Lecture 25
(Galilean and Special Relativity)

Physics 2310-01 Spring 2020

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The Concept of Relativity

- From Wikipedia:
  - **Galilean invariance** or **Galilean relativity** states that ***"The laws of motion are the same in all inertial frames"***. Galileo Galilei first described this principle in 1632 in his *Dialogue Concerning the Two Chief World Systems* using the example of a ship travelling at constant velocity, without rocking, on a smooth sea; any observer doing experiments below the deck would not be able to tell whether the ship was moving or stationary. The fact that the Earth orbits around the sun at approximately 30 km/s offers a somewhat more dramatic example.

What he actually said: "The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line… A clear proof of this we have from the experiment of a ship; where all motions happen after the same manner, whether the ship is at rest, or is carried forwards in a right line.

Careful Descriptions

The laws of motion are the same in all inertial frames.

- The first several lectures on Special Relativity will be focused on just defining some terms so that we can understand their meaning and the consequences of the Principle of Relativity.
- First: The Principle of Relativity is a postulate (assumption).
  - It cannot be proved (you would have to test it in all possible inertial frames.
  - But it has yet to be disproved. Careful measurements have yet to find any discrepancies.
  - The consequences of losing the assumption are difficult to swallow.
- So, we will take Galilean Relativity as an unproved, but consistent assumption.

The Laws of Motion

The laws of motion are the same in all inertial frames.

- While these weren't derived at the time, we will use Newton's Laws as our tests for Galilean Relativity.
- First Law: When no forces act on a body, it's motion will remain unchanged – if it is at rest, it will remain at rest, if it is moving, it will remain moving with the same speed and direction.
- Second Law: The vector sum of all forces acting on a body with mass m, will cause that body to accelerate with a relationship between the net force, the mass and the acceleration given by: $\vec{F} = m\vec{a}$
- Third Law: When one body acts on another with a force, the second body acts back on the first with a force equal in magnitude to the first, but in opposite directions.

Spacetime Coordinates

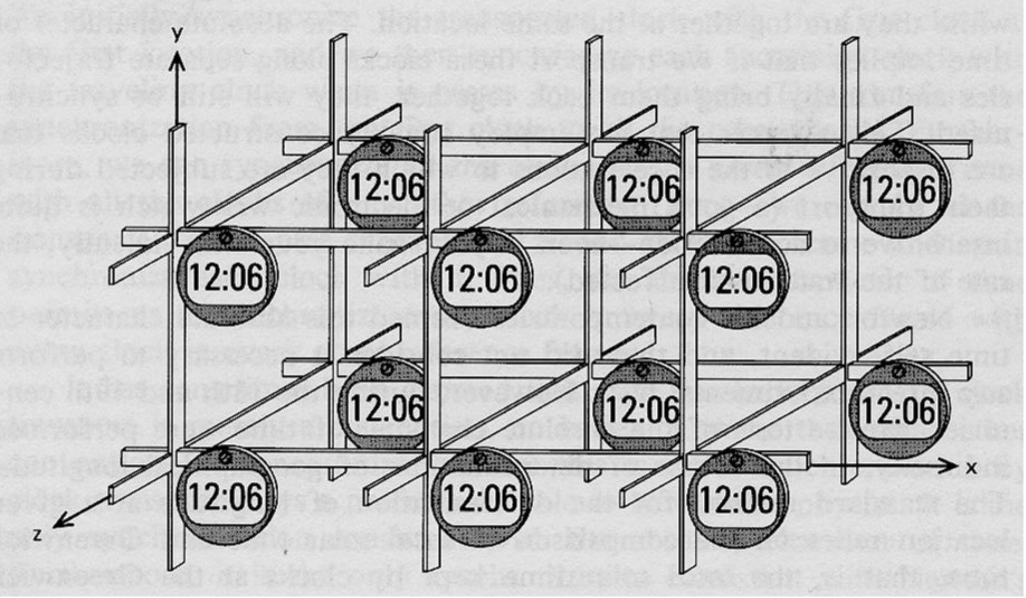
The laws of motion are the same in all inertial frames.

- In order to do anything at all (experimentally, at least) with Newton's Laws, we need a way to record ***events***.
- An event is a physical occurrence that takes place at a specific point in space and at a specific time.
- An event is then characterized by a set of four numbers – its three spatial coordinates and the time of the event.
- We will refer to these four numbers as the spacetime coordinates of the event and generally place them in order: (t,x,y,z), or more generally (x_0,x_1,x_2,x_3).
- But how do we measure the location and time of events?

Reference Frames

The laws of motion are the same in all inertial frames.

