Lecture 23
(Diffraction II
Multiple-Slit Diffraction)

Physics 2310-01 Spring 2020

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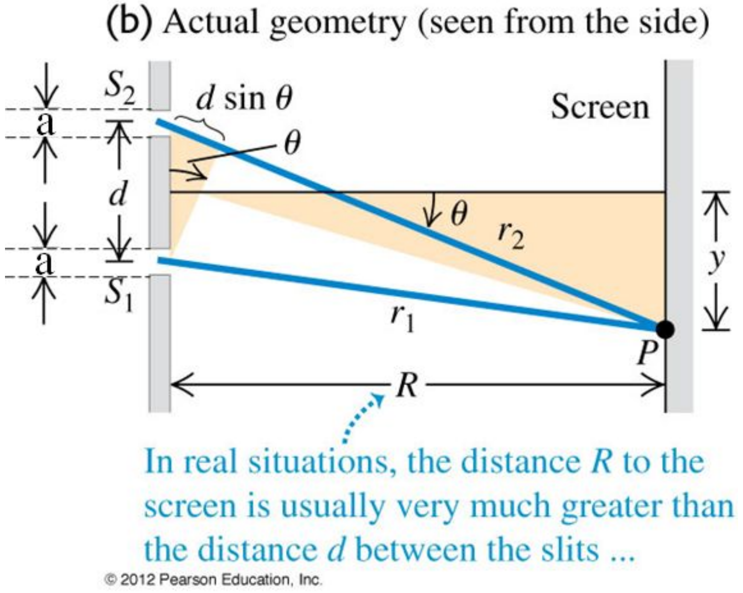
Realistic Double-Slit Diffraction

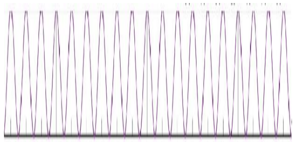
- Okay, now let's return to the double slit situation, and take into account that the widths of the slits are finite.
- Set the widths of the slits as a, the space between them as d…



Realistic Double-Slit Diffraction

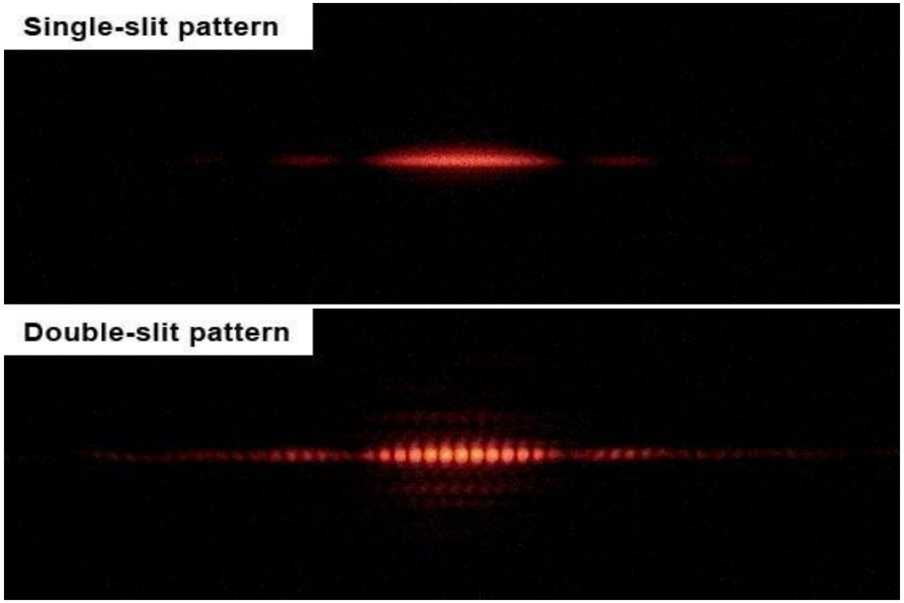
- Then the realistic intensity pattern is just the ideal pattern modulated by the single slit diffraction pattern.

$$I = \frac{2E^2}{\mu_0 c}\cos^2\left(\frac{\phi}{2}\right) = \frac{2E^2}{\mu_0 c}\cos^2\left(\frac{kd\sin\theta}{2}\right)$$



$$I = I_0\left\{\frac{\sin\left[(\pi a\sin\theta)/\lambda\right]}{(\pi a\sin\theta)/\lambda}\right\}^2$$

$$I = I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)\left\{\frac{\sin\left[(\pi a\sin\theta)/\lambda\right]}{(\pi a\sin\theta)/\lambda}\right\}^2$$



