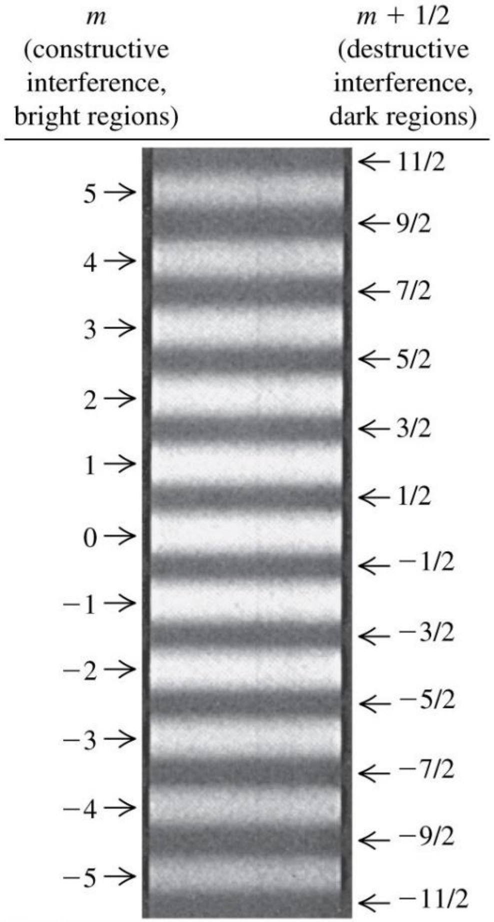


# Describing the Intensity Pattern

$$y_m = R\frac{m\lambda}{d}, \quad m = 0, \pm 1, \pm 2\ldots \quad \text{Constructive}$$

$$y_m = R\frac{\left(m+\frac{1}{2}\right)\lambda}{d}, \quad m = 0, \pm 1, \pm 2\ldots \quad \text{Destructive}$$

- OK, so now we know:
  - where the bright and dark spots are,
  - what happens if we change the distance between slits,
  - what happens when we change the wavelength of light
- But can we quantitatively describe the intensity pattern that we see?
- Yes, but we will need a tool that you saw earlier:
- Phasors.



# What is the intensity at some point?

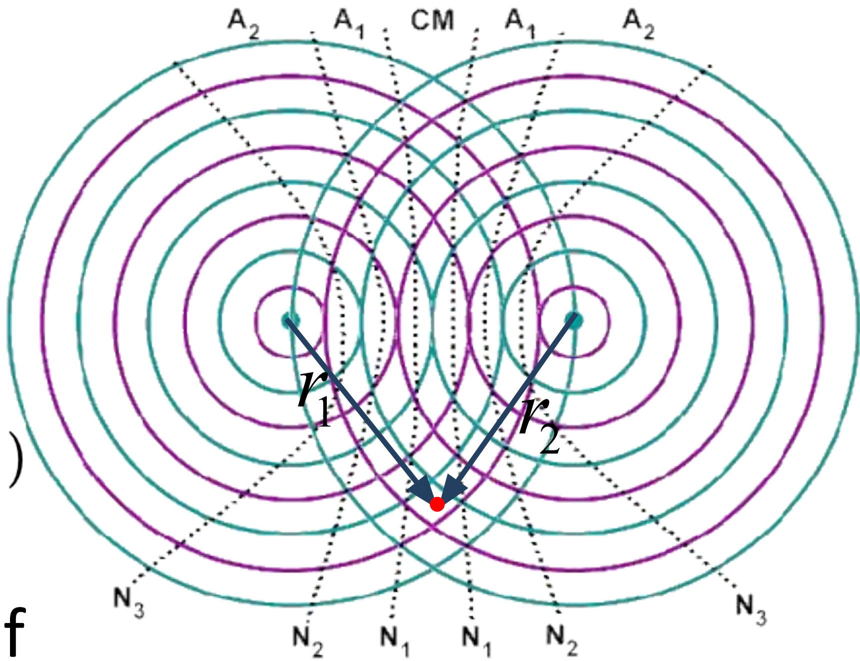
- The electric field (of an EM wave) is written generally as:

$$E_1(r,t) = E_1(r)cos(kr - \omega t + \phi_1^0)$$

- But, if we want to look at the intensity at a point:

$$E_1(r_1,t) = E_1(r_1)cos(kr_1 - \omega t + \phi_1^0)$$

- Then $r_1$ is a constant and $kr_1$ can be rolled into the phase of the wave at that point in space (it doesn't change with time).




