Lecture 21
(Interference III
Thin Film Interference and the Michelson Interferometer)

Physics 2310-01 Spring 2020

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Thin Films

- We are all relatively familiar with the phenomena of thin film interference.
- We've seen it in an oil slick or in a soap bubble.
- But how is this interference?
- What are the two sources?
- Why are there colors instead of bright and dark spots?

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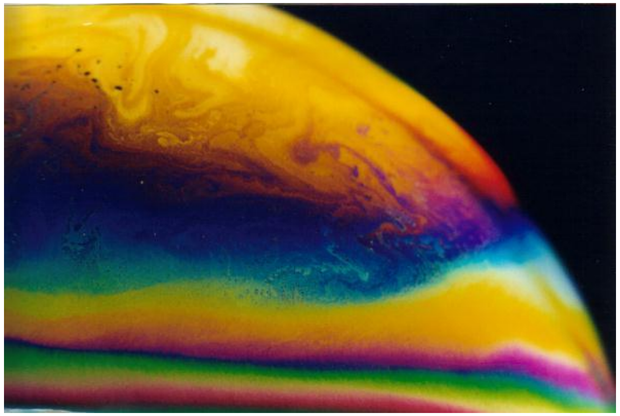

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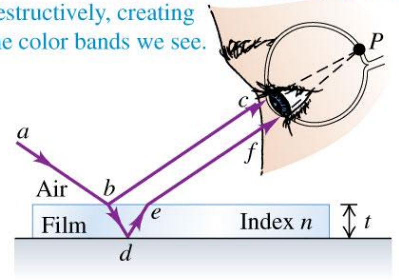
The two sources…

- In thin film interference, the two sources of coherent light come from reflections from two different interfaces.
- Different geometries will provide different types of interference patterns.
- The “thin film” could just be an air gap, for instance.

(a) Interference between rays reflected from the two surfaces of a thin film

Light reflected from the upper and lower surfaces of the film comes together in the eye at *P* and undergoes interference.

Some colors interfere constructively and others destructively, creating the color bands we see.



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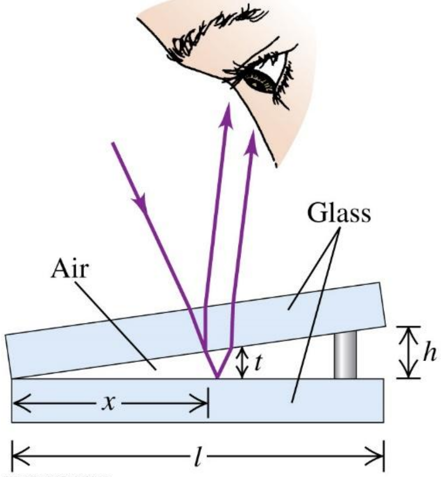