An Example

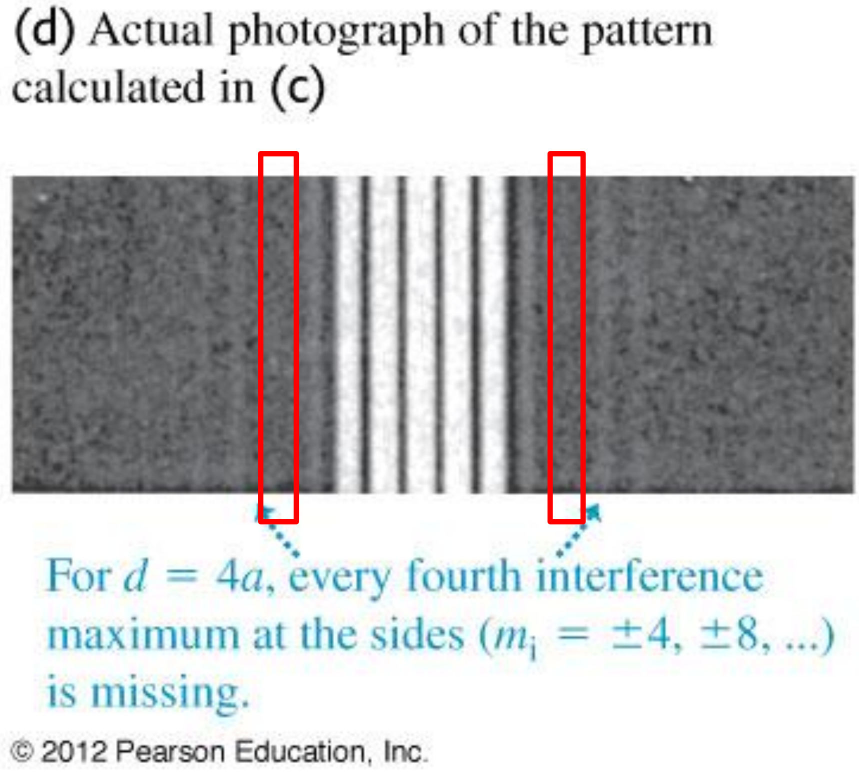
- Since for the two-slit interference we have for the ***maxima***:

$$d \sin\theta = m\lambda$$

- But, for diffraction we have for the ***minima***:

$$a \sin\theta = m\lambda$$

- Then, in this picture, where d=4a, every fourth interference maxima will align with a diffraction minimum.

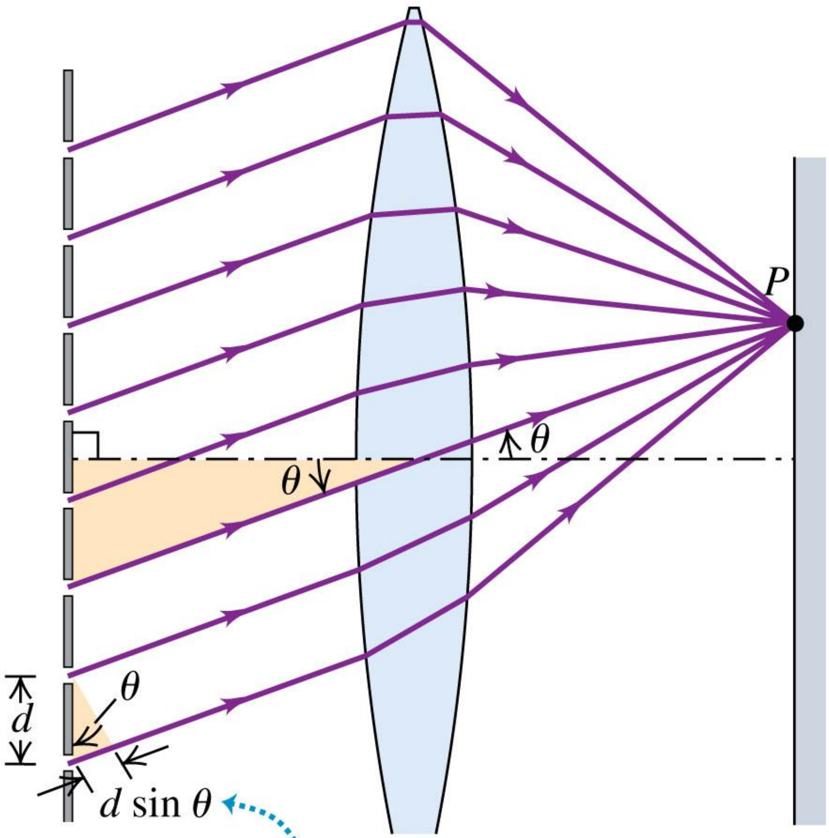


Multi-Slit Diffraction

- Eight very narrow slits (compared to wavelength of light) spaced *d* apart – can ignore diffraction effects.
- Maxima will occur when $\frac{2\pi}{\lambda} d \sin\theta = m 2\pi \Rightarrow$

