Lecture 26 (Spacetime Diagrams & Time Measurement)

Physics 2310-01 Spring 2020

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Clicker Question

- Are you here?
  - A) Yes
  - B) No
  - C) I don’t know
  - D) I’m both here and not here (superposition state)
  - E) What does here actually mean?

Units in Special Relativity

- It is useful to use a special set of units in Special Relativity, known as SR units.
- The only difference is the unit of distance – instead of measuring distance in meters, one measures in the amount of time it takes light to travel a distance.
- So, the unit of distance is the second (or year, or hour, or…).
- You probably have heard distance quoted in terms of light-years (especially in astronomy).
- We will leave off the term "light-" and just refer to distance in units of time.

SR Unit Conversion

- If you want to convert from a distance in SR units to a distance in SI units, you have to use the conversion factor: $D[m]_{SI} = D[s]_{SR} \times 299{,}792{,}458 \frac{m}{s} \approx D[s]_{SR} \times 3\times10^{8} \frac{m}{s}$
- Or, the inverse conversion: $D[s]_{SR} = \dfrac{D[m]_{SI}}{3\times10^{8} \frac{m}{s}}$
- So, for instance, the distance between the earth and the sun, $D[m]_{SI} = 1.4960\times10^{11}$ m:

$$D[s]_{SR} = \frac{1.4960\times10^{11}\, m}{3\times10^{8} \frac{m}{s}} = 499 s = 8.31 \min$$

- Or, 1 m to SR units:

$$D[s]_{SR} = \frac{1.0\, m}{3\times10^{8} \frac{m}{s}} = 3.34 ns$$

SR Unit Conversion

- What are the SI units for energy?

$$1\left[J\right]_{SI} = 1\left[\frac{kg \cdot m^2}{s^2}\right]_{SI} = 1\frac{kg \cdot m^2}{s^2} \times \left(\frac{1}{3\times 10^8 \frac{m}{s}}\right)^2 = 1.11\times 10^{-17}\left[kg\right]_{SR}$$

$$or,$$

$$1\left[kg\right]_{SR} = 1kg \times \left(3\times 10^8 \frac{m}{s}\right)^2 = 8.99\times 10^{16}\left[J\right]_{SI}$$

- Note that in SR units, mass and energy have the same units…
- What about momentum?

$$1\left[kg\frac{m}{s}\right]_{SI} = 1\frac{kg \cdot m}{s} \times \frac{1}{3\times 10^8 \frac{m}{s}} = 3.34\times 10^{-9}\left[kg\right]_{SR}$$

- Also has the same units…

SR Unit Conversion

- What is the value of the speed of light in SR units?

$$3\times10^{8}\left[\frac{m}{s}\right]_{SI} = 3\times10^{8}\,\frac{m}{s}\times\frac{1}{3\times10^{8}\,\frac{m}{s}} = 1[\ \ ]_{SR}$$

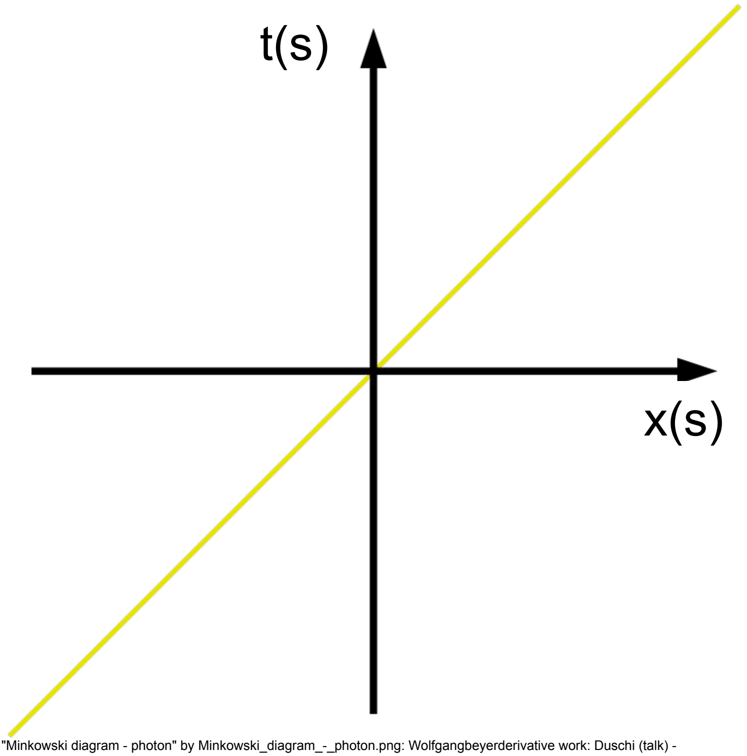
- No units!
- In fact, all velocities in SR units are unit-less.
- Let's say something is going 1.49896249e+8m/s,
- How fast is it going in SR units?
  - A: 0.5
- All velocities of objects are just fractions of the speed of light (and always <1)!

Spacetime Diagrams

- Okay, now that we have our units for spatial distances and time, we want to introduce another tool – the spacetime diagram.
- You will find this extremely useful, IF you understand how to set it up and use it…
- We will begin with spacetime diagrams in general, and then consider spacetime diagrams for two different frames in Galilean Relativity, and then consider spacetime diagrams for two different frames in Special Relativity.