$$E_1(r_1,t) = E_1(r_1)cos[-\omega t + (kr_1 + \phi_1^0)]$$

# Superposition with Phasors

- Now, if we want to add two waves (superimpose) we have to add the phasors ***vectorially***, where the only important phase is the ***phase difference***:

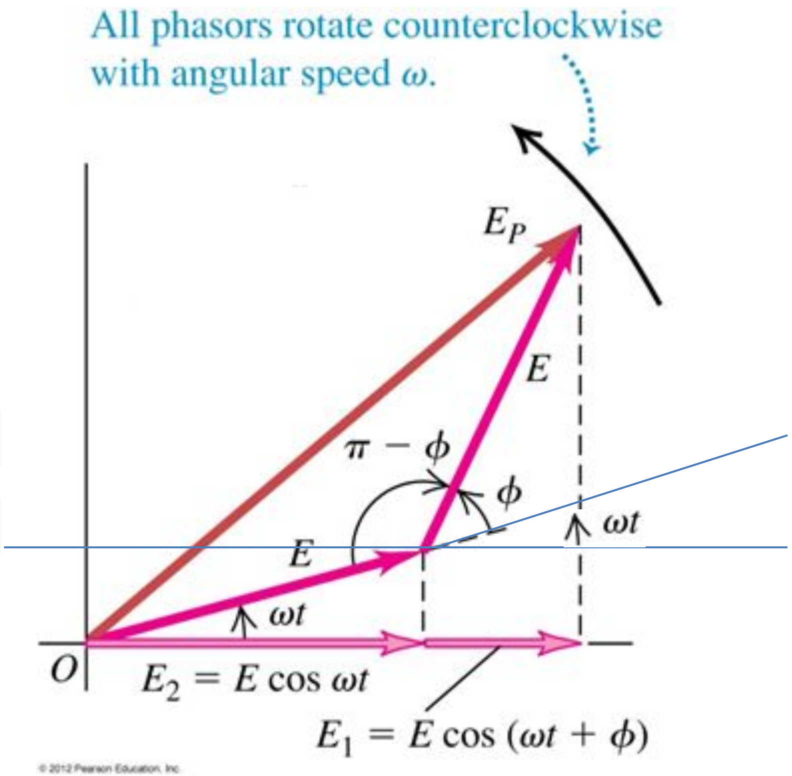
$$E_1(r_1,t) = E_1(r_1)cos[-\omega t + (kr_1 + \phi_1^0)]$$

$$E_2(r_2,t) = E_2(r_2)cos[-\omega t + (kr_2 + \phi_2^0)]$$

$$\Rightarrow$$

$$\delta\phi = (kr_2 + \phi_2^0) - (kr_1 + \phi_1^0)$$

$$\delta\phi = (kr_2 - kr_1) + (\phi_2^0 - \phi_1^0)$$



# Superposition with Phasors

- Then, we can find the amplitude of the resultant wave as the vector sum of the two phasors:

