Lecture 22
(Diffraction I
Single-Slit Diffraction)

Physics 2310-01 Spring 2020

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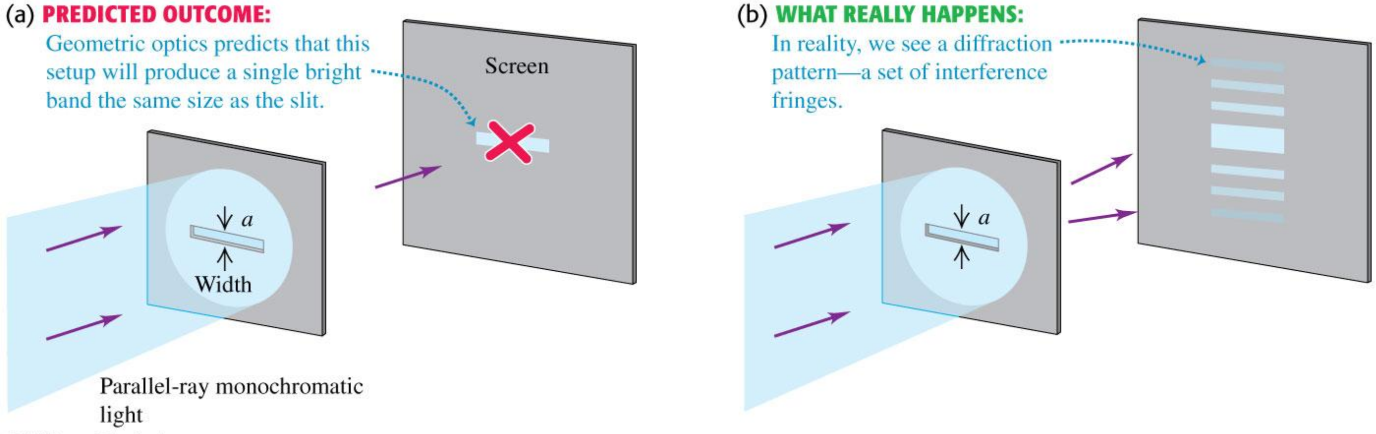
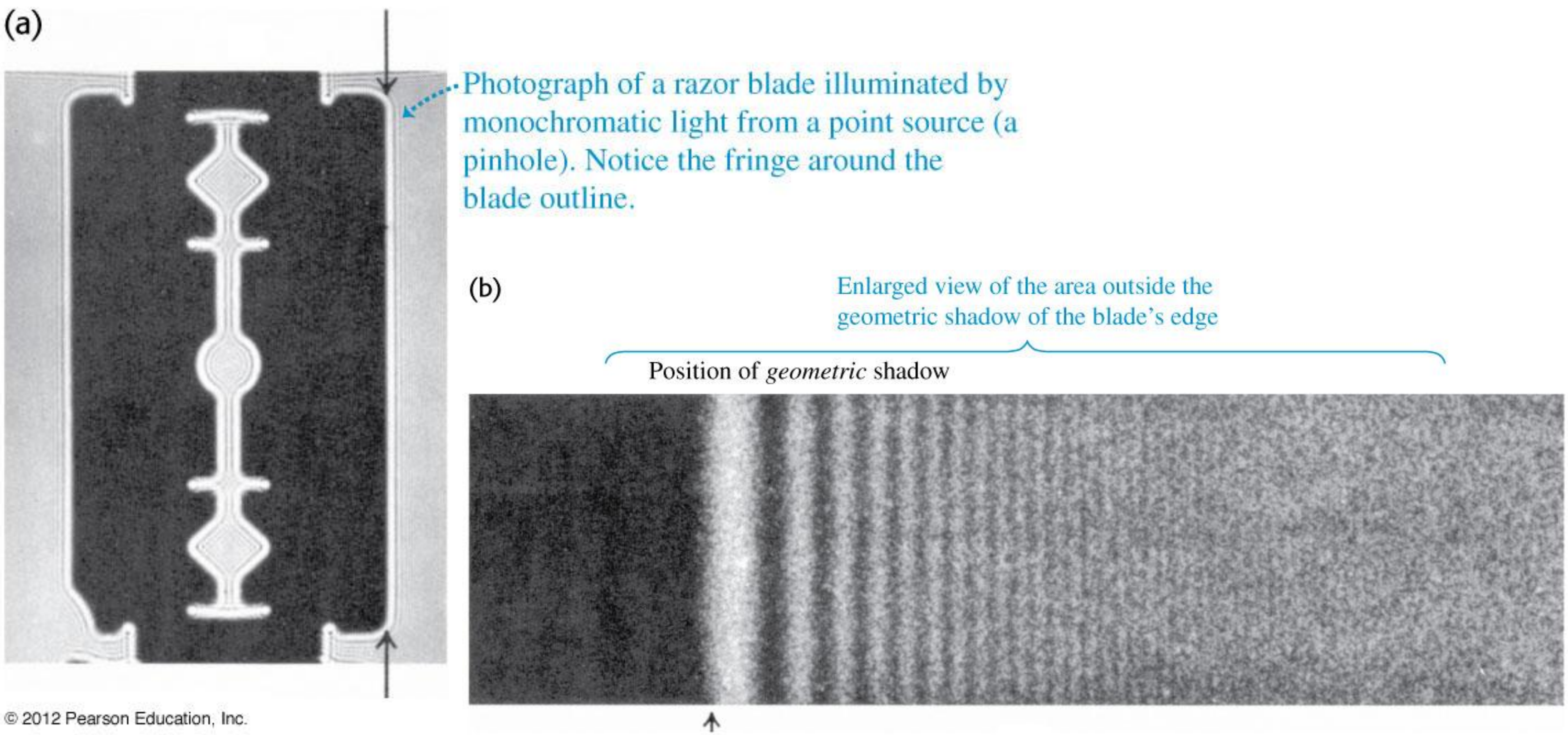
Single-Slit Diffraction