$$d \sin\theta = m\lambda$$

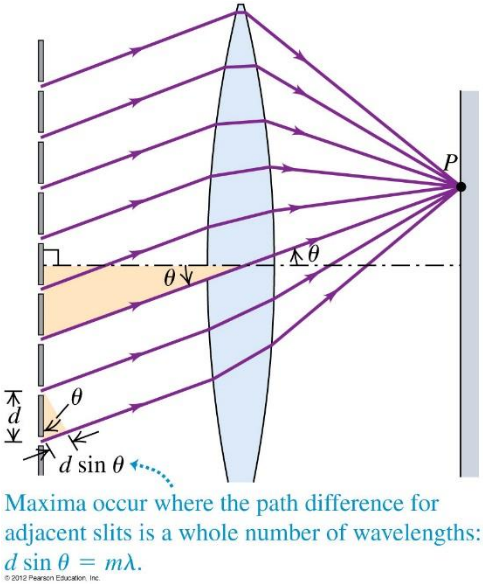
- Same as two-slit pattern!
- Not exactly…

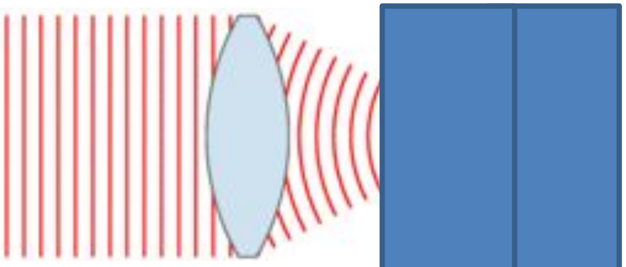


Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d \sin\theta = m\lambda$.

What about the lens???

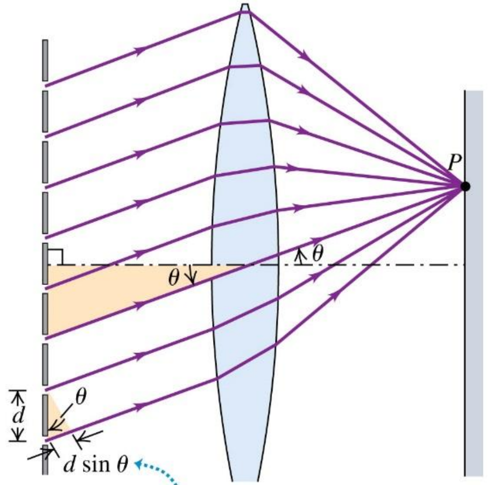
- Is no one else bothered by the presence of the lens?
- Doesn’t it affect the phase of the individual rays, and hence affect the interference pattern?
- It turns out that the phase change for each part of the plane waves that arrive at the lens is the same ***at the focal plane***.
- You can see this more clearly by putting a screen at a different location…



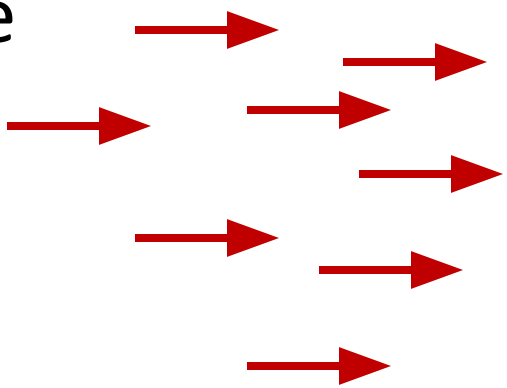


Phasors in Multi-Slit Diffraction

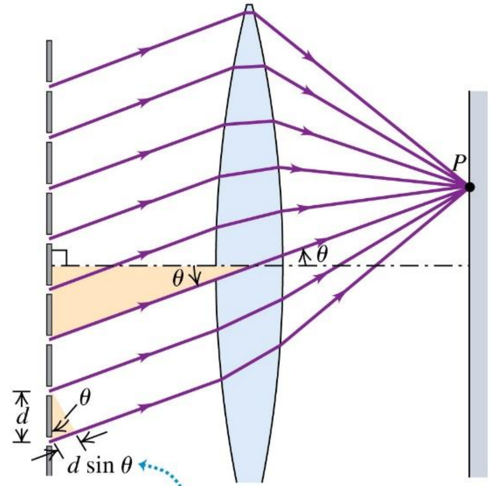
- Let's try to use phasors to understand the 8-slit situation.
- Here are our phasors.
- How do we arrange them when we are looking at the central bright spot?



Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d \sin\theta = m\lambda$.