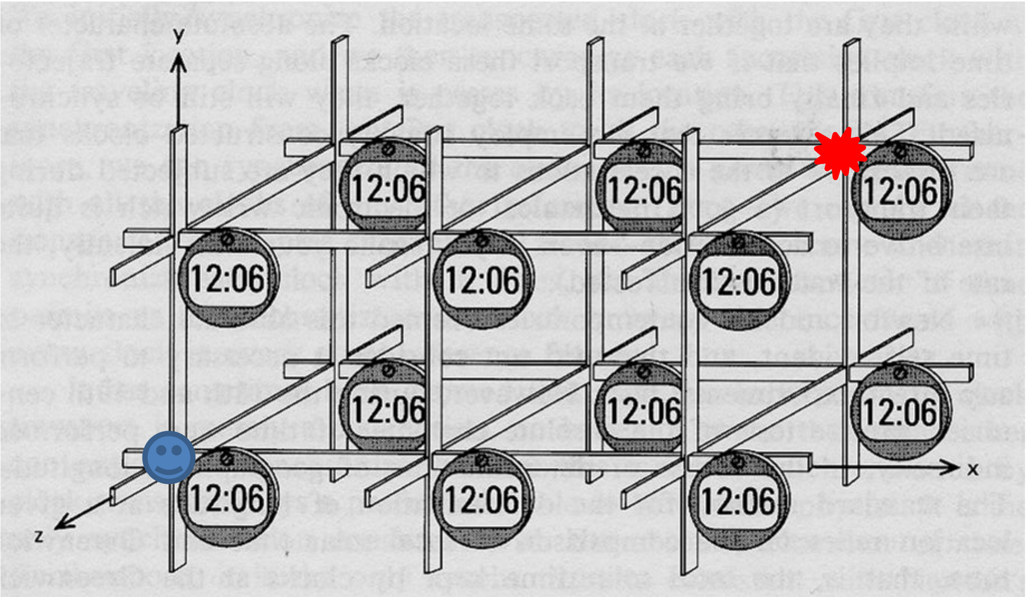
- A reference frame is a cubic lattice of measuring sticks (or any other means of measuring distance), plus an associated lattice of synchronized clocks.
- We will have to get back to the question of how one goes about synchronizing the clocks later.
- A reference frame is associated with an observer (or an event or series of events).



Observation

The laws of motion are the same in all inertial frames.

- We even have to be careful about what we mean when we say that the "observer" observes!
- Let's say the observer is at the origin, and an explosion happens at a distance away.
- The observer would only hear the explosion after the sound reaches her, but that isn't what we mean by an observation!
- The observation of the event is the spacetime coordinates of the event as it occurred, not as the observer "sees" it.



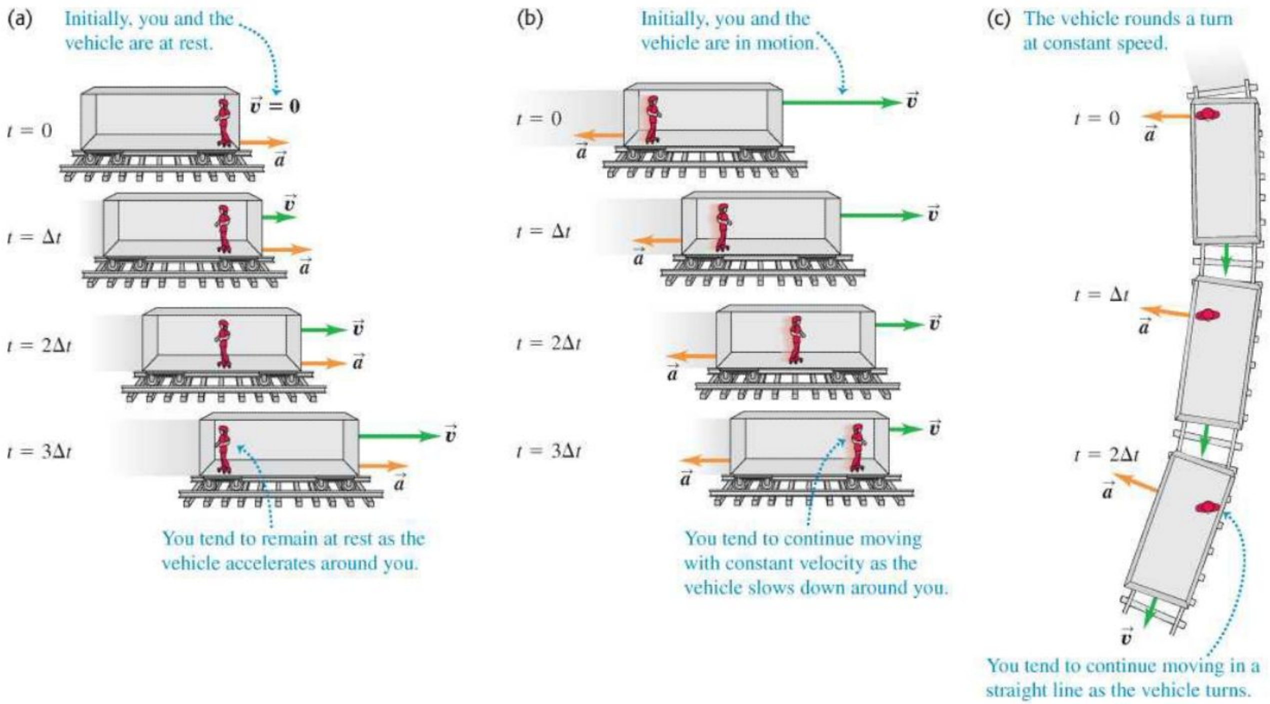
Inertial Frames

The laws of motion are the same in all inertial frames.

