Lecture 24
(Diffraction III
Circular Apertures and Holography)

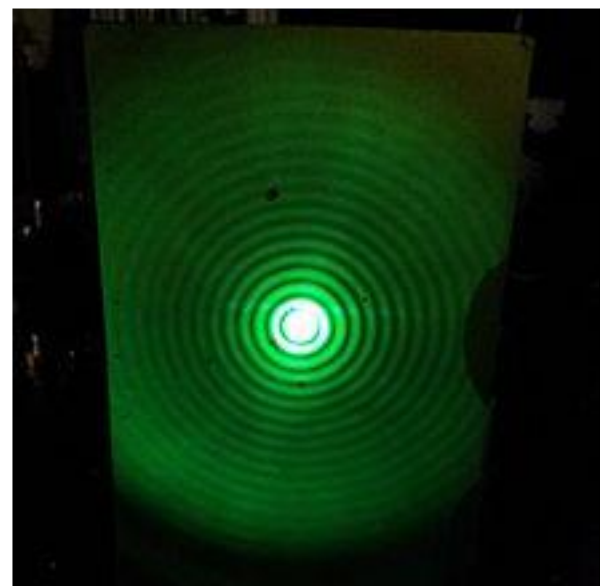
Physics 2310-01 Spring 2020

Douglas Fields

Circular Hole Diffraction

- Finally, I want to discuss the diffraction from a circular aperture.
- Why?
  - Telescopes, binoculars, cameras, eyes…
- Diffraction limits the resolution of all (high resolution) optical instruments.
- There is no getting around this, except by increasing the aperture (one way or another).

The central bright spot of this pattern is called an Airy disk



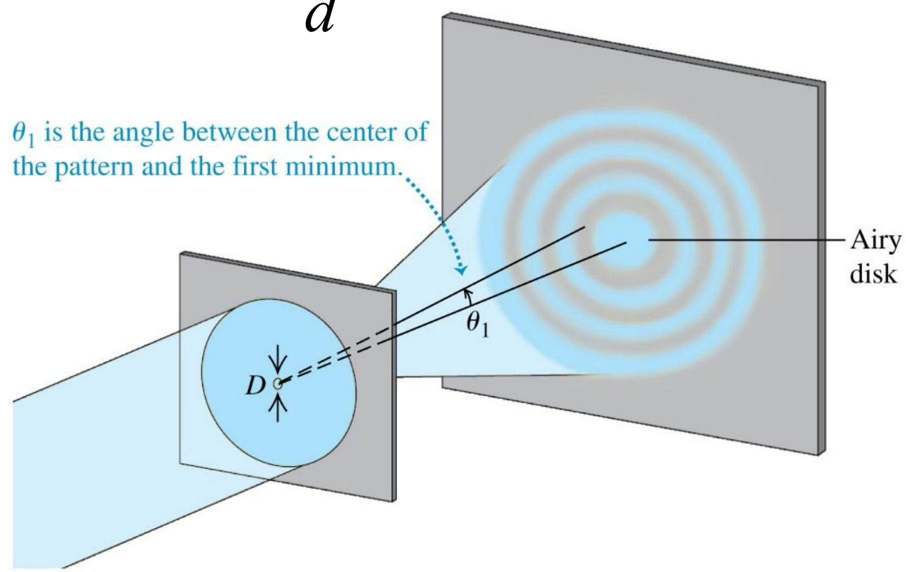
"Airy disk created by laser beam through pinhole" by Anaqreon (talk) (Uploads) - Anaqreon (talk) (Uploads). Licensed under CC0 via Commons - https://commons.wikimedia.org/wiki/File:Airy_disk_created_by_laser_beam_through_pinhole.jpg#/media/File:Airy_disk_created_by_laser_beam_through_pinhole.jpg