Phasors in Multi-Slit Diffraction



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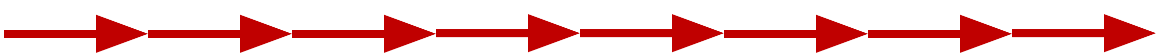
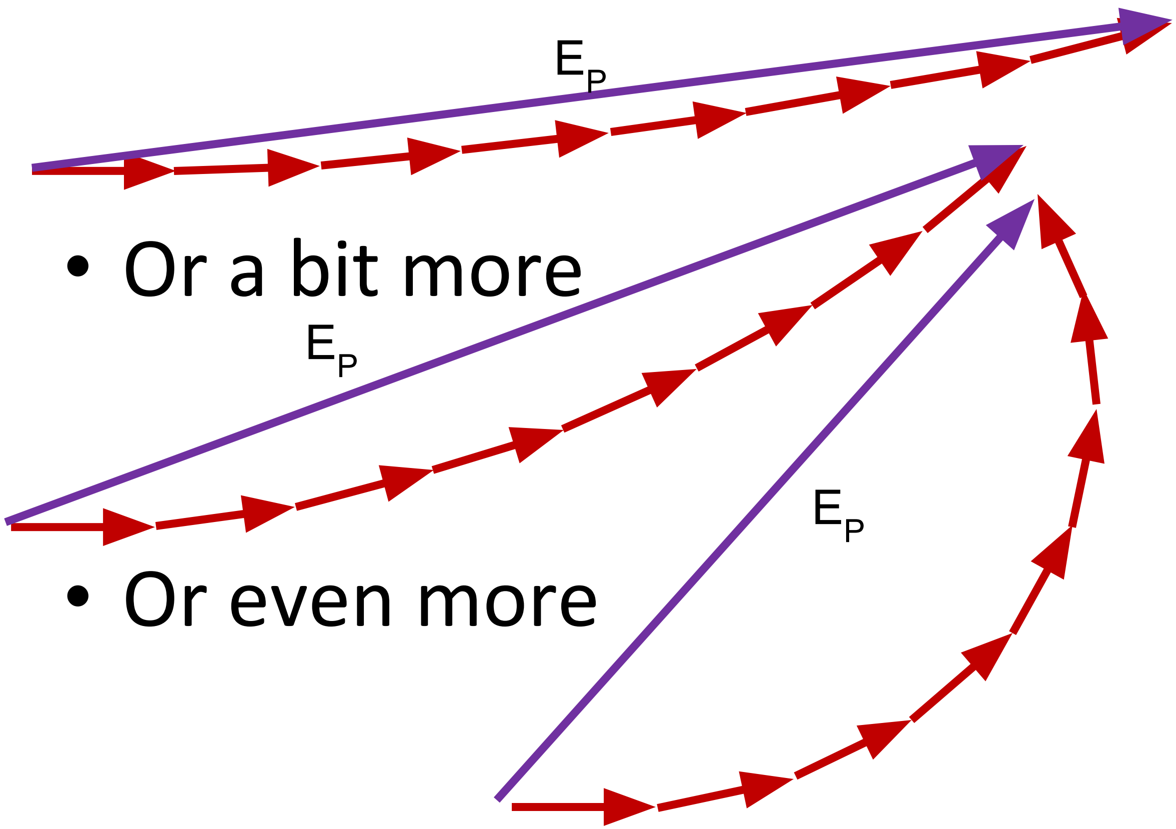


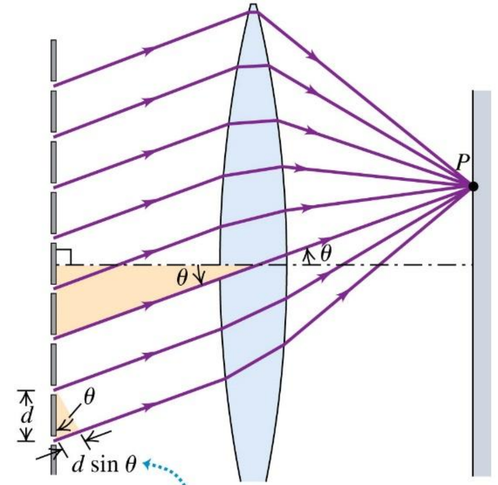
- In fact, that is what it will look like for all maxima of the interference pattern.

Phasors in Multi-Slit Diffraction

- What about when we go slightly off of the center of the maximum?



- Or a bit more
- Or even more



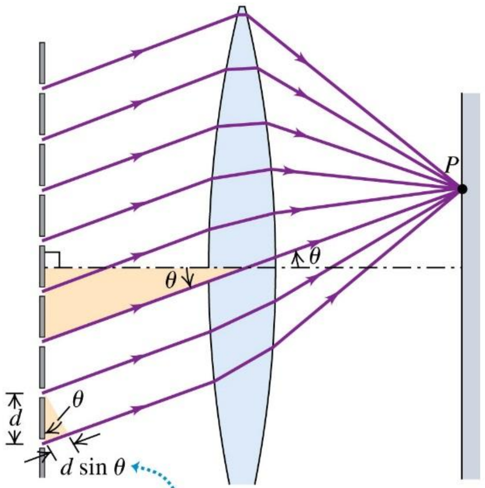
Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d \sin\theta = m\lambda$.