- Finally, we must define what we mean by an inertial frame.
- An inertial frame is a frame in which an isolated object (no forces acting on it) is seen to move with constant (maybe zero) velocity.
- In other words, an inertial reference frame is one in which Newton's First Law holds.

Non-Inertial Frame Example

- In each of these cases, a reference frame tied to the train contains a person on roller skates.
- Using Newton's First Law, how would an observer in this frame account for the motion of the person on the skates?



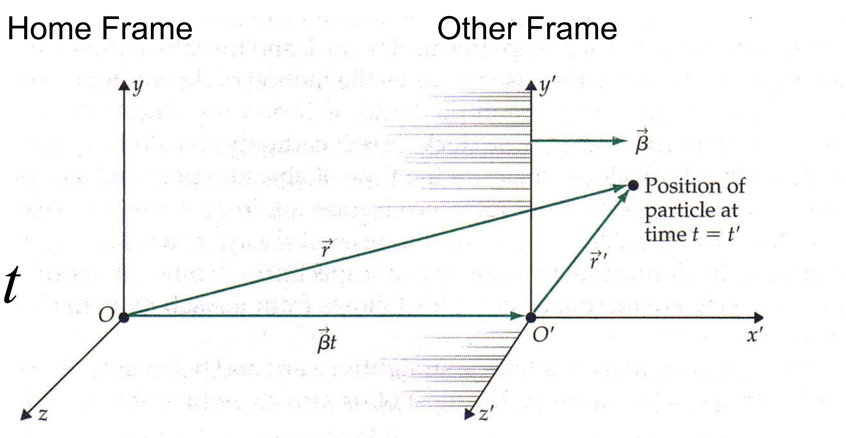
Testing Relativity

- In order to test our assumption that the laws of motion are the same in all inertial frames, we have to come up with an experiment, and understand how two observers see the same set of events.
- We will begin by having two frames, one at rest (the Home Frame), and the other moving at constant velocity $\beta=\beta_x$ relative to the Home Frame (the Other Frame).
- Some object resides at rest at a point in the Other Frame.
- What is the relationship between the spatial coordinates of the frames?

$$\vec{r}'(t') = \vec{r}(t) - \vec{\beta}t$$

$$x'(t') = x(t) - \beta_x t$$
$$y'(t') = y(t) - \beta_y t$$
$$z'(t') = z(t) - \beta_z t$$

$$x' = x - \beta t$$
$$y' = y$$
$$z' = z$$



Galilean Time

- Now, we also have to assign times in both the Home and Other Frame.
- Galilean Relativity posits that time is universal, so therefore t = t’.

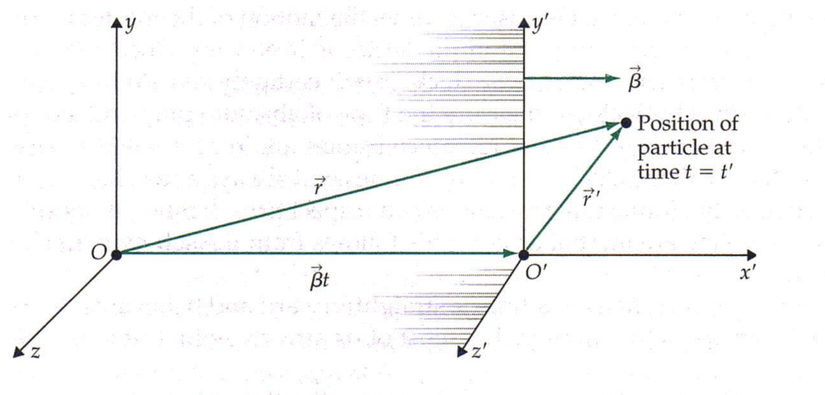
$$x' = x - \beta t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Galilean transformation equations



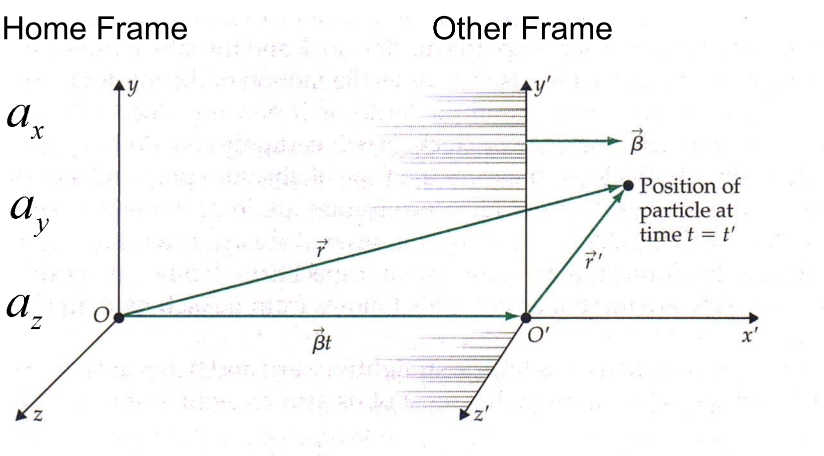
Newton’s Law’s in the Other Frame

- From the Galilean transformation equations, we can take the first and second derivatives of the spatial coordinates in the Other Frame, to get the acceleration of the object in the Other Frame.
- We find that the two frames agree on the acceleration, and since they also agree on the mass and the forces involved (in Galilean Relativity), Newton’s Second Law holds in both frames.