Airy's Formula

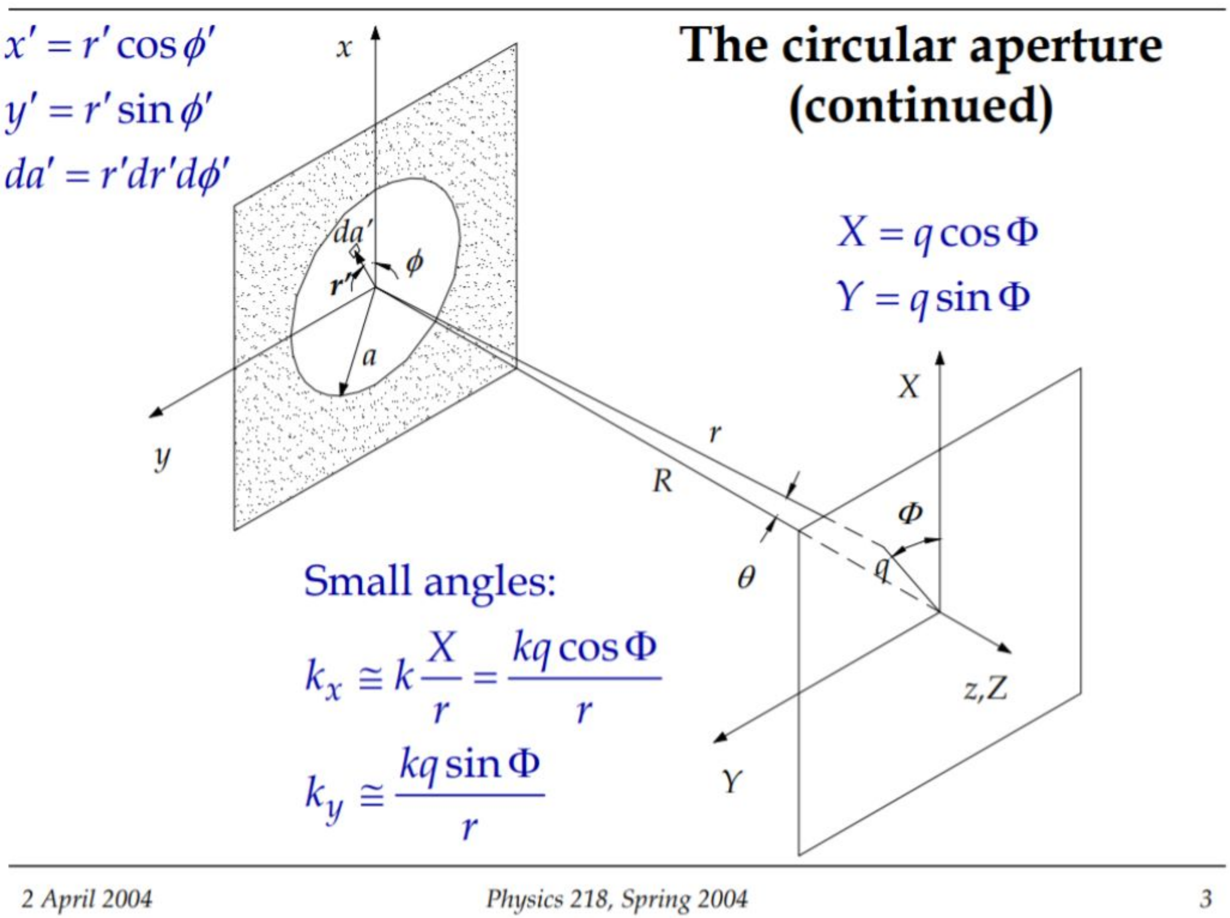
- Sir George Airy formulated the condition for the location of the first minimum in a circular aperture diffraction pattern.
- The book just gives you the solution for the angle of the first minimum:

$$\sin\theta \approx 1.22\frac{\lambda}{d}$$

- But why?



Must Add E-fields From All Point Sources



http://www.pas.rochester.edu/~dmw/phy218/Lectures/Lect_72b.pdf

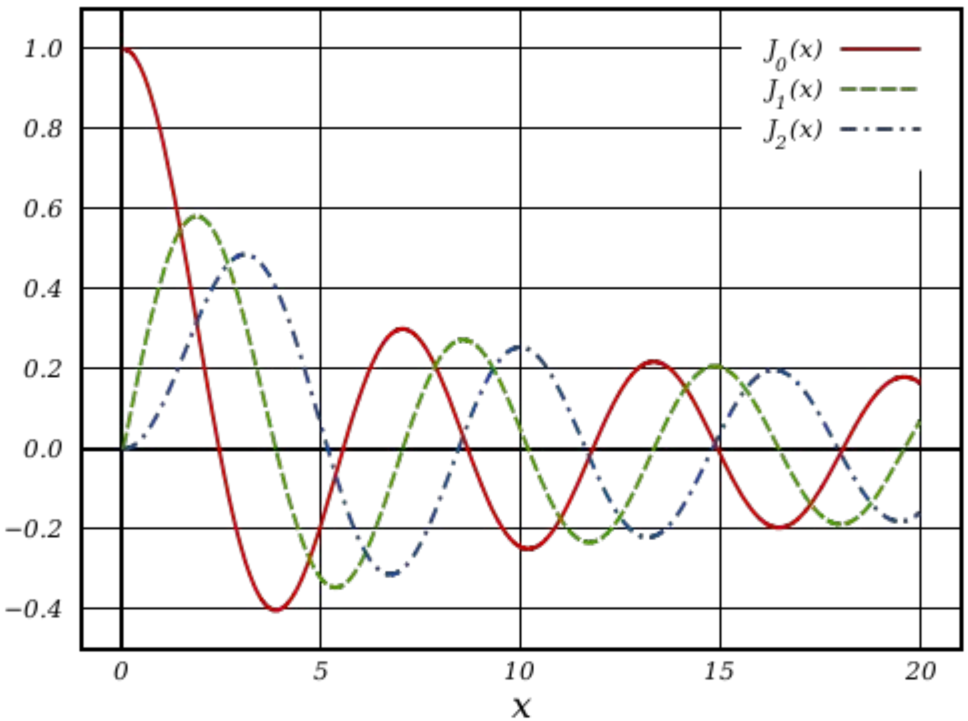
Bessel's Functions

- Bessel's functions are solutions to the differential equation:

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2)y = 0$$

- The solutions are:

$$y(x) = J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\Gamma(m+\alpha+1)}\left(\frac{x}{2}\right)^{2m+\alpha}$$