- As we have already hinted at, and seen, waves don't behave as we might have expected from our study of geometric optics.
- We can see interference fringes even when we only have a single slit!
- To understand this phenomena, we have to go back to Huygens' Principle and phasor diagrams.



Doesn't have to be a slit…

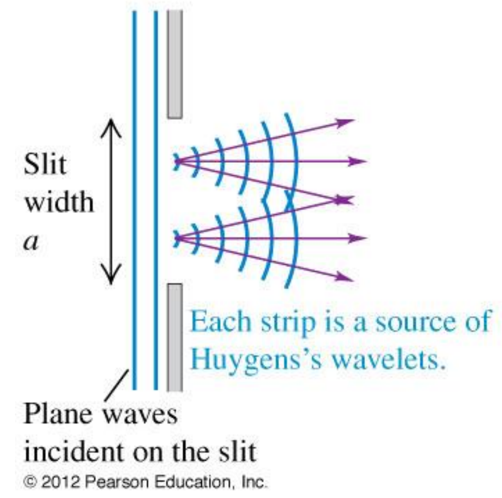
- Any obstruction can cause an interference pattern.



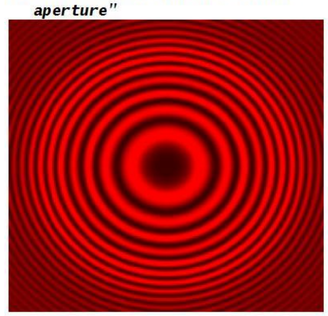
Single-Slit Diffraction

- Because real slits have finite width, then there is not just a single source of Huygens wavelets, but many (infinite?).
- As we saw with the two-slit problem, the geometry of the problem is much easier when we go to the limit that the distance to the screen (where the interference pattern is viewed) is much larger than the width of the slit. – Fraunhofer or far-field diffraction.
- When this isn't the case, you still get diffraction, it just looks different. – Fresnel or near-field diffraction.