$$\mathbf{E_P} = \mathbf{E_1} + \mathbf{E_2}$$

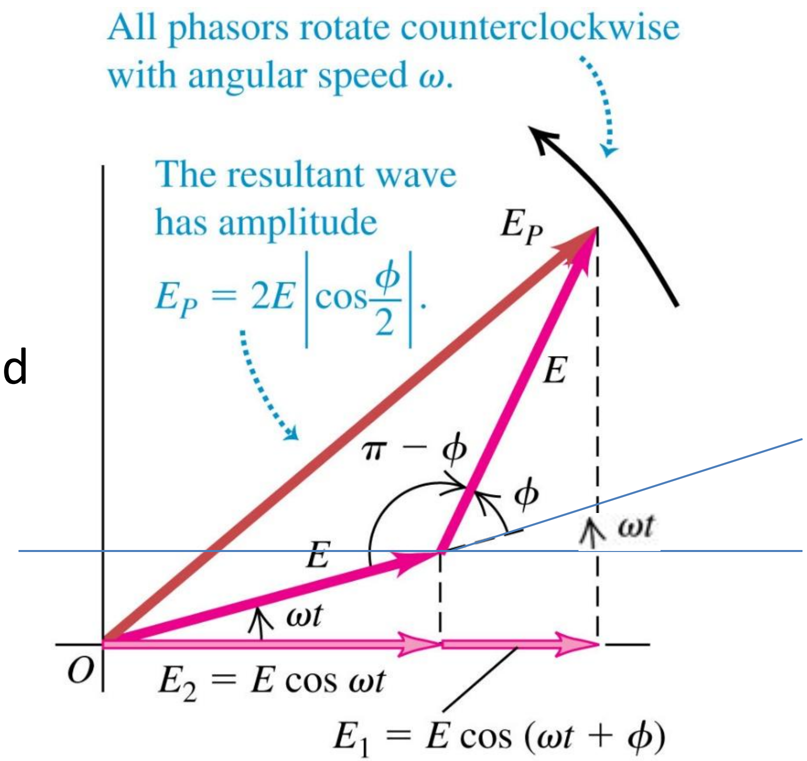
- Now, if $|E_1|$ is not equal to $|E_2|$, regardless of the phase difference, there will be no (complete) destructive interference.
- With $|E_1| = |E_2| = E$, we can use trig to find $E_P$:

$$\begin{aligned} E_P^2 &= E^2 + E^2 - 2E^2 cos(\pi - \delta\phi) \\ &= E^2 + E^2 + 2E^2 cos(\delta\phi) \\ &= 2E^2(1 + cos(\delta\phi)) \\ &= 4E^2 cos^2(\frac{\delta\phi}{2}) \end{aligned}$$

$$E_P = 2E \mid cos(\frac{\delta\phi}{2}) \mid$$

where,

$$\delta\phi = (kr_2 - kr_1) + (\phi_2^0 - \phi_1^0)$$ Phase DIFFERENCE!



# Amplitude and Intensity

- Now, $\boldsymbol{E_P}$ is the **amplitude** of the resultant phasor (it also rotates with angular frequency ω).

$$E_P(t) = E_P cos\omega t = 2E \mid cos(\frac{\delta\phi}{2}) \mid cos\omega t$$

- The intensity of light is just:

$$I = S_{Avg} = \frac{1}{2\mu_0 c} E^2_{Max}$$

$$E_{Max} = E_P = 2E \mid cos(\frac{\delta\phi}{2}) \mid \Rightarrow$$

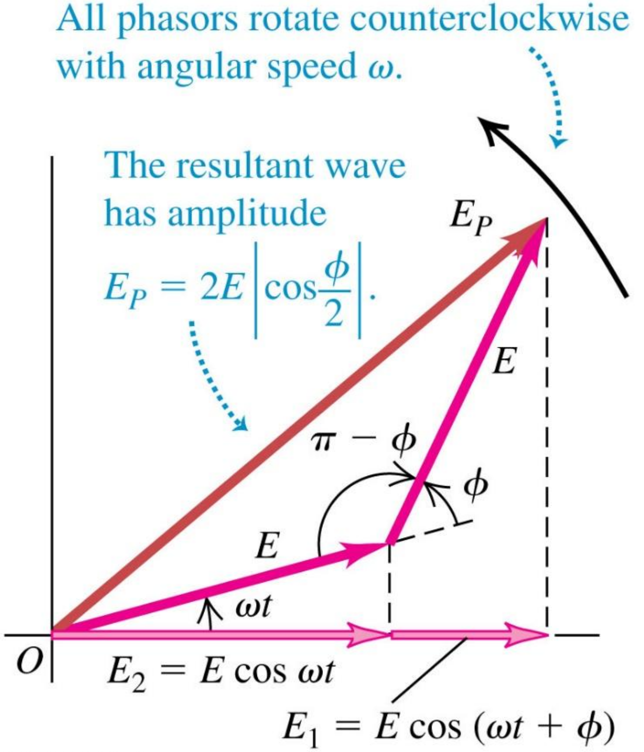
$$I = \frac{1}{2\mu_0 c}[2E cos(\frac{\delta\phi}{2})]^2$$