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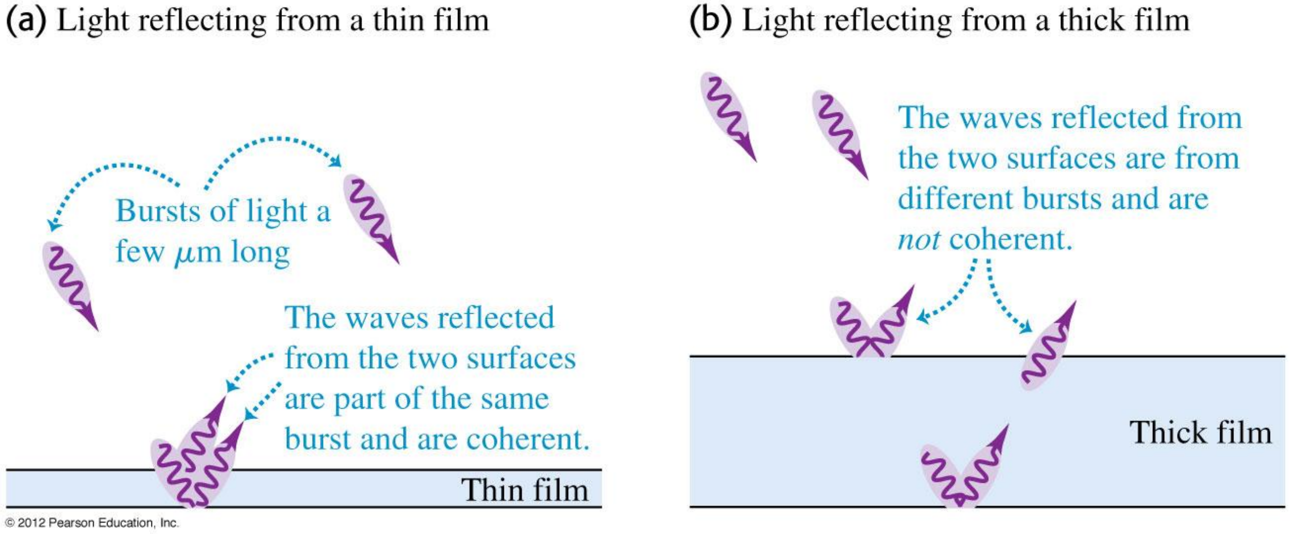
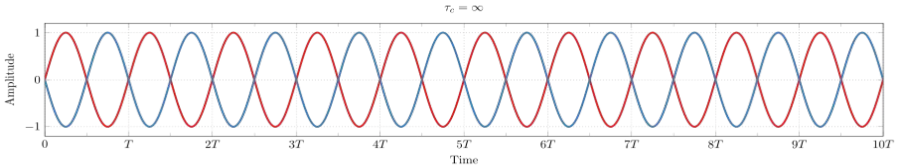
Coherence and Thin Film Interference

- Why "thin film"?
- Generally, when talking about thin film interference, the source light is what we would normally call incoherent – sunlight or room light.
- However, most light is coherent on a small enough distance/time scale…
- The book describes it like this:



Coherence - Wikipedia

- Perfectly coherent:



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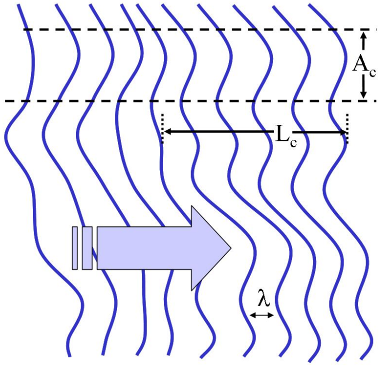
- Spatially partially coherent:



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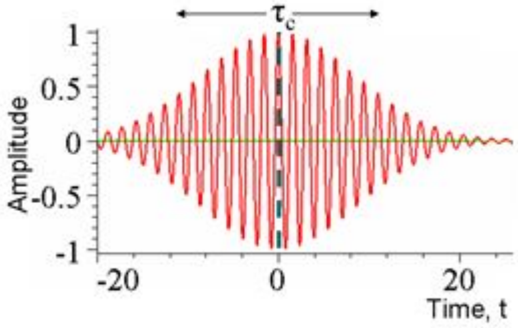
- Temporally partially coherent:



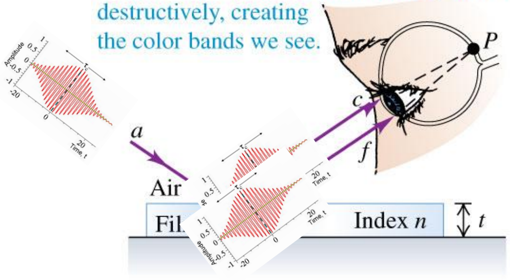
Optical Path Difference and Coherence Length

- If the difference in the amount of time it takes for each of the two paths is small compared to the coherence length, then the two sources can interfere.
- The difference in that time is referred to as the optical path difference (OPD).
- It depends on both the difference in path ***length***, and the ***index of refraction*** of the material.