(a) A slit as a source of wavelets

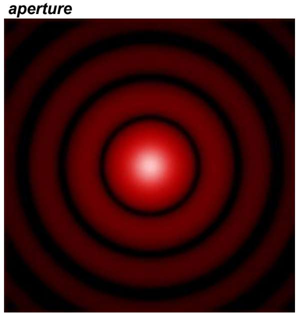


Fresnel diffraction of circular aperture"



"Circular Aperture Fresnel Diffraction high res" by Gisling - Own work. Licensed under CC BY-SA 3.0 via Commons - https://commons.wikimedia.org/wiki/File:Circular_Aperture_Fresnel_Diffraction_high_res.jpg#/media/File:Circular_Aperture_Fresnel_Diffraction_high_res.jpg

Fraunhofer diffraction of circular aperture



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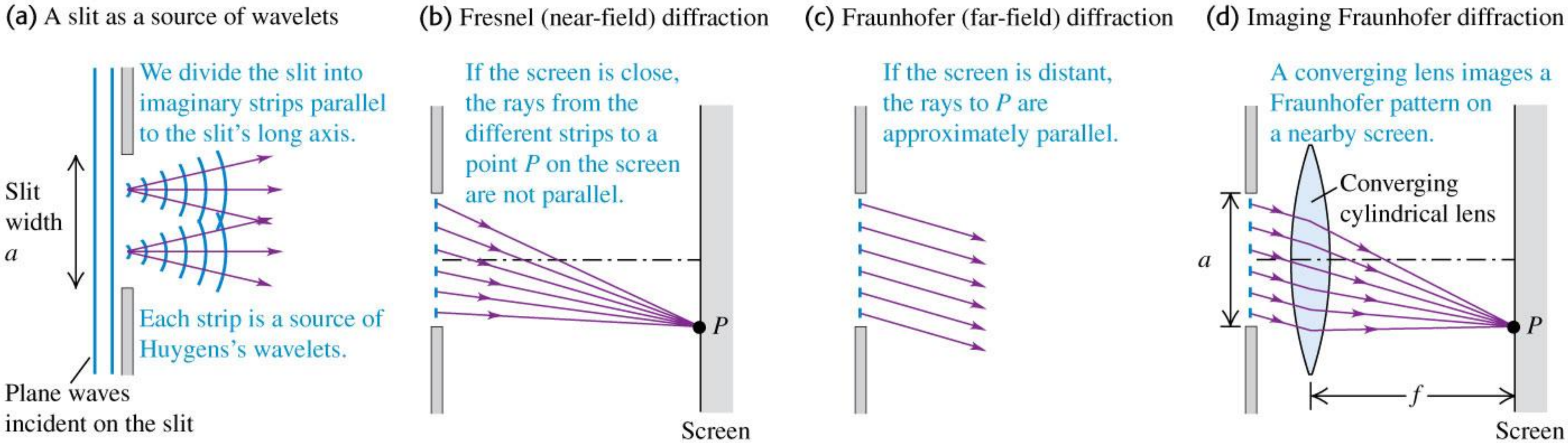
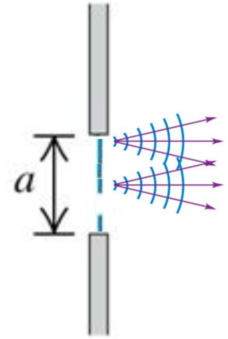
Fresnel and Fraunhofer Diffraction

- The only distinction is the distance between the slit and the screen.
- In Fraunhofer diffraction the distance is large enough to consider the rays to be parallel.
- We will limit our discussions to Fraunhofer diffraction (to make the geometry simpler).