$$I = \frac{2E^2}{\mu_0 c} cos^2(\frac{\delta\phi}{2})$$

# Sources in Phase

- If the two sources are in phase, $\phi_1^0 = \phi_2^0 = \phi_0$ and then the phase difference at the position of interest is just:

$$E_1(r_1,t) = E_1(r_1)cos[-\omega t + (kr_1 + \phi_0)]$$
$$E_2(r_2,t) = E_2(r_2)cos[-\omega t + (kr_2 + \phi_0)]$$
$$\Rightarrow$$
$$\delta\phi = (kr_2 + \phi_0) - (kr_1 + \phi_0)$$
$$\delta\phi = (kr_2 - kr_1) \Rightarrow$$
$$\delta\phi = \frac{2\pi}{\lambda}(r_2 - r_1)$$

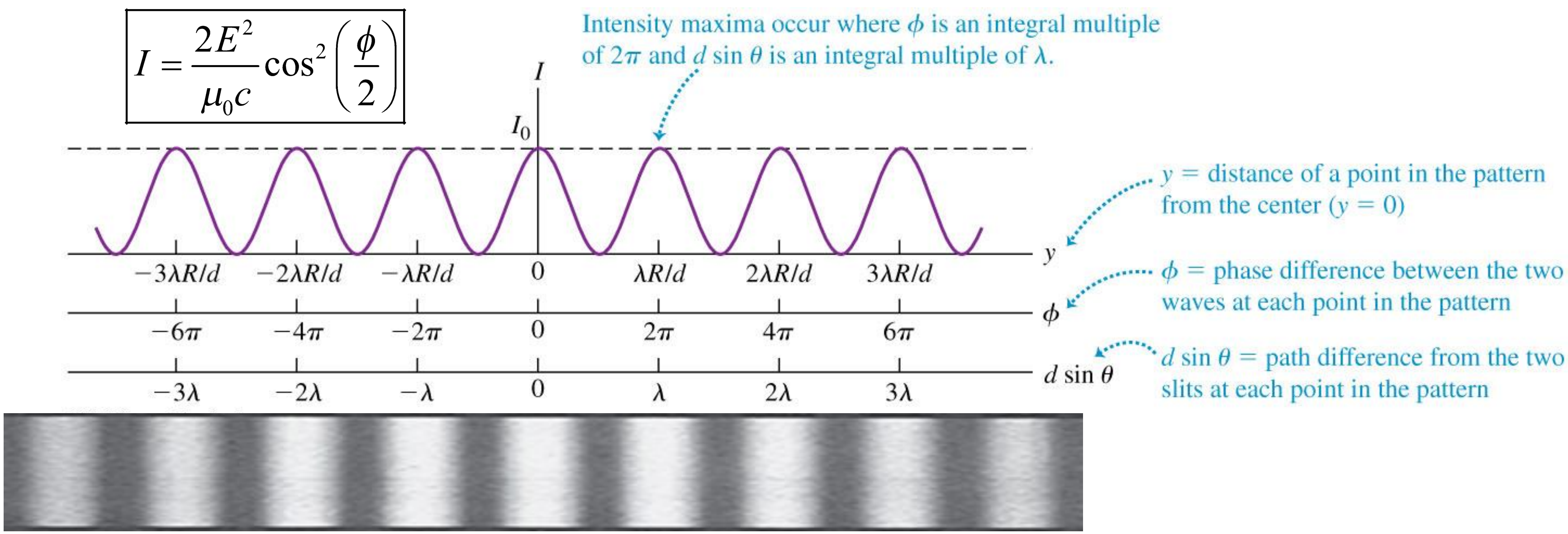


# Sources in Phase

- Then, the conditions for constructive and destructive interference can be put in terms of the overall phase difference:
  - Constructive: $\phi = \cancel{(\phi_2 - \phi_1)} + (kr_2 - kr_1) = 2m\pi$
  - Destructive: $\phi = \cancel{(\phi_2 - \phi_1)} + (kr_2 - kr_1) = (2m+1)\pi$