Phasors in Multi-Slit Diffraction

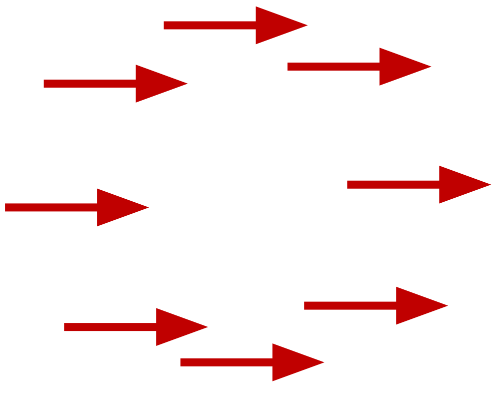
- When we finally get to a phase change between adjacent rays of π/4, the phasors will add to zero.
- This means that the magnitude of the oscillating field is zero at all times.
- For eight slits, we can solve the equation:

$$8\phi_{adj} = m(2\pi), \quad m = 1,2,3\ldots7$$

to find all unique phases (less than 2π) that result in total destructive interference.



Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d \sin\theta = m\lambda$.

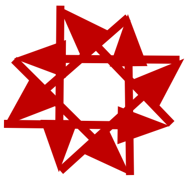


$E_P = 0$

Phasors in Multi-Slit Diffraction

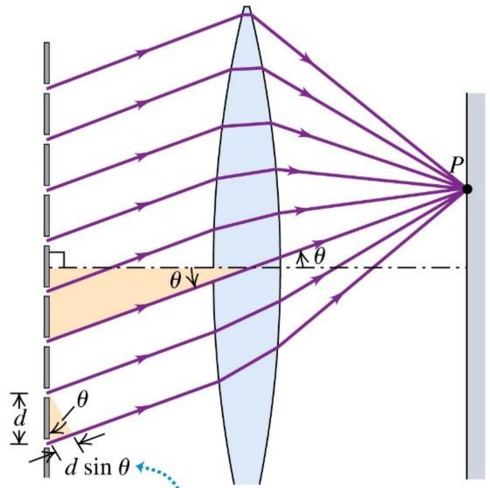
$$8\phi_{adj} = m(2\pi), \quad m = 1,2,3\ldots7$$

- Let's try the m=3 value, 3π/4:



$E_P = 0$

- Notice that the m=8 solution is actually another maximum.



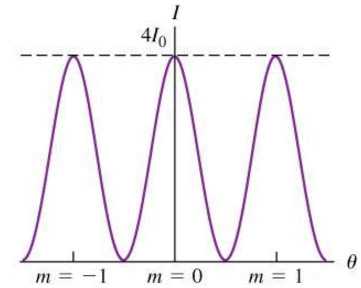
Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d \sin\theta = m\lambda$.

Multi-slit Intensity Patterns

- Notice that the intensity maxima are in the same place for different numbers of slits, but there are more and more minima in between.
- The maxima are also more and more narrow.

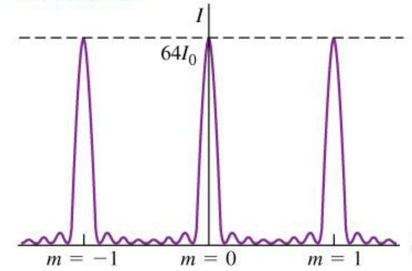
$N = 2$

(a) $N = 2$: two slits produce one minimum between adjacent maxima.



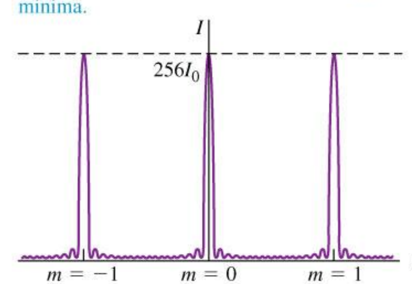
$N = 8$

(b) $N = 8$: eight slits produce taller, narrower maxima in the same locations, separated by seven minima.



$N = 16$

(c) $N = 16$: with 16 slits, the maxima are even taller and narrower, with more intervening minima.