Fraunhofer Diffraction

- Let's first start by looking at our slit of width ***a***.
- We then look at two sources of Huygen's wavelets located at the top and the middle of the slit, separated by distance ***a/2***.
- We will analyze these two wavelets,
- But our analysis will be the same for any two wavelets from "slits" separated by the same distance.



Fraunhofer Diffraction

- So, just as before, the path difference is just the slit spacing times the sin of the angle:

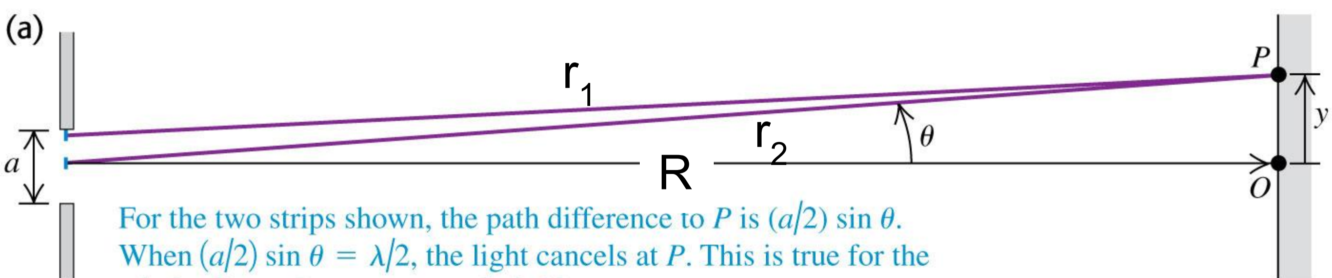
$$\left(r_1 - r_2\right) = \frac{a}{2}\sin\theta$$

- And, we use the same small angle approximations as before:

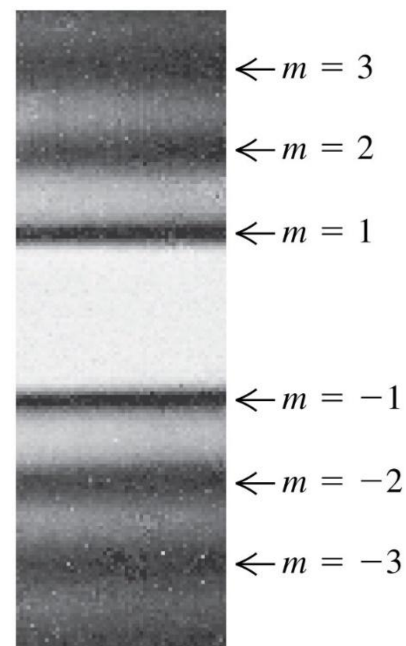
$$\tan\theta \approx \sin\theta = \frac{y}{R}$$

- So, the phase difference is given by:

$$\delta\phi = k(r_1 - r_2) = \frac{2\pi}{\lambda}\frac{a}{2}\frac{y}{R}$$



For the two strips shown, the path difference to P is $(a/2)\sin\theta$. When $(a/2)\sin\theta = \lambda/2$, the light cancels at P. This is true for the whole slit, so P represents a dark fringe.



Fraunhofer Diffraction

- The condition for a dark band is that the two sources are out of phase, so

$$\delta\phi = \frac{2\pi}{\lambda}\frac{a}{2}\frac{y}{R} = \pi$$

- Solving for y, we get:

$$y_{min} = \frac{R\lambda}{a}$$

- Now, instead of dividing the slit into pairs that start at the halfway point, we could have started at the quarter-way point (since to cover the entire slit requires some factor of two) and repeat the process. This would give:

$$y_{min} = \frac{2R\lambda}{a}$$

- Doing it at the one-sixth-way point, would give us:

$$y_{min} = \frac{3R\lambda}{a}$$

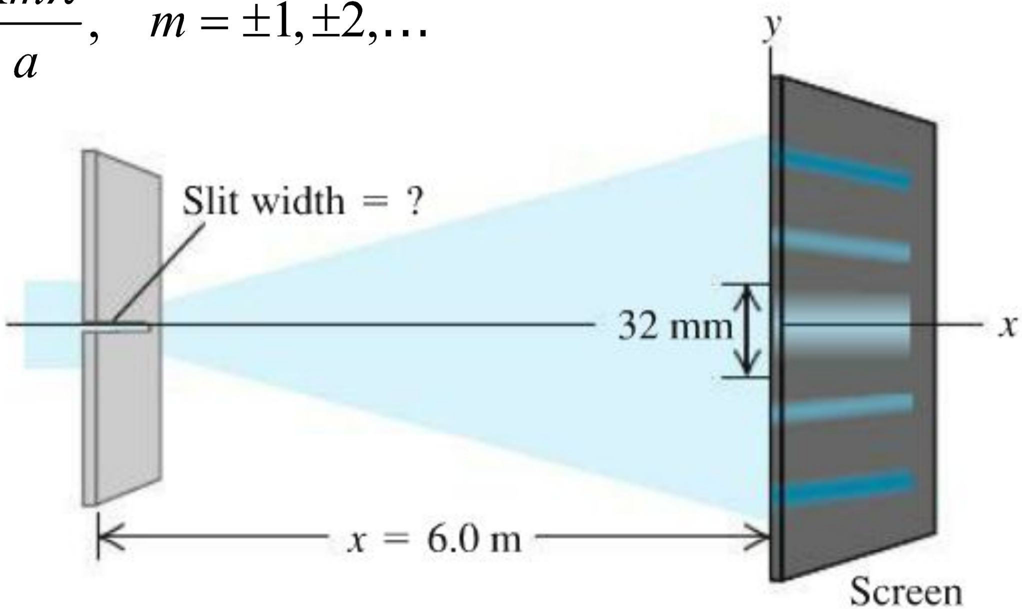
- And so on, so that in general,

$$y_{min} = \frac{mR\lambda}{a}, m = \pm 1, \pm 2, \pm 3...$$

Example 36.1 Single-slit diffraction

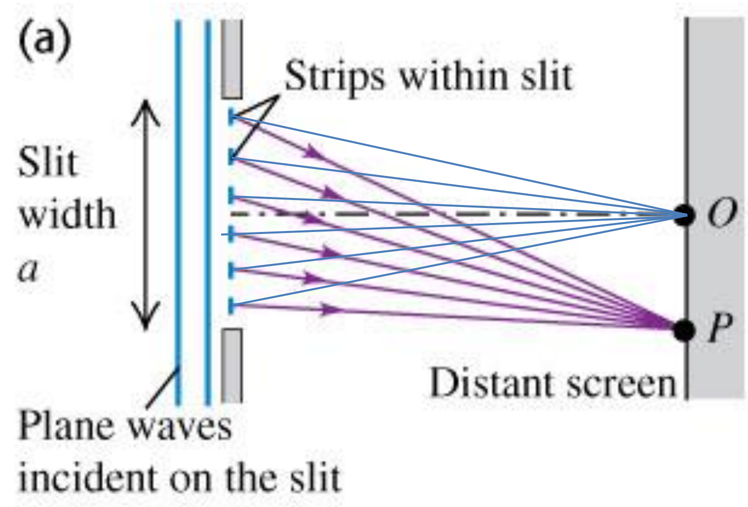
You pass 633-nm laser light through a narrow slit and observe the diffraction pattern on a screen 6.0 m away. The distance on the screen between the centers of the first minima on either side of the central bright fringe is 32 mm (Fig. 36.7). How wide is the slit?