(a) Interference between rays reflected from the two surfaces of a thin film

Light reflected from the upper and lower surfaces of the film comes together in the eye at P and undergoes interference.

Some colors interfere constructively and others destructively, creating the color bands we see.



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OPD

- Remember, what is important to interference effects is the ***phase difference of the light at the observer***, so we will need to calculate how the phase, ***kr***, is different for the two sources.

$$kr = \frac{2\pi}{\lambda} r$$

  - where *λ* is the wavelength **in the material** that the wave is travelling the distance *r*:

$$\lambda_1 = \frac{\lambda_0}{n_1} \qquad k_1 r_1 = \frac{2\pi}{\lambda_1} r_1 = \frac{2\pi n_1}{\lambda_0} r_1$$

$$\Rightarrow$$

$$\lambda_2 = \frac{\lambda_0}{n_2} \qquad k_2 r_2 = \frac{2\pi}{\lambda_2} r_2 = \frac{2\pi n_2}{\lambda_0} r_2$$

OPD

- So, $$k_2 r_2 - k_1 r_1 = \frac{2\pi n_2}{\lambda_0} r_2 - \frac{2\pi n_1}{\lambda_0} r_1 = \frac{2\pi}{\lambda_0}\left(n_2 r_2 - n_1 r_1\right)$$

$$OPD = n_2\left(\overline{AB} + \overline{BC}\right) - n_1\left(\overline{AD}\right)$$

$$\overline{AB} = \overline{BC} = \frac{d}{\cos\left(\theta_2\right)}$$

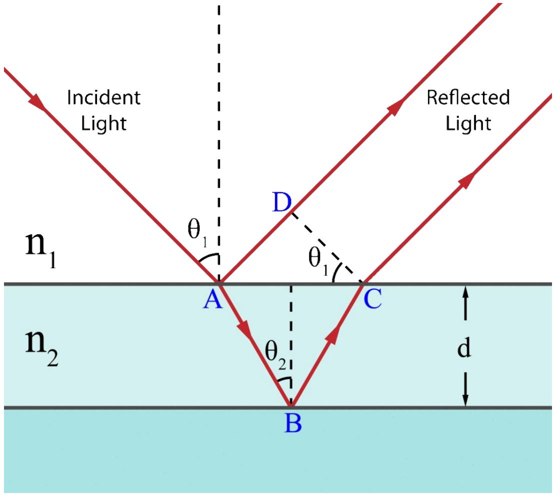
$$\overline{AD} = 2d\tan\left(\theta_2\right)\sin\left(\theta_1\right)$$

Using Snell's Law, $n_1\sin(\theta_1) = n_2\sin(\theta_2)$

$$OPD = n_2\left(\frac{2d}{\cos\left(\theta_2\right)}\right) - 2d\tan\left(\theta_2\right) n_2\sin\left(\theta_2\right)$$

$$OPD = 2n_2 d\left(\frac{1-\sin^2\left(\theta_2\right)}{\cos\left(\theta_2\right)}\right)$$

$$OPD = 2n_2 d\cos\left(\theta_2\right)$$



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Then, for destructive interference,

$$\frac{2\pi}{\lambda_0}\left[2n_2 d\cos\left(\theta_2\right)\right] = m\pi, \quad m = 1, 3, 5\ldots \Rightarrow$$

$$2d\cos\left(\theta_2\right) = \frac{m}{2}\left(\frac{\lambda_0}{n_2}\right), \quad m = 1, 3, 5\ldots$$