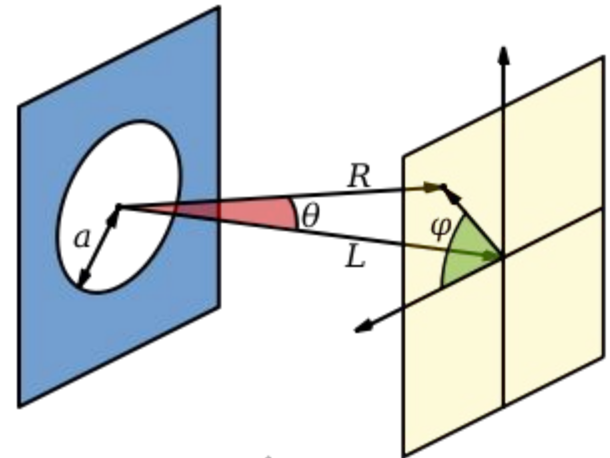


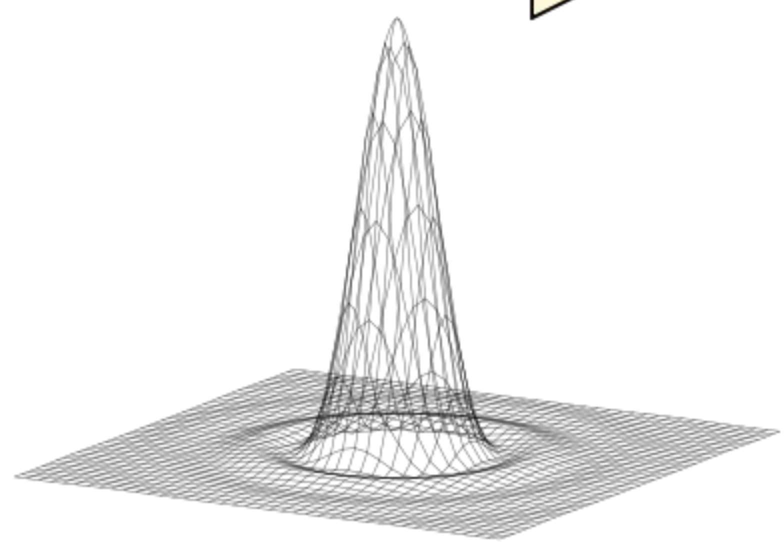
"Bessel Functions (1st Kind, n=0,1,2)" by Inductiveload - Own work, made with Inkscape. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:Bessel_Functions_(1st_Kind,_n%3D0,1,2).svg#/media/File:Bessel_Functions_(1st_Kind,_n%3D0,1,2).svg

Airy's Formula

- The actual solution that Airy determined was:

$$I(\theta) = I_0 \left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2 = I_0 \left(\frac{2J_1(x)}{x} \right)^2$$





"Airy-3d" by Sakurambo 桜ん坊 - Point coordinates calculated using Chipmunk Basic and converted into SVG format using Adobe Illustrator. Licensed under Public Domain via Wikipedia - https://en.wikipedia.org/wiki/File:Airy-3d.svg#/media/File:Airy-3d.svg

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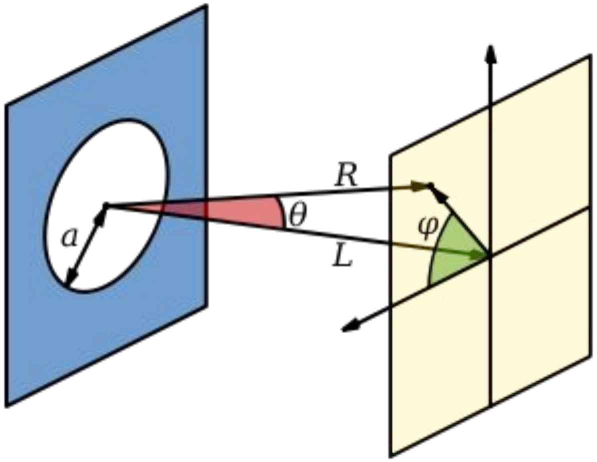
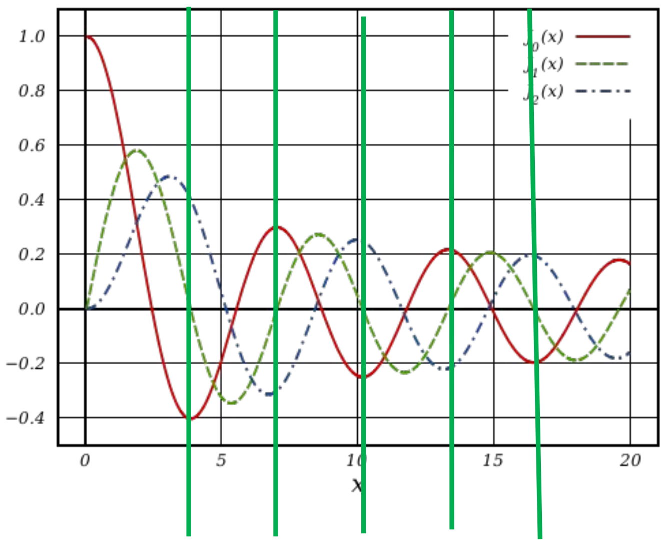