$$x' = x - \beta t \quad \Rightarrow \quad v_x{}' = v_x - \beta \quad \Rightarrow \quad a_x{}' = a_x$$

$$y' = y \quad \Rightarrow \quad v_y{}' = v_y \quad \Rightarrow \quad a_y{}' = a_y$$

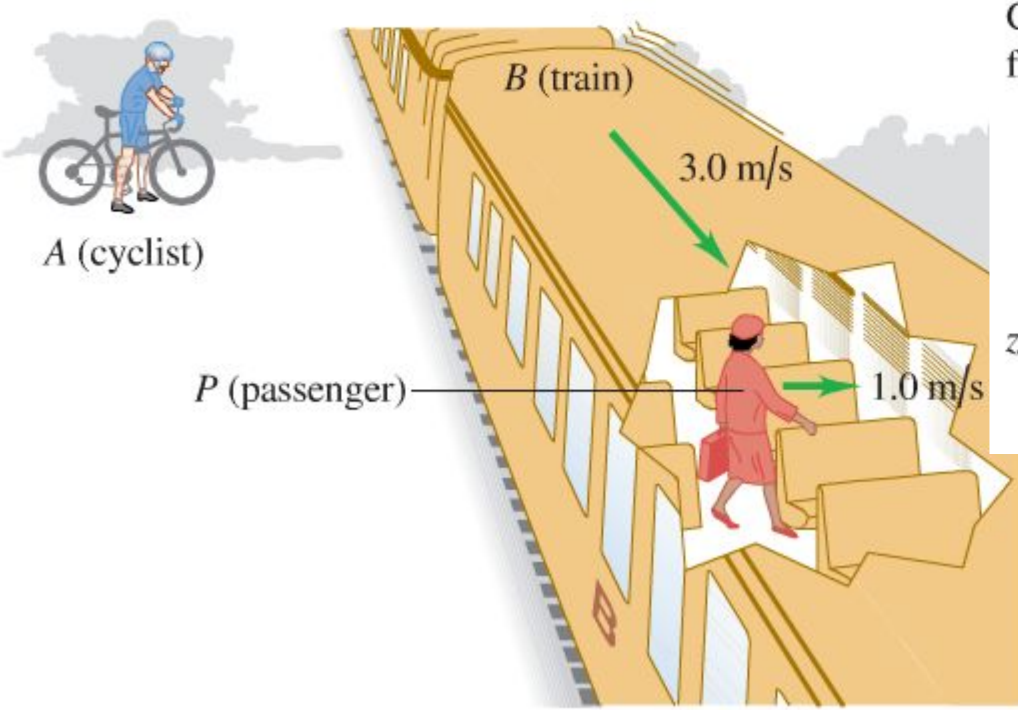
$$z' = z \quad \Rightarrow \quad v_z{}' = v_z \quad \Rightarrow \quad a_z{}' = a_z$$