Generalization to N-slit Diffraction

Minima:

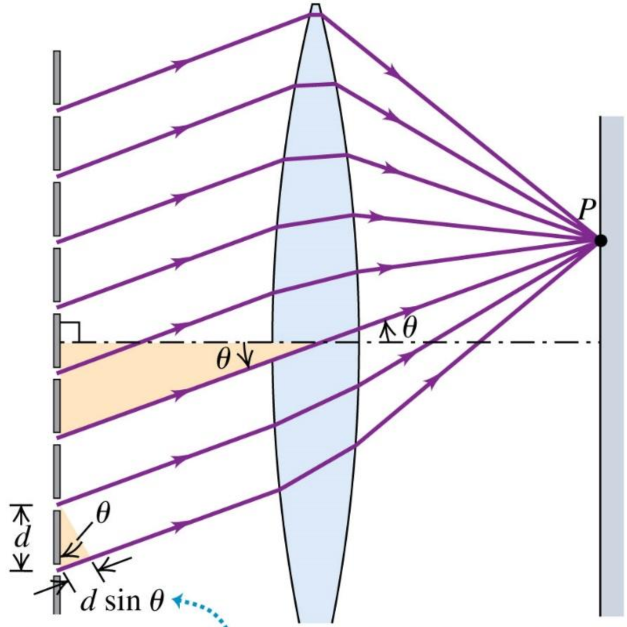
$$N\phi_{adj} = m2\pi, \quad m = 1,2,3\ldots(N-1) \Rightarrow$$

$$\phi_{adj} = \frac{m2\pi}{N}, \quad m = 1,2,3\ldots(N-1)$$

$$\therefore kd\sin\theta = \frac{m2\pi}{N}, \quad m = 1,2,3\ldots(N-1) \Rightarrow$$

$$\frac{2\pi}{\lambda}d\sin\theta = \frac{m2\pi}{N}, \quad m = 1,2,3\ldots(N-1) \Rightarrow$$

$$d\sin\theta = \frac{m\lambda}{N}, \quad m = 1,2,3\ldots(N-1)$$



Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d\sin\theta = m\lambda$.

Maxima:

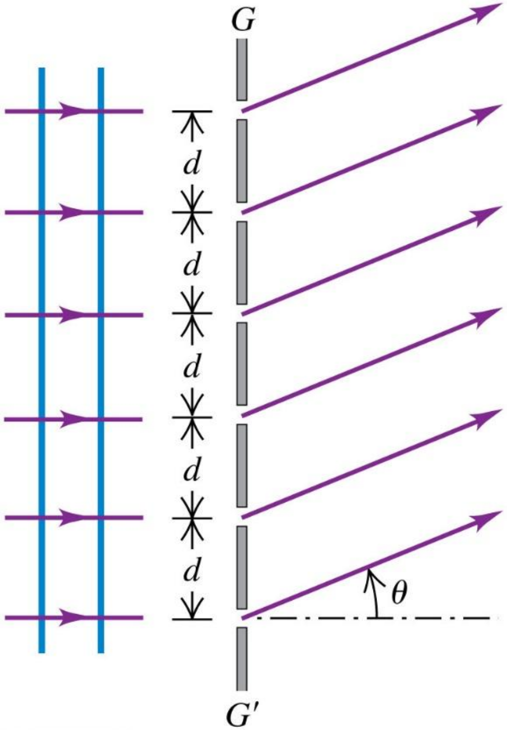
$$\phi_{adj} = m2\pi$$

$$\therefore kd\sin\theta = m2\pi \Rightarrow$$

$$\frac{2\pi}{\lambda}d\sin\theta = m2\pi \Rightarrow$$

$$d\sin\theta = m\lambda, \quad m = 1,2,3\ldots$$

Coherent light passing through six (6) slits separated by a distance d produces a pattern of dark and bright areas on a distant screen. There will be a **dark** area on the screen at a position where the path difference to the screen from adjacent slits is



A. $\lambda/2$.

B. $\lambda/3$.

C. $\lambda/6$.

D. any of these.

Generalization to N-slit Diffraction

$$N\phi_{adj} = 2m\pi, \quad m = 1,2,3\ldots(N-1) \Rightarrow$$

$$\phi_{adj} = \frac{2m\pi}{N}, \quad m = 1,2,3\ldots(N-1)$$

$$\therefore kd\sin\theta = \frac{2m\pi}{N}, \quad m = 1,2,3\ldots(N-1) \Rightarrow$$

$$\frac{2\pi}{\lambda}d\sin\theta = \frac{2m\pi}{N}, \quad m = 1,2,3\ldots(N-1) \Rightarrow$$

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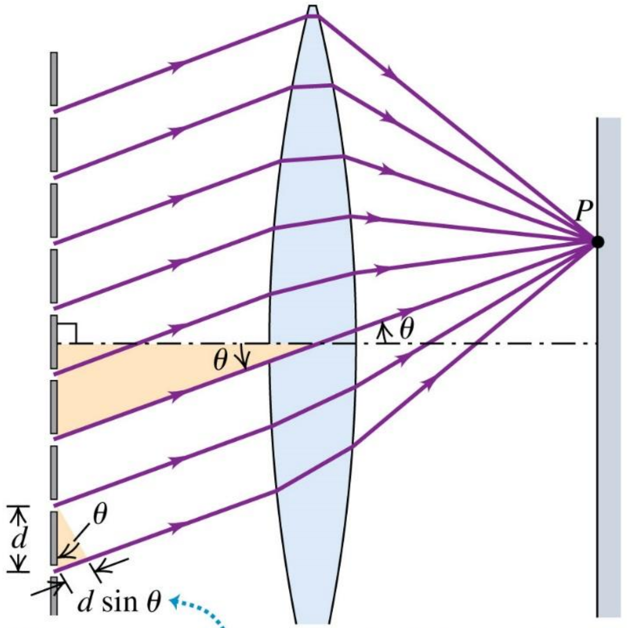