- The factor of 1.22 comes from the zeros of the first order Bessel function,

$$\frac{2J_1(x)}{x} = 0 \Rightarrow$$

$$x \approx 3.8317, 7.0156, 10.1735, 13.3237, 16.4706\ldots$$



- And with *a=d/2* and *k=2π/λ*,

$$ka\sin\theta \approx 3.8317 \Rightarrow$$

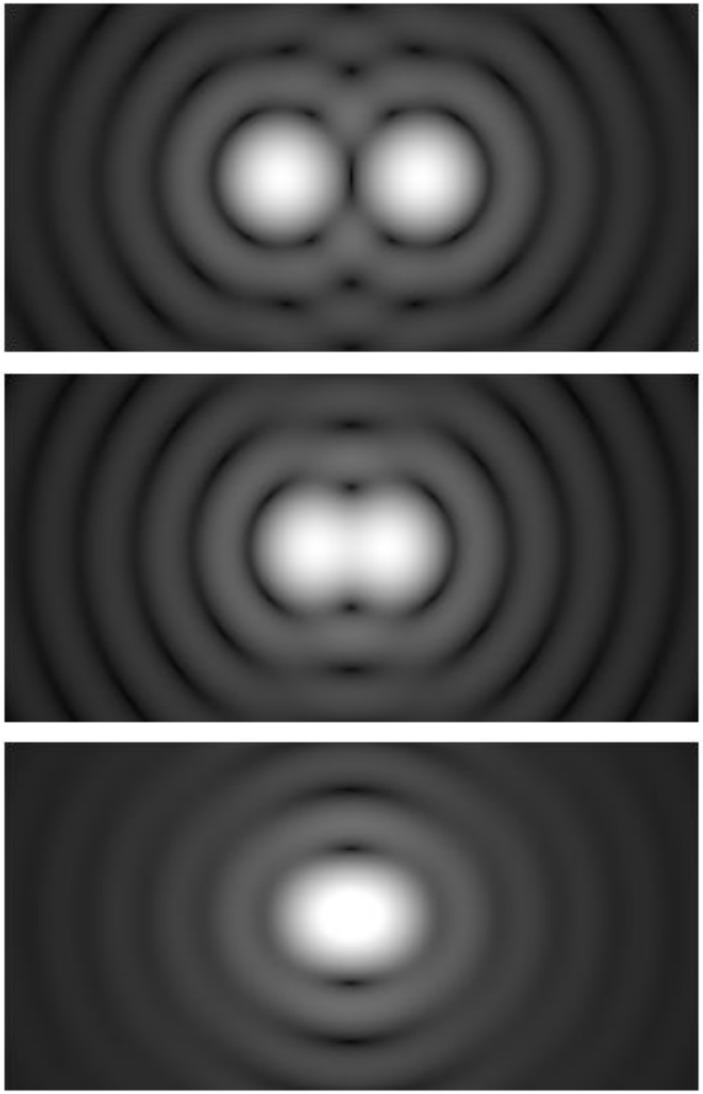
$$\sin\theta \approx 1.22\frac{\lambda}{d}$$

Rayleigh Criterion

- Lord Rayleigh (actually: John William Strutt, 3rd Baron Rayleigh) used the Airy formula to constitute the resolving power of an optical device:

$$\theta \approx 1.22\frac{\lambda}{d}$$

- Known as the Rayleigh Criterion.
- When the angular separation of two objects is less than this, the device cannot resolve them as two objects.
- A humorous quotation, from Lord Rayleigh:
  - "When I was bringing out my *Scientific Papers* I proposed a motto from the Psalms, *"The Works of the Lord are great, sought out of all them that have pleasure therein."* The Secretary to the Press suggested with many apologies that the reader might suppose that **I** was the Lord."