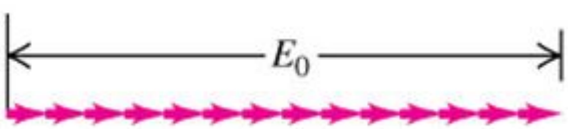
$$y = \frac{Rm\lambda}{a}, \quad m = \pm 1, \pm 2, \ldots$$



Intensity and Phasors

- OK, I don't know about you, but this method seems a little sketchy. Let's try to be a bit more like scientists, and just use phasors for some number of sources in the slit, which we will then let go to infinity.
- We'll start with 14 sources, and begin by looking at the phasors at point O, where, for long distance to the screen, they are all in phase:

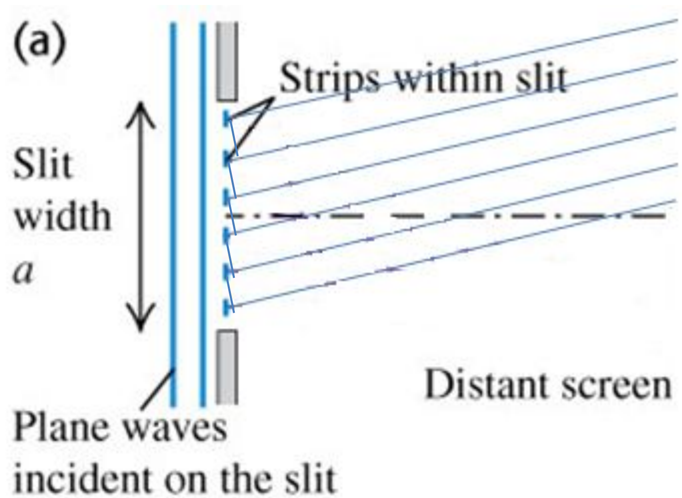


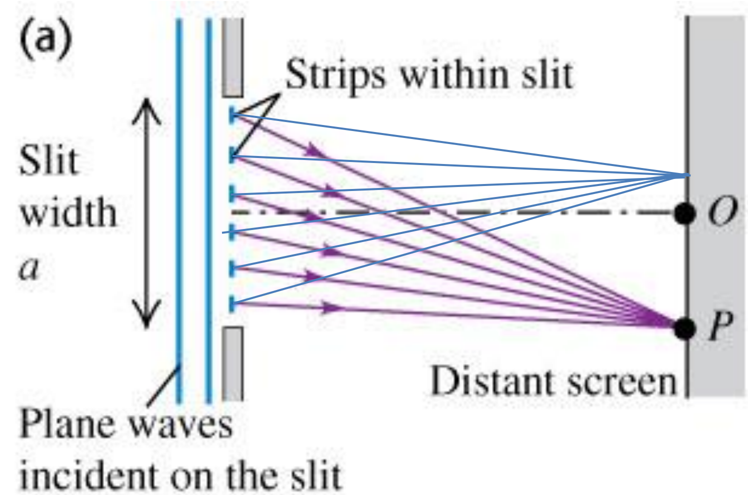


- We take the magnitude of the resultant wave at *O* to be E_0.

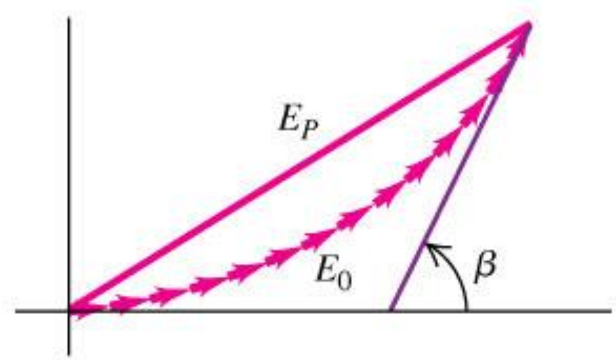
Slightly off center…

- If we move slightly off-center, each ray is slightly out of phase with the next.
- Remember that this is far-field, so that we can assume that each ray is parallel.





(c) Phasor diagram at a point slightly off the center of the pattern; β = total phase difference between the first and last phasors.



To infinity, and beyond!