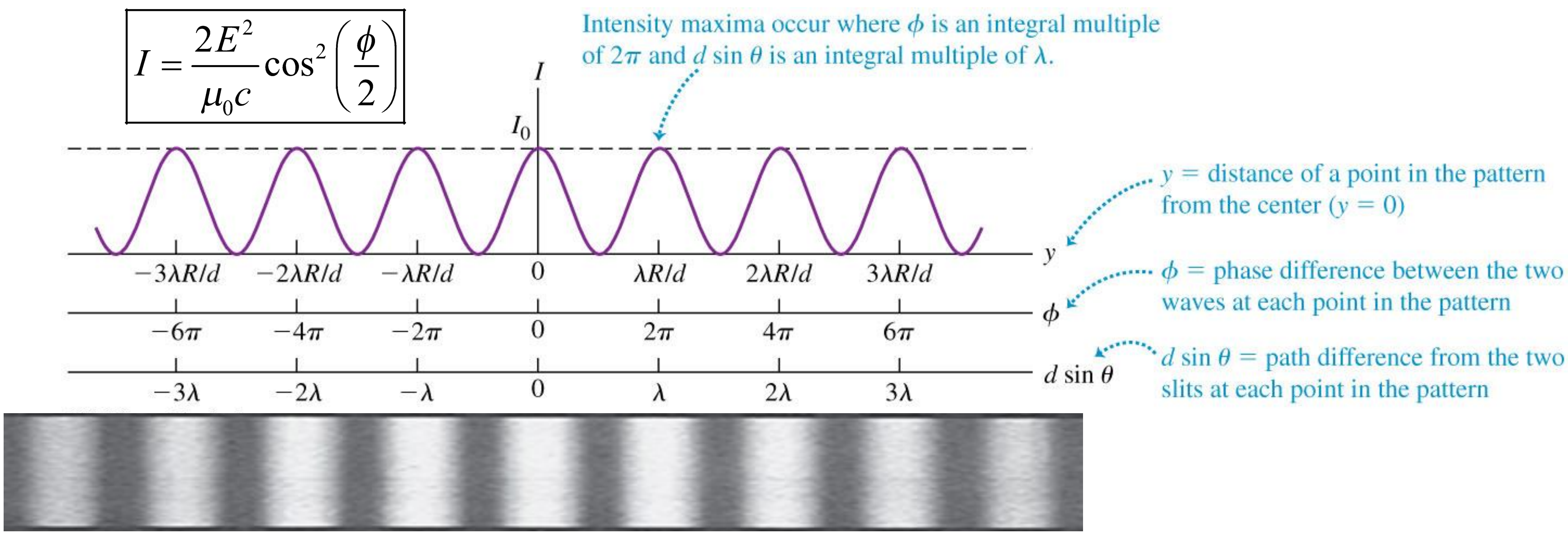
$$m = 0, \pm 1, \pm 2, \ldots$$

$$\boxed{I = \frac{2E^2}{\mu_0 c}\cos^2\left(\frac{\phi}{2}\right)}$$



# Assumptions again…

- What do you see that is inconsistent between the picture and the graph?



# Phasors, really?

- We will see in the coming lectures that you get interference patterns ***even when you only have one slit!***
- To understand this phenomena, and others, you have to understand the phasor diagrams obtained when you break up the source wave front into a continuous set of sources (a la Huygens).
- We won't begin this discussion today, but know that you need to understand how to set up a phasor diagram.