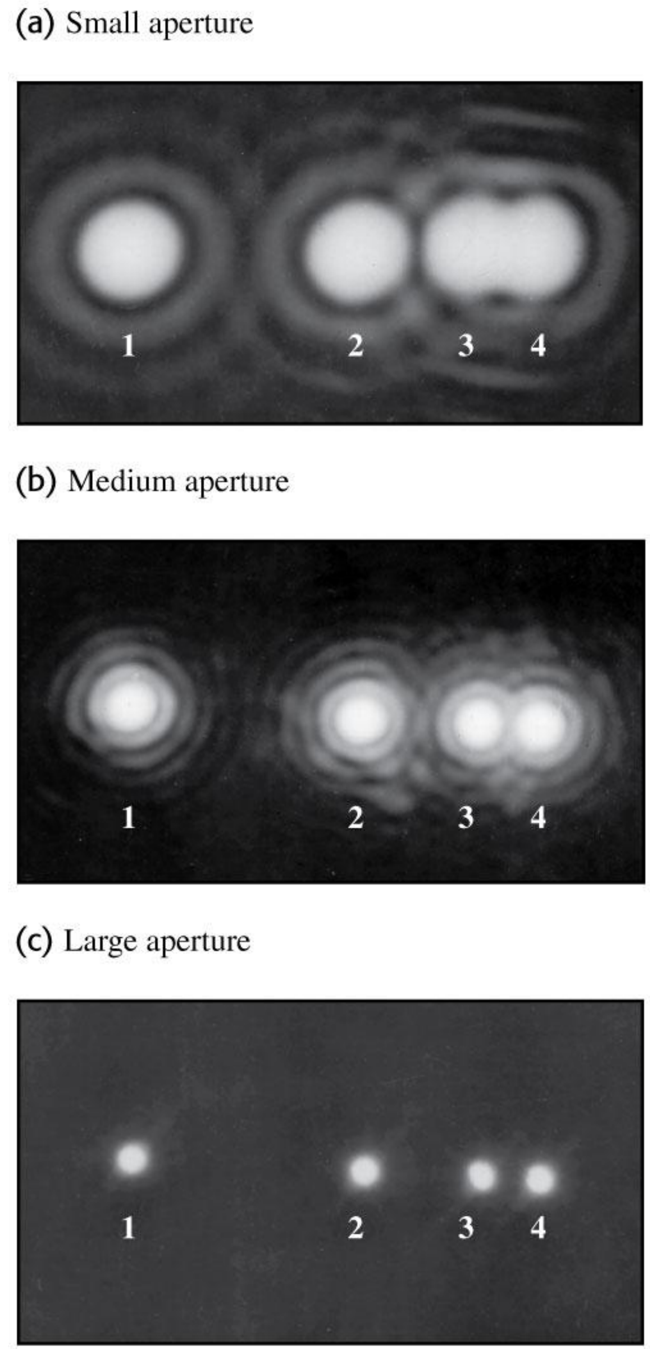


Rayleigh's Criterion

- Notice that the angular resolution gets better with the size of the aperture:

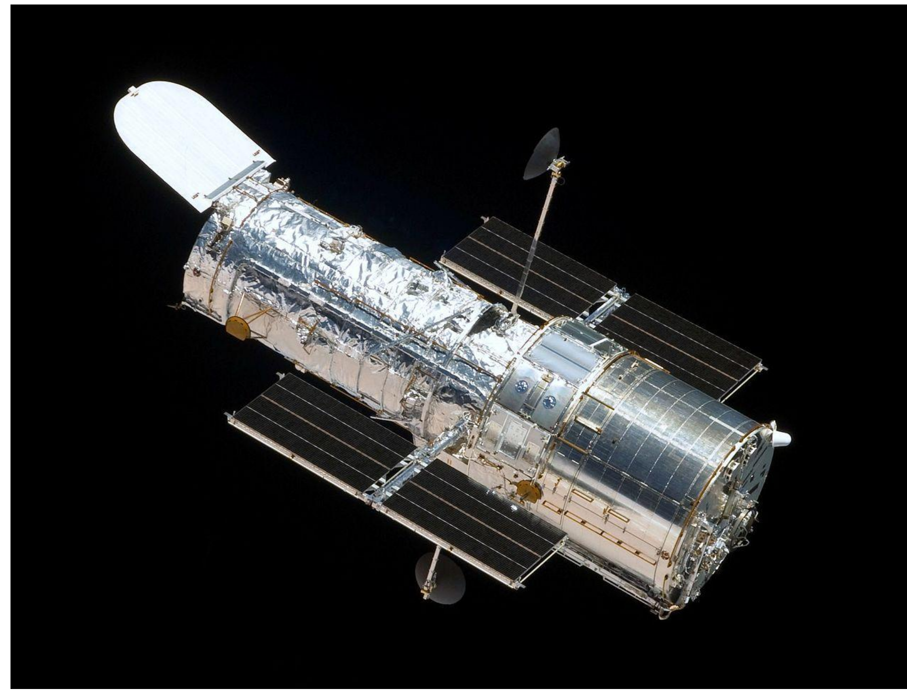
$$\theta \approx 1.22\frac{\lambda}{d}$$

- Hence, if you want to be able to look at the details of something (say, the universe), you need large apertures.