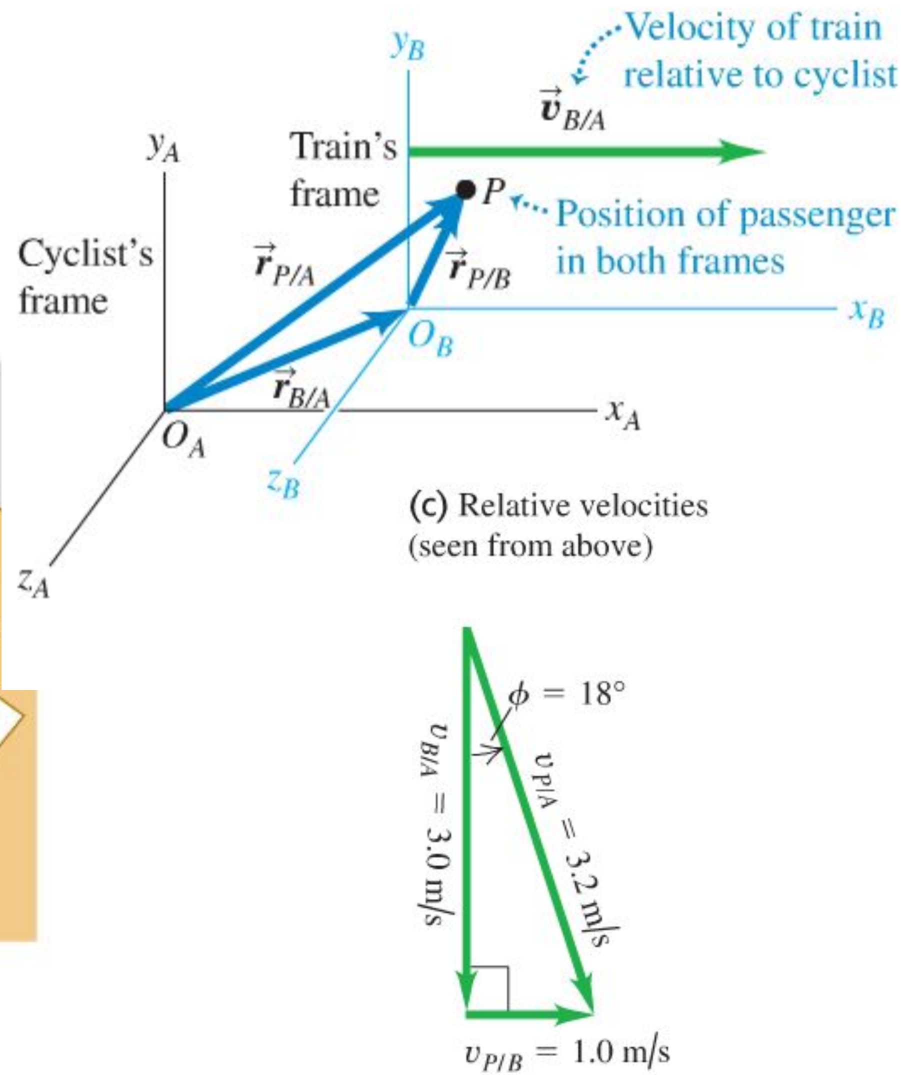


$$\vec{F} = m\vec{a} \Rightarrow$$
$$\vec{F} = m\vec{a}'$$

Relative Velocity

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

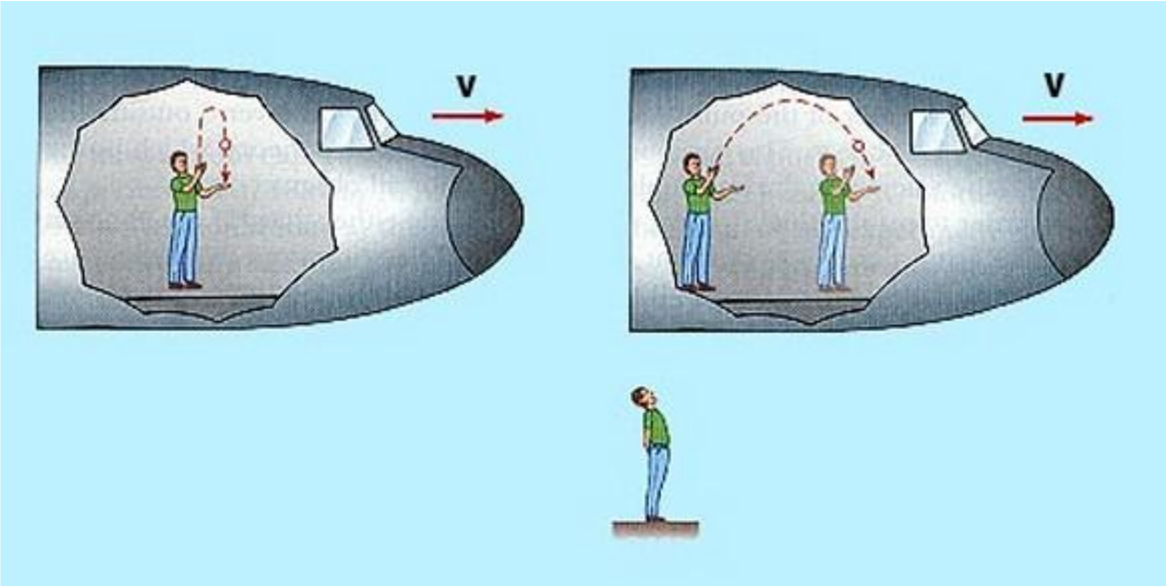




(c) Relative velocities (seen from above)

Example

- Even though the paths look very different to an observer in the different frames, for both observers, the laws of physics hold.
- **That** is the essence of the **principle of relativity**.



What about Momentum?
What about Kinetic Energy?

So, what’s wrong with Galilean Relativity?

The laws of motion are the same in all inertial frames.

- Nothing, as stated above.
- But, in the late 19th century, Maxwell set the world a-light…
- And Maxwell’s equations are not invariant under Galilean transformations.
- Proof: https://pdfs.semanticscholar.org/42cc/1fbb7c73ced83dae0886291366e112511fed.pdf
- Non-rigorous explanation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{d^2 B}{dt^2}$$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}$$

$$\oint_{\partial V} \vec{E} \cdot d\vec{S} = 0$$

$$\oint_{\partial V} \vec{B} \cdot d\vec{S} = 0$$

$$\oint_{\partial S} \vec{B} \cdot d\vec{r} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint_{\partial S} \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t}\Phi_B$$

Lorentz Transformations

- However, it was known before Einstein that Maxwell’s equations were invariant under a different transformation called the Lorentz transformation:
- But, no one paid much attention to this, as it involved a transformation of TIME!!!
- Do note though, that the Lorentz transformations become exactly the Galilean transformations at (the trivial) v = 0.
- But, since c is very large, small v also is basically the Galilean transformation.
- We will return to this…

$$
\begin{aligned}
t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\
x' &= \gamma\left(x - vt\right) \\
y' &= y \\
z' &= z
\end{aligned}
$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

But what about matter waves?