or

$$2d\cos\left(\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2\ldots$$

Treatise on University Physics

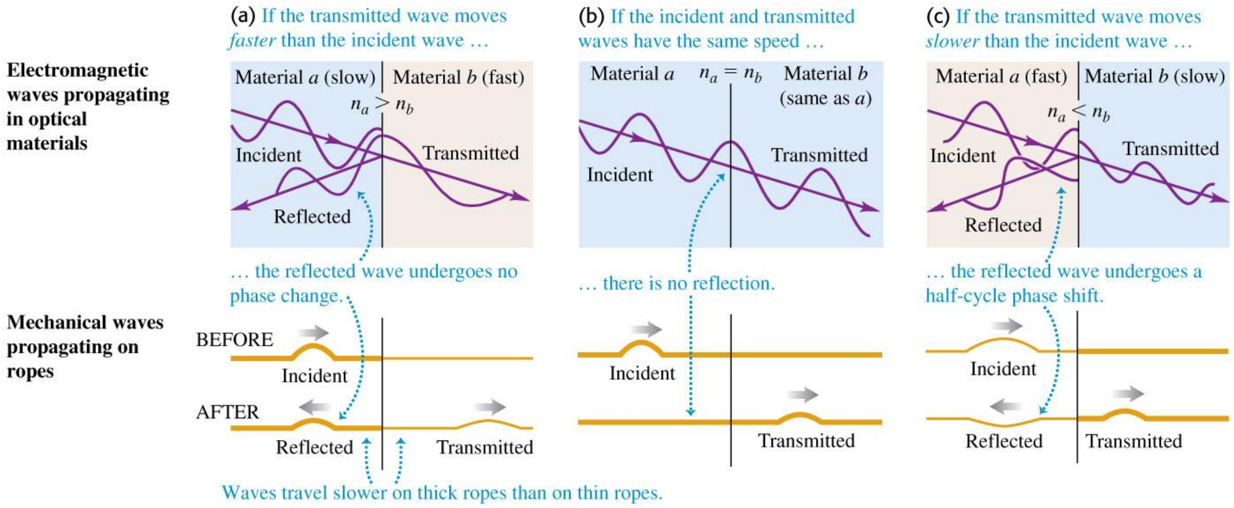
- The book kind of glosses over all of this and then pastes a big yellow colored equation in front of you to memorize:

$$\boxed{\begin{array}{ll} 2t = m\lambda & \text{Constructive reflective interference} \\ & \quad m = 0, \pm 1, \pm 2, \ldots \\ 2t = \left(m + \frac{1}{2}\right)\lambda & \text{Destructive reflective interference} \end{array}}$$

- But what is λ?
- What happened to the cosθ term?
- Don’t memorize; understand.

Phase Change at Boundaries

- To complete our understanding of thin film interference, we have to keep in mind that when there is reflection at a boundary, the phase can change by π if the wave goes from a lower to a higher index of refraction.



Conditions for Constructive and Destructive Interference

- If no phase changes, or both phases change,

$$2n_2 d\cos\left(\theta_2\right) = m\lambda_0$$ For constructive interference

$$2n_2 d\cos\left(\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda_0$$ For destructive interference