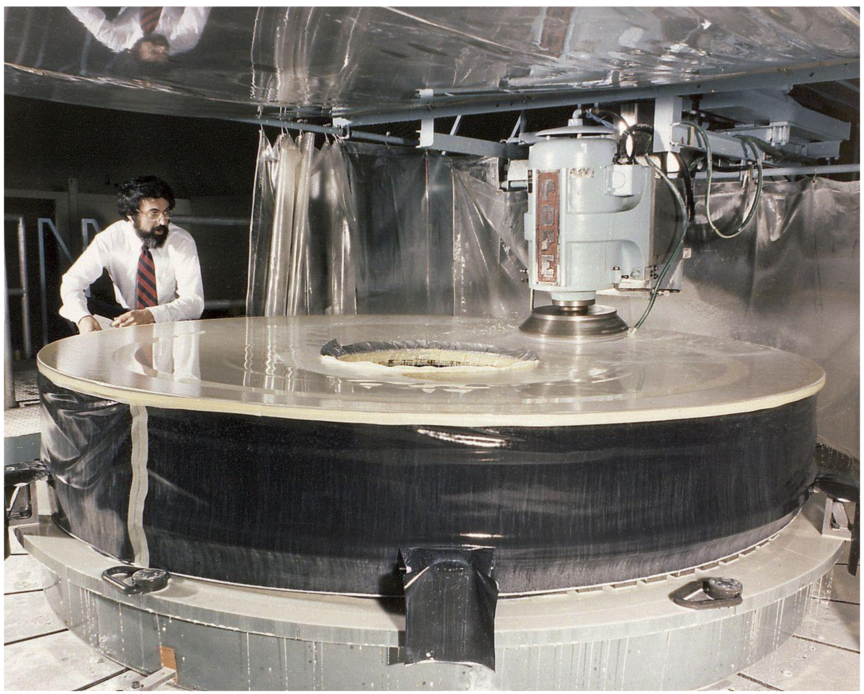


Somewhat Large Aperture

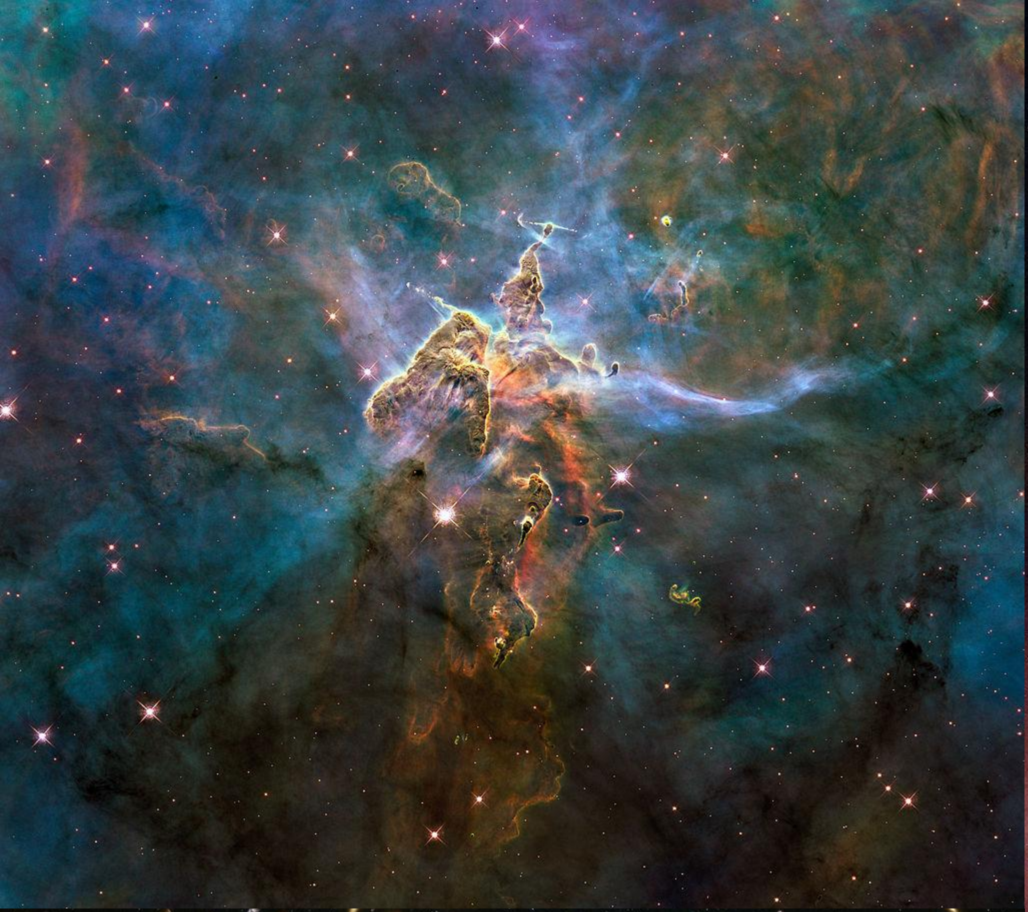
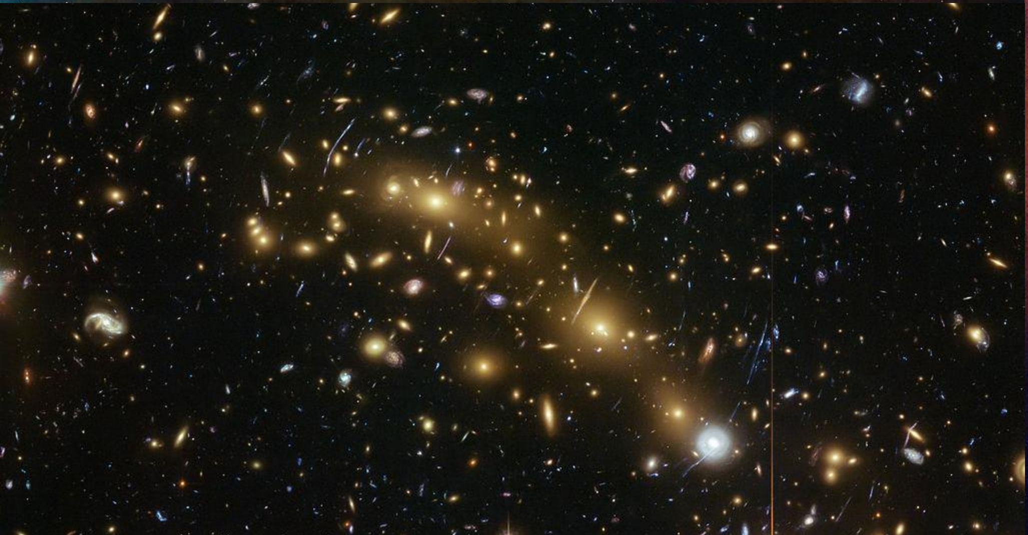
- Hubble Space Telescope



"HST-SM4" by Ruffnax (Crew of STS-125) - http://spaceflight.nasa.gov/gallery/images/shuttle/sts-119/hires/s125e011848.jpg. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:HST-SM4.jpeg#/media/File:HST-SM4.jpeg