- Matter waves have a preferred reference frame.

$$y(x,t) = A\cos(kx - \omega t)$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y$$

$$\frac{\partial^2 y}{\partial x^2} = -k^2 y$$

$$but, \quad \omega = vk \Rightarrow v^2 = \frac{\omega^2}{k^2} \Rightarrow$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Waves on a string

$$v = \sqrt{\frac{T}{\mu}}$$

E&M waves on ?

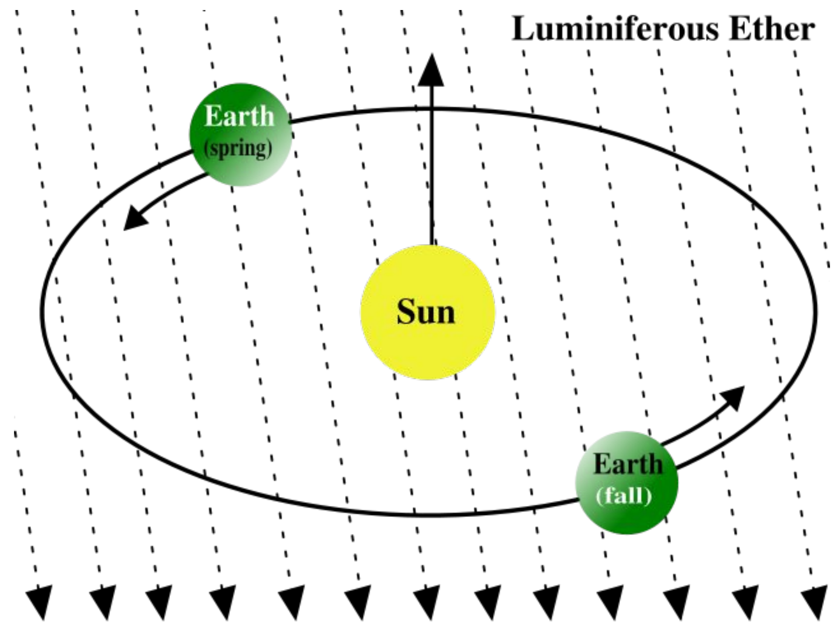
$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{d^2 E}{dt^2}$$

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{d^2 B}{dt^2}$$

$$v = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}} = \frac{1}{\sqrt{\left(4\pi \times 10^{-7}\, H/m\right) \times \left(8.854 \times 10^{-12}\, F/m\right)}} = 2.998 \times 10^8\, m/s = c$$

So, couldn't E&M waves have a medium that is waving?

- This was the conclusion of early 20th century physicists.
- They called it the ***ether***.
- The frame of the local ether would then be the preferred frame for E&M waves.
- What if the local ether were moving relative to the ground?
- In principle, since the earth is moving through space (around the sun, for instance), then the ether might be moving relative to us.
- Then we should be able to tell the difference in the velocity of light relative to the ground, if the velocity of light is constant relative to the ether.



Swimmer analogy

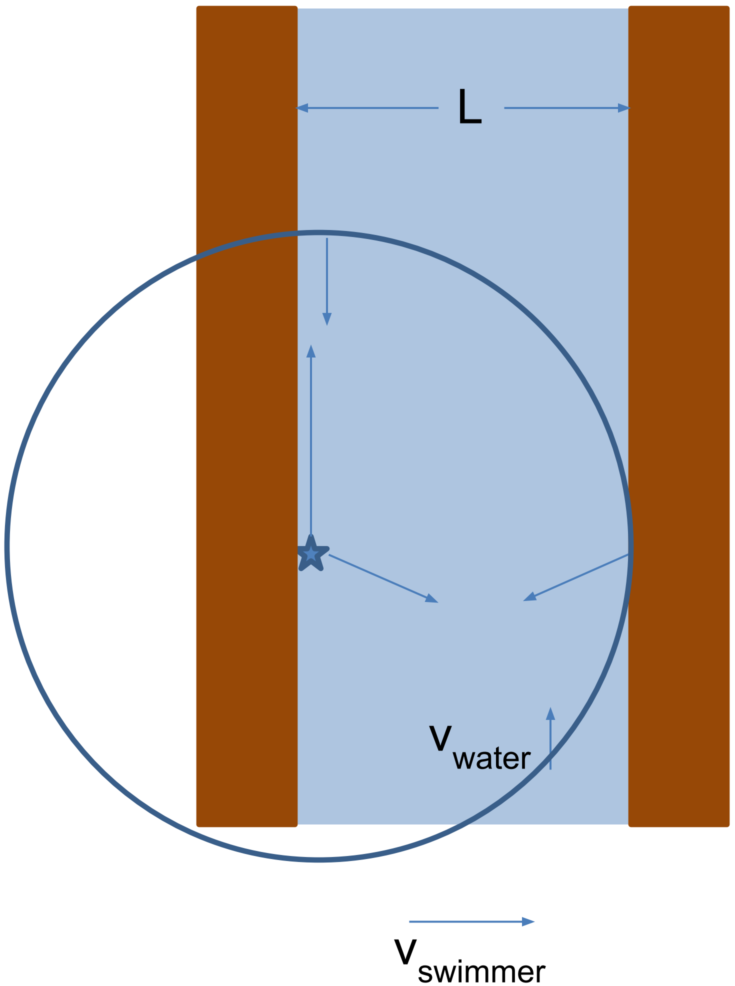
- Consider two waves that travel along the ether, and across the ether between a source and a detector.
- If the velocity of light is different (relative to the ground, for instance) along these two paths, then the phase of the light should also be different!
- Think about two swimmers going from one point on a shore swimming a fixed distance L, and then returning to the starting point.
- One swimmer swims across the current, the other swims initially in the direction of the current, but then has to swim back against the current.
- Each swimmer has the same speed relative to the water.
- How much time does each swimmer take?