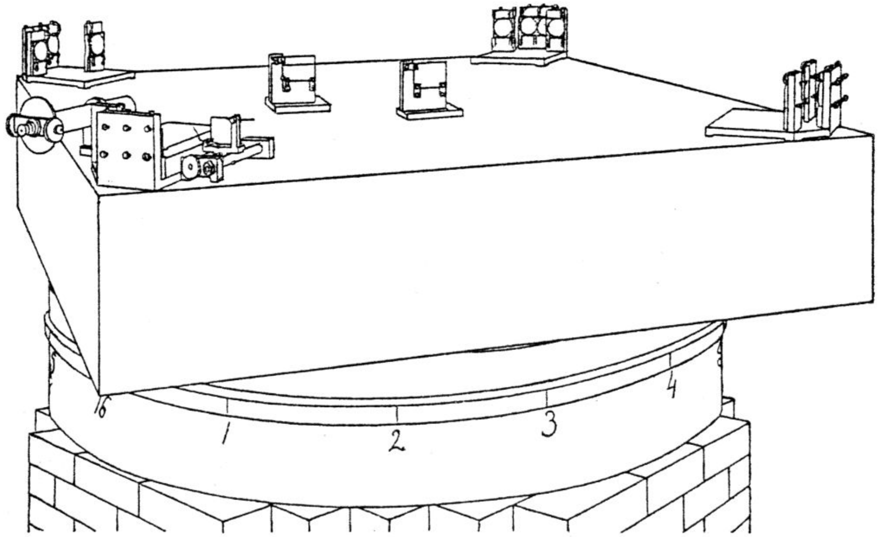
- If one phase changes,

$$2n_2 d\cos\left(\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda_0$$ For constructive interference

$$2n_2 d\cos\left(\theta_2\right) = m\lambda_0$$ For destructive interference

Michelson Interferometer

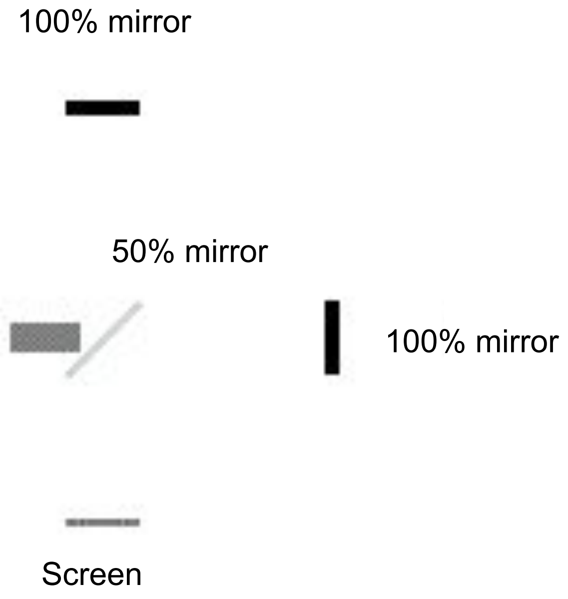
- In the late 1800s, Albert Michelson built an interferometer based on splitting the light from a single source, sending the light down perpendicular paths, and then recombining them.
- An interferometer can have many uses, but the most famous (and the one for which he earned a Nobel Prize) had to do with the dependence of speed of light on the direction it travelled.
- We will come back to this at the end of this semester, but for now let’s just see how this experimental apparatus works…



"On the Relative Motion of the Earth and the Luminiferous Ether - Fig 3" by Albert Abraham Michelson (1852 – 1931) with Edward Morley (1838 -1923) - http://www.aip.org/history/exhibits/gap/Michelson/Michelson.html#michelson1 aip.org]. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:On_the_Relative_Motion_of_the_Earth_and_the_Luminiferous_Ether_-_Fig_3.png#/media/File:On_the_Relative_Motion_of_the_Earth_and_the_Luminiferous_Ether_-_Fig_3.png

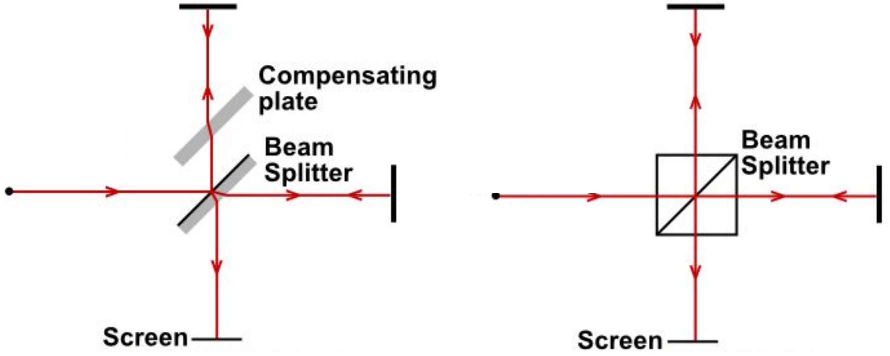
Basic Premise

- The basic idea for the Michelson Interferometer is to use a beam splitter (a mirror that reflects 50% of the light and transmits the rest) to create two sources.
- Each source then reflects again from 100% mirrors, and then returns to the beam splitter, partially combining and then going to a detector (or screen).
- When all of the phase changes are accounted for (see next slide), the difference in phase should just be the difference in total path length for the two beams of light.