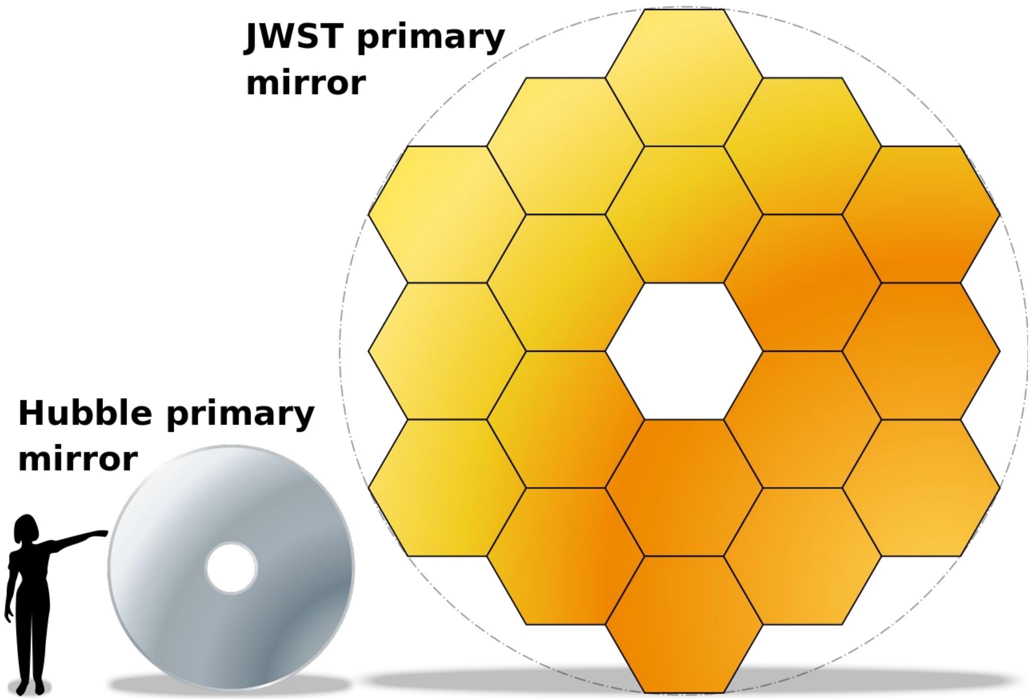


"Hubble mirror polishing" by NASA Marshall Space Flight Center - NASA Marshall Space Flight Center Collection (NIX MSFC-7995584). Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:Hubble_mirror_polishing.jpg#/media/File:Hubble_mirror_polishing.jpg


The successor…

- James Webb Space Telescope



Even Larger Aperture

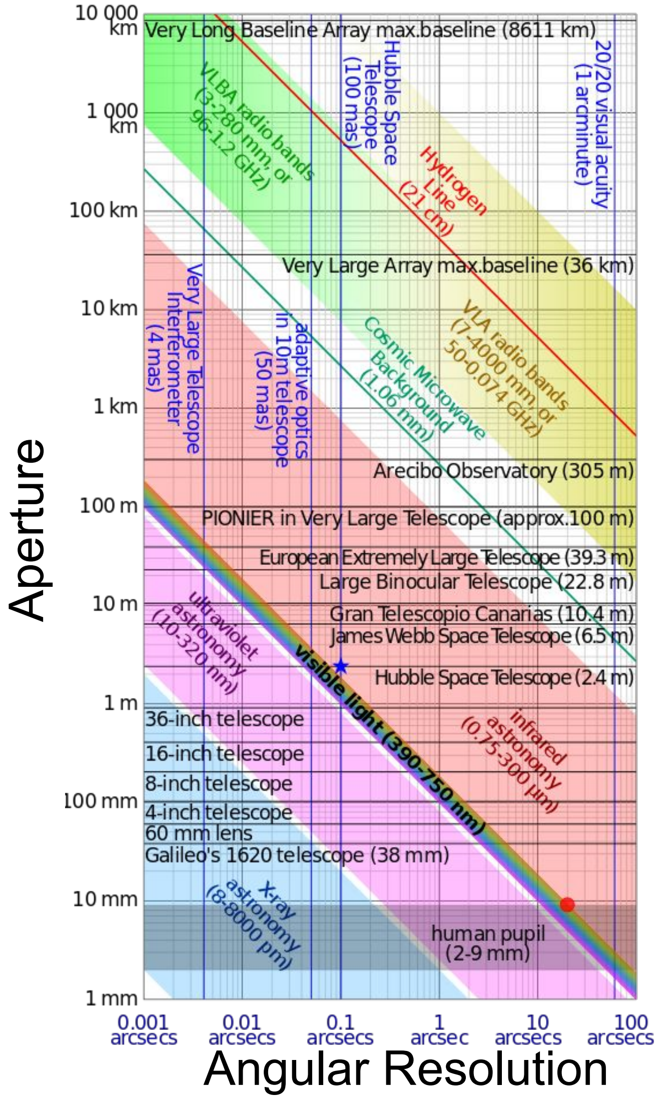
- The Very Large Array (VLA) is located just southwest of here.
- The angular resolution that can be reached is between 0.2 and 0.004 arcseconds.
- While it has a much larger aperture, it has to deal with background noise and distortions from the atmosphere.

Even Larger Apertures

- There are many programs to expand the aperture:
  - https://en.wikipedia.org/wiki/Square_Kilometre_Array.
  - https://en.wikipedia.org/wiki/Long_Wavelength_Array.
  - https://en.wikipedia.org/wiki/LOFAR.
  - https://en.wikipedia.org/wiki/MeerKAT.
  - https://en.wikipedia.org/wiki/Murchison_Widefield_Array.

Comparisons

- Just for future reference…



You use a telescope lens to form an image of two closely spaced, distant stars. Which of the following will increase the resolving power?