$$v_{across} = \sqrt{v_{swimmer}^2 - v_{water}^2} \Rightarrow t_{across} = \frac{2L}{v_{across}} = \frac{2L}{\sqrt{v_{swimmer}^2 - v_{water}^2}}$$

$$v_{down} = v_{swimmer} + v_{water} \Rightarrow t_{down} = \frac{L}{v_{swimmer} + v_{water}}$$

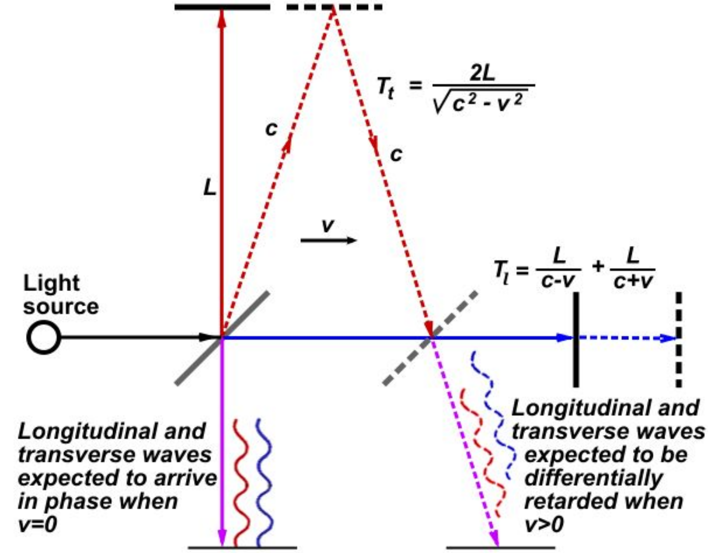
$$v_{up} = v_{swimmer} - v_{water} \Rightarrow t_{up} = \frac{L}{v_{swimmer} - v_{water}}$$

$$t_{along} = t_{down} + t_{up} = \frac{L}{v_{swimmer} + v_{water}} + \frac{L}{v_{swimmer} - v_{water}}$$



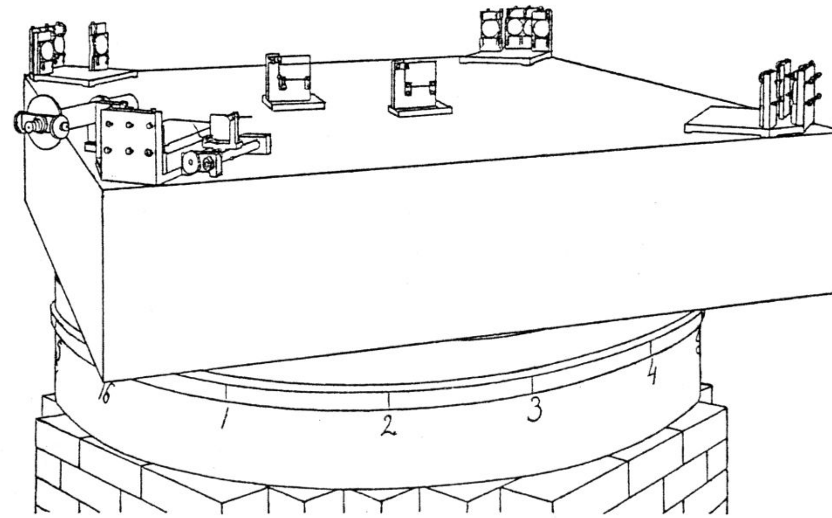
Michelson-Morley Experiment

- And, we have an excellent way to see this.
- Let's use a Michelson interferometer.
- If we have it set up so that one arm is parallel to the direction the ether travel, and the other is perpendicular, the two wave will have a phase difference caused by the difference in travel time.
- If we then rotate the interferometer so that the roles are reversed, we should see interference fringes whiz by as the phases change!



Michelson-Morley Experiment

- Michelson and Morley (another physicist) did just that experiment by setting up an interferometer on a rotating table.
- The experiment became what has been called the most famous failed experiment in history.
- What they saw was consistent with an ether velocity of less than 1/40 of what was expected.
- Later experiments refined their technique and observed even less of an affect.
- So, if no ether…?