Getting the Optical Paths Equal

- Because the beam splitter is made of mirrored glass, one of the beams passes through the glass three times, while the other only once.
- We add a compensating plate to correct for that:



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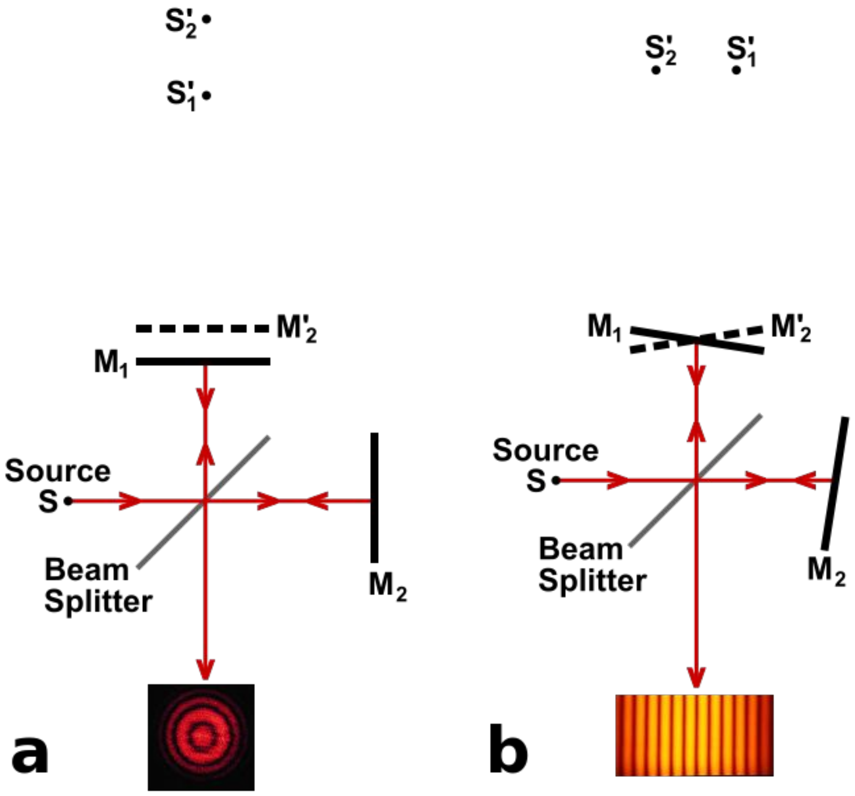
Two Virtual Sources

- Depending on the orientation of the mirrors, we can get different interference patterns that reflect the positions of the virtual sources.
- If the sources are in line, one behind the other, we get circular fringes.
- If you tilt the mirrors, you can get the two virtual sources next to each other, and you get our familiar two-slit diffraction pattern.



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Uses

- If you move one of the mirrors by $\lambda/2$, then the optical path for that beam is increased by one wavelength, and one should see the same pattern on the screen, but the fringes will move across the screen as you move the mirror.
- So, by counting how many times a fringe passes by, you can count the number of half-wavelengths you move the mirror.
- Because you use visible light, you can then measure a movement of the mirror to one half (or less) a wavelength:

$$\lambda_{visible} \sim 400nm - 700nm$$

- The takeaway is that you have a very precise way to measure changes in optical path distances.
- Michelson and Morley used this to blow away the prevailing theory of propagation of light.
- More on that at the end of this semester…