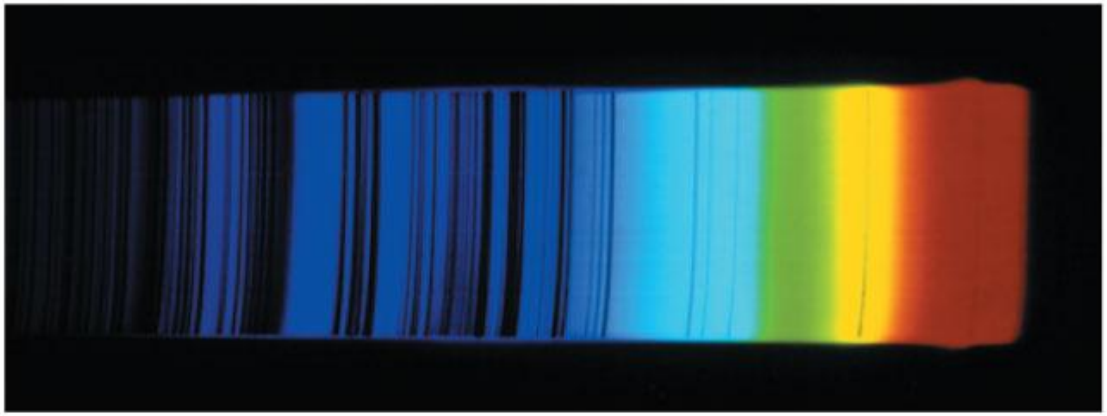
Maxima occur where the path difference for adjacent slits is a whole number of wavelengths: $d\sin\theta = m\lambda$.

What would happen if we let the number of slits get really large?

Diffraction Gratings

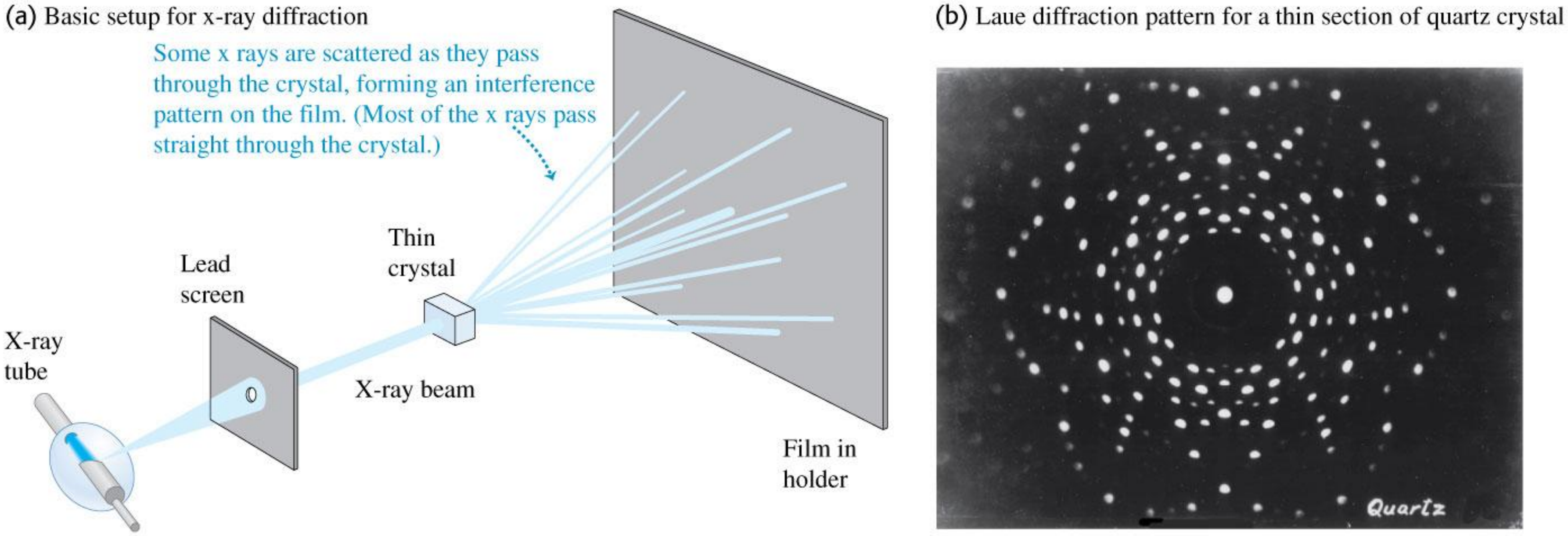
- Because the location of the maxima are dependent on the wavelength, and for a given wavelength are very narrow, a diffraction grating is very useful for spectroscopy…