Einstein’s Dilemma

- Albert Einstein knew about the Michelson-Morley results, but he was much more concerned about our previous problem of the invariance of Maxwell’s equations.
- He wrote: “*After ten years of reflection such a principle resulted from a paradox upon which I had already hit at the age of sixteen: If I pursue a beam of light with velocity c (velocity of light in a vacuum), I should observe such a beam as an electromagnetic field, constant in time, periodic in space. However, there seems to exist no such thing, neither on the basis of experience, nor according to Maxwell’s equations*…”
- Since he also knew of the necessity of Relativity as a principle of science, his next step, was obvious, at least to him…

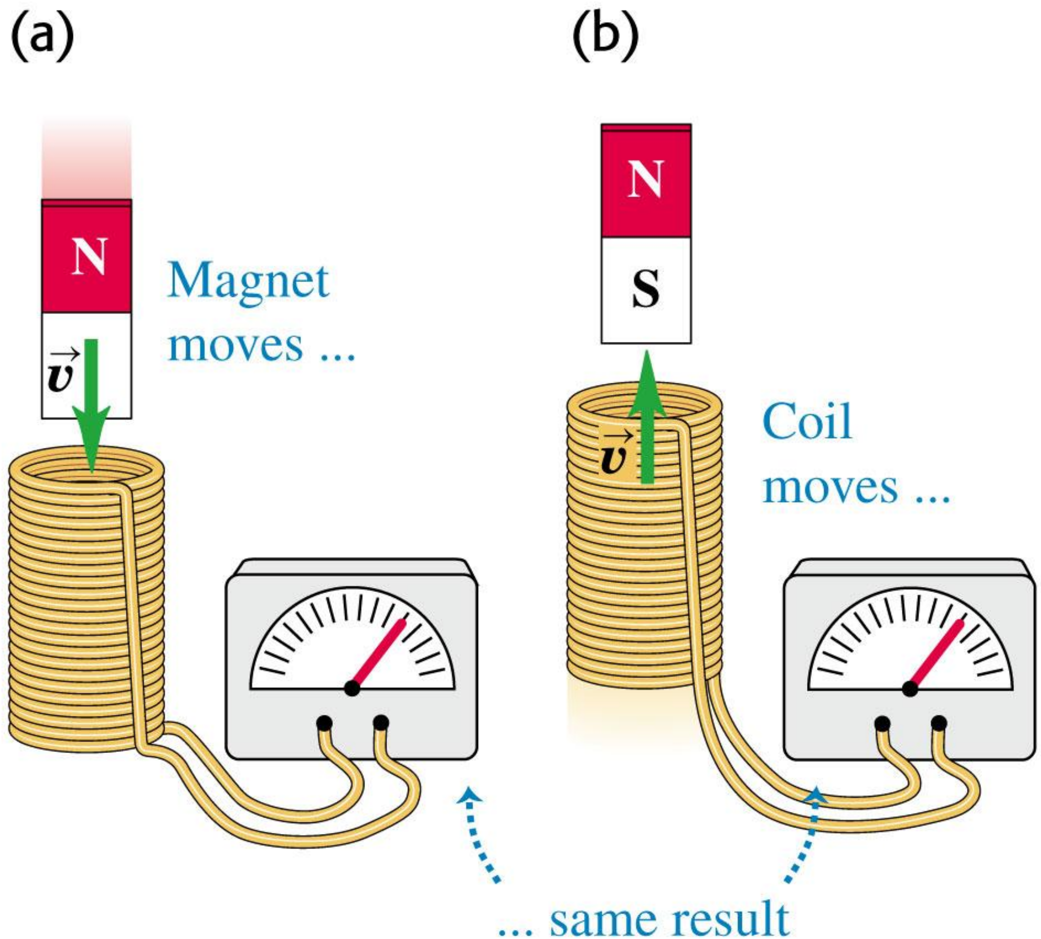
Galilean Relativity vs. Special Relativity

The laws of motion are the same in all inertial frames.

All laws of physics are the same in all inertial frames.

- Einstein's great brilliance, in my opinion anyway, was his willingness to accept a principle (the principle of relativity), for **ALL** laws of physics, and then see where the consequences of that led him.
- So, his restatement of the Galilean Principle of Relativity included three ideas:
  - All the laws of physics are the same in any inertial reference frame.
  - All the constants of physics are the same in any inertial reference frame.
  - Nothing can travel faster than the speed of light in vacuum.
- The last of these is not really independent, but was stated as an important consequence of the first two.

Case in point:



Special Relativity

All laws of physics are the same in all inertial frames.

- For the remainder of our study of Special Relativity, we will try to understand the physical ramifications of this simple Law.
- In order to do that we will need to develop some tools…

Clock Synchronization in Special Relativity

- We need to return to the question of how we synchronize our clocks.
- With the assumption that the speed of light is the same in all inertial reference frames, it is rather straightforward:
  - We send a light beam from some location (generally the origin of our reference frame) to each clock.
  - We then set that clock to the time we had at the origin when we sent the beam, plus the amount of time it took for the light beam to get there:

$$t = t_0 + \frac{d}{c}$$

  - Another way to look at this is: each clock in a reference frame is synchronized if you “see” the face of that clock, and the difference in the time that you read and the time that you have locally is related to the distance between you and the clock you are reading by:

$$c\left(t - t_0\right) = d$$

Clock Synchronization in Special Relativity

- If you are uncomfortable with the idea of using the constancy of the speed of light to synchronize clocks in your reference frame, there are other ways to do it…
- You could require that clocks on either side of you, an equal distance away have the same reading.
  - This doesn't require that the speed of light = c, just that the speed of light doesn't depend on direction.