A. Use a filter so that only the blue light from the stars enters the lens.

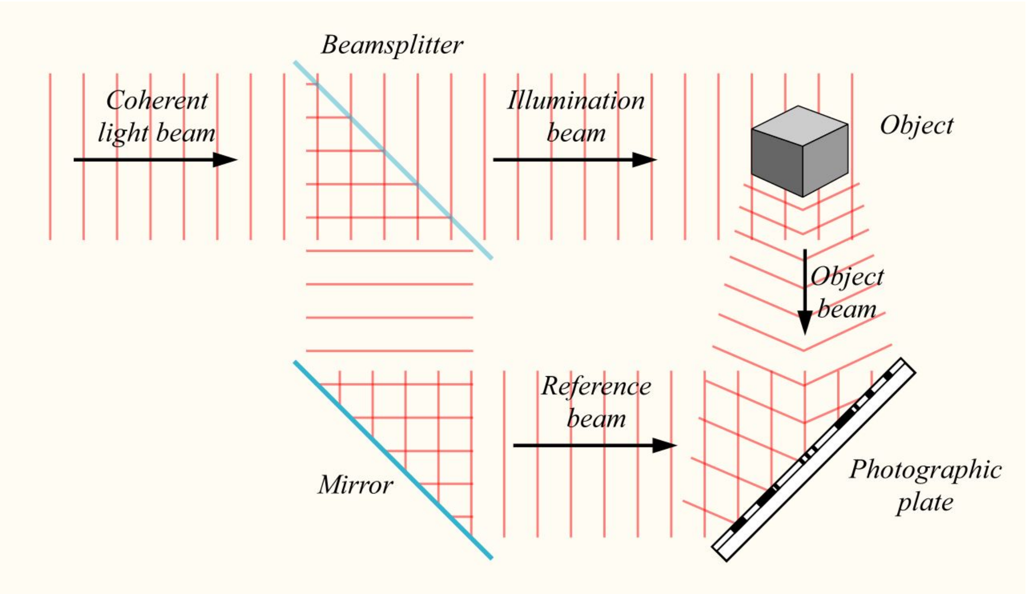
B. Use a filter so that only the red light from the stars enters the lens.

C. Use a lens of smaller diameter.

D. more than one of the above

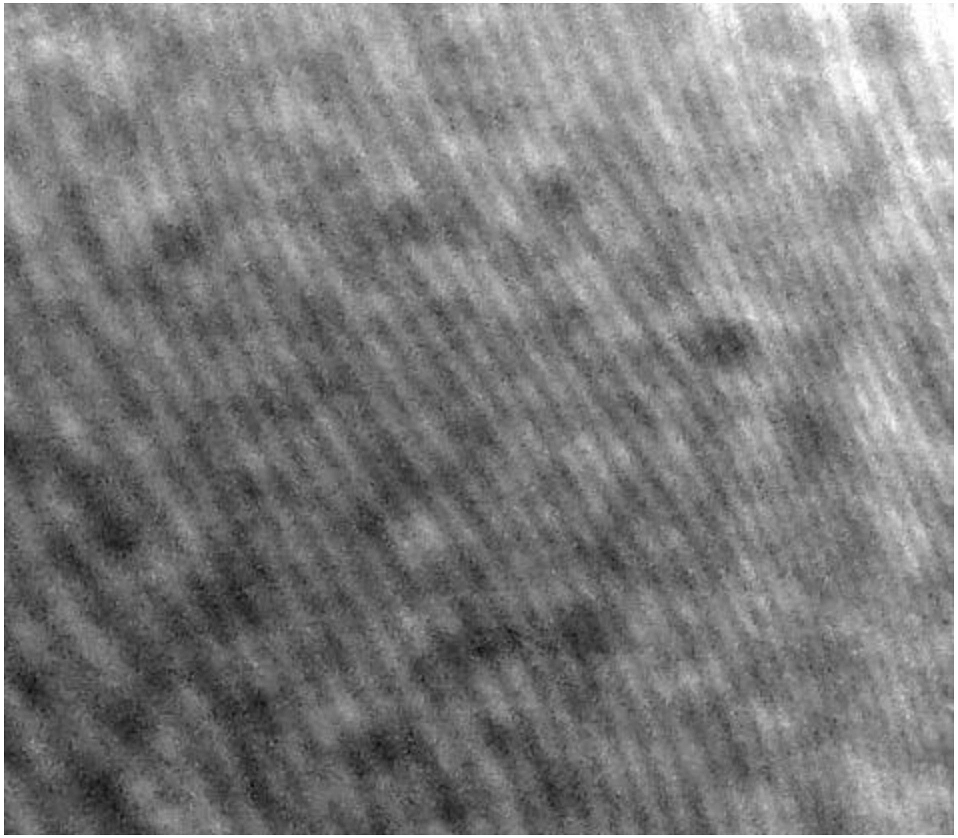
Holography

- One interesting use of interference is the creation of an interference pattern which encodes the information contained in a beam of light coming from an object.



Holography

- The image recorded on the film is just the interference pattern between the reference beam and object beam.
- Each area on the film encodes the entire image, but from an increasingly narrow (with decreasing film area) point of view.



"Holographic recording" by Epzcaw (talk) - Own work. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:Holographic_recording.jpg#/media/File:Holographic_recording.jpg

Holography

- To view the holograph, the reference beam must be used as a reconstruction beam.
- The wave fronts produced are then exactly the wave fronts of the object beam, creating a virtual image.