(b)



X-ray Diffraction

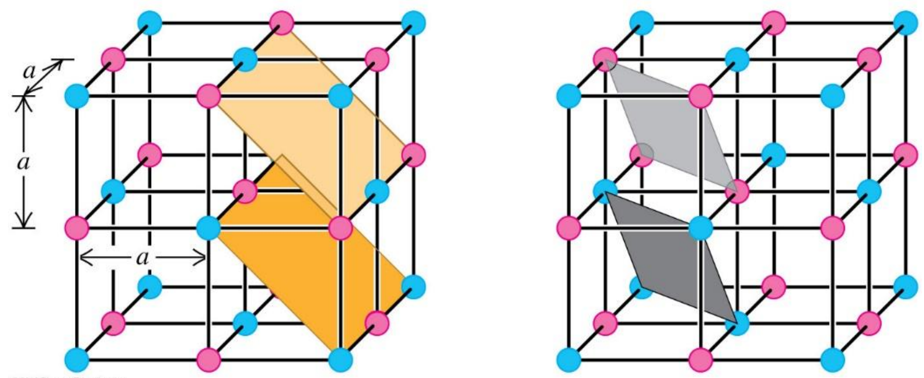
- Also known as X-ray crystallography.
- Can be used to determine the structure of the crystal.
- Why use x-rays?

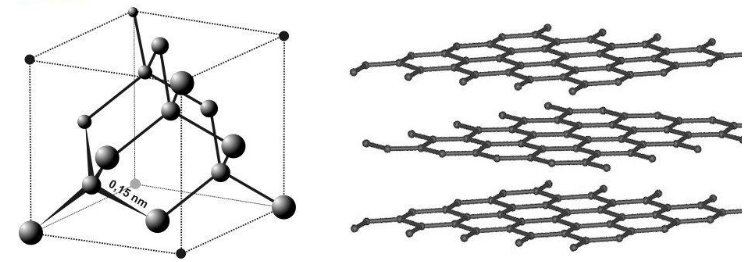


X-ray Diffraction

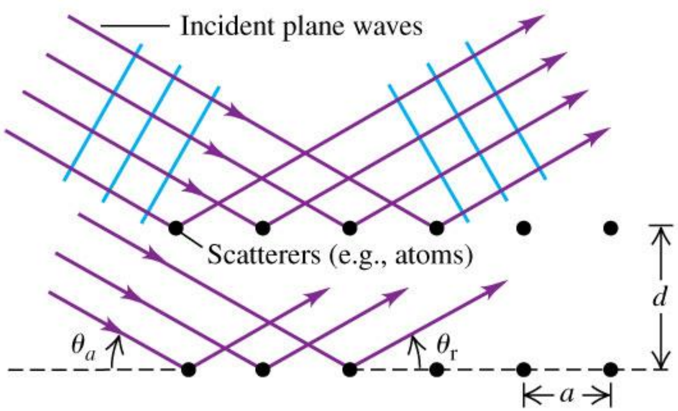
(a) Spacing of planes is $d = a/\sqrt{2}$.

(b) Spacing of planes is $d = a/\sqrt{3}$.

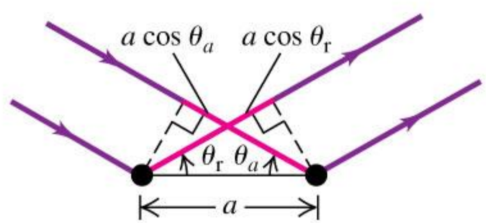




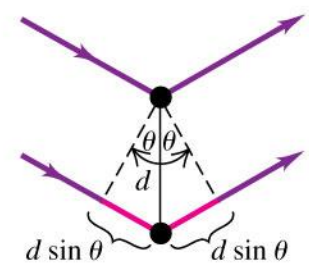
(a) Scattering of waves from a rectangular array



(b) Scattering from adjacent atoms in a row
Interference from adjacent atoms in a row is constructive when the path lengths $a \cos\theta_a$ and $a \cos\theta_r$ are equal, so that the angle of incidence θ_a equals the angle of reflection (scattering) θ_r.



(c) Scattering from atoms in adjacent rows
Interference from atoms in adjacent rows is constructive when the path difference $2d \sin\theta$ is an integral number of wavelengths, as in Eq. (36.16).



Bio-physics

- “An X-ray diffraction pattern of a crystallized enzyme. The pattern of spots (*reflections*) and the relative strength of each spot (*intensities*) can be used to determine the structure of the enzyme.” - Wikipedia