- In the limit of infinite numbers of strips each infinitesimally long, it becomes an arc of a circle, with angular extent β – the difference between the phase of the first and last rays.
- Since the length of an arc is given by:

$$\ell = \beta r$$

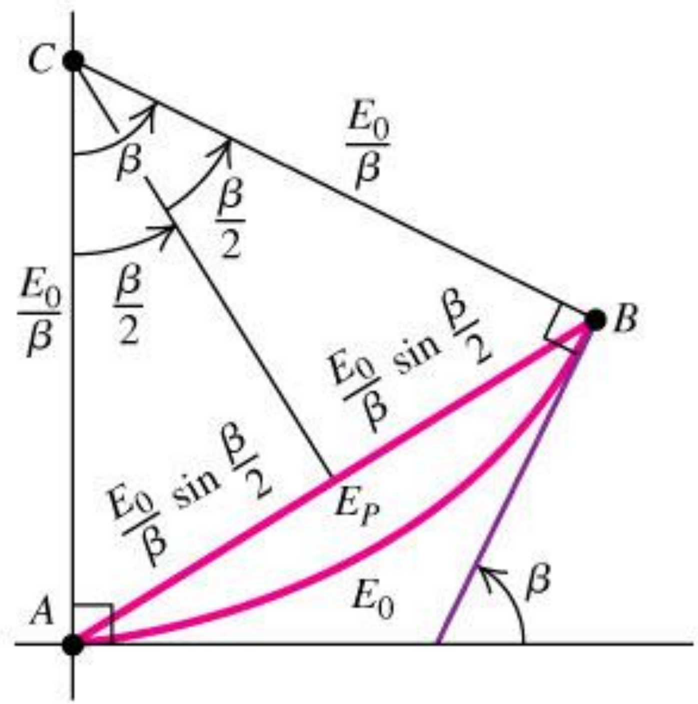
- And we know what the arc length is, E_0, so:

$$E_0 = \beta r \Rightarrow r = \frac{E_0}{\beta}$$

- Then, with just a little simple trig, we can get the length of the resultant phasor:

$$E_P = 2\frac{E_0}{\beta}\sin\frac{\beta}{2} = E_0\frac{\sin(\beta/2)}{\beta/2}$$

(d) As in (c), but in the limit that the slit is subdivided into infinitely many strips



Resultant Amplitude

- Now, the total phase change between the first and last rays is just:

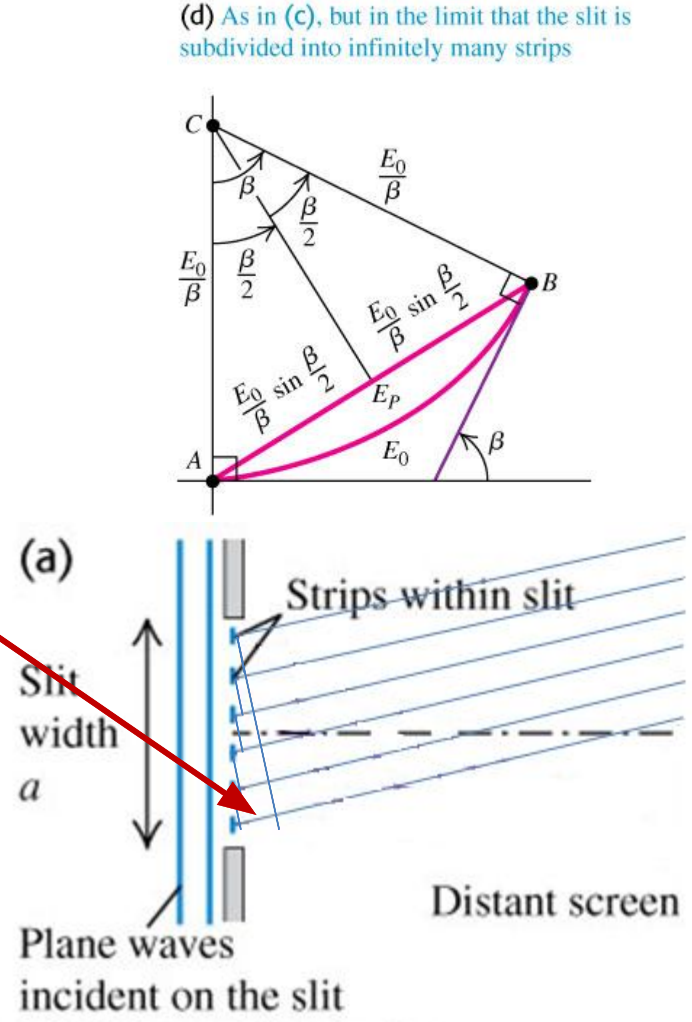
$$\beta = ka\sin\theta$$

- So,

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2}$$

$$= E_0 \frac{\sin\left[(ka\sin\theta)/2\right]}{(ka\sin\theta)/2}$$

$$= E_0 \frac{\sin\left[(\pi a\sin\theta)/\lambda\right]}{(\pi a\sin\theta)/\lambda}$$



Intensity

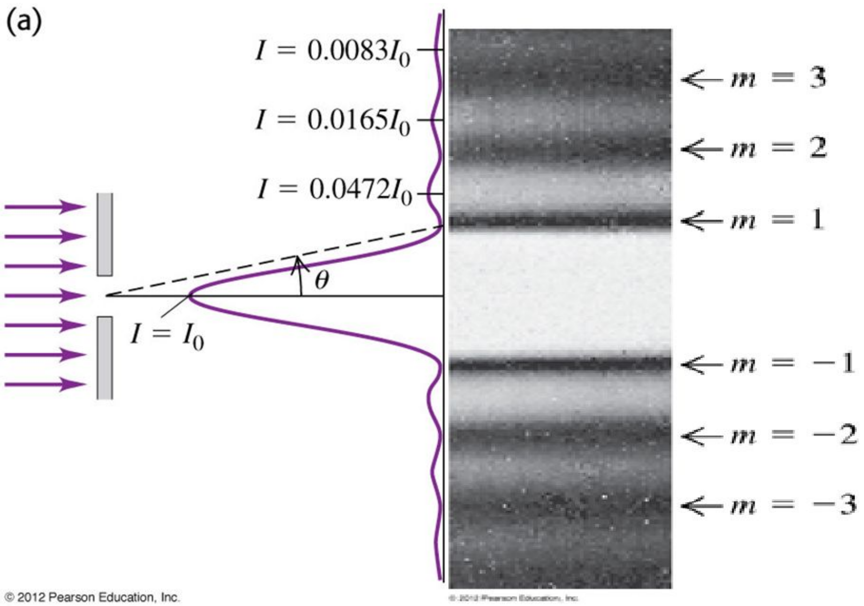
- Since the intensity is just proportional to the square of the electric field, then,

$$E_P = E_0 \frac{\sin\left[(\pi a \sin\theta)/\lambda\right]}{(\pi a \sin\theta)/\lambda} \Rightarrow$$

$$I = I_0 \left\{ \frac{\sin\left[(\pi a \sin\theta)/\lambda\right]}{(\pi a \sin\theta)/\lambda} \right\}^2$$

- Now, we have a complete picture of the diffraction pattern:

$$y = \frac{Rm\lambda}{a}, \quad m = \pm 1, \pm 2, \ldots$$



Width of Central Peak

- The positions of the first ***minima*** are given by: $\sin\theta = \dfrac{m\lambda}{a}, \quad m = \pm 1$
- For small angles, $\sin\theta = \theta$, the width of the central peak is

$$\theta_1 - \theta_{-1} = \frac{\lambda}{a} - \frac{-\lambda}{a} = \frac{2\lambda}{a}$$

- Also, as ***a*** approaches the wavelength, there are no minima, and the diffraction pattern is just one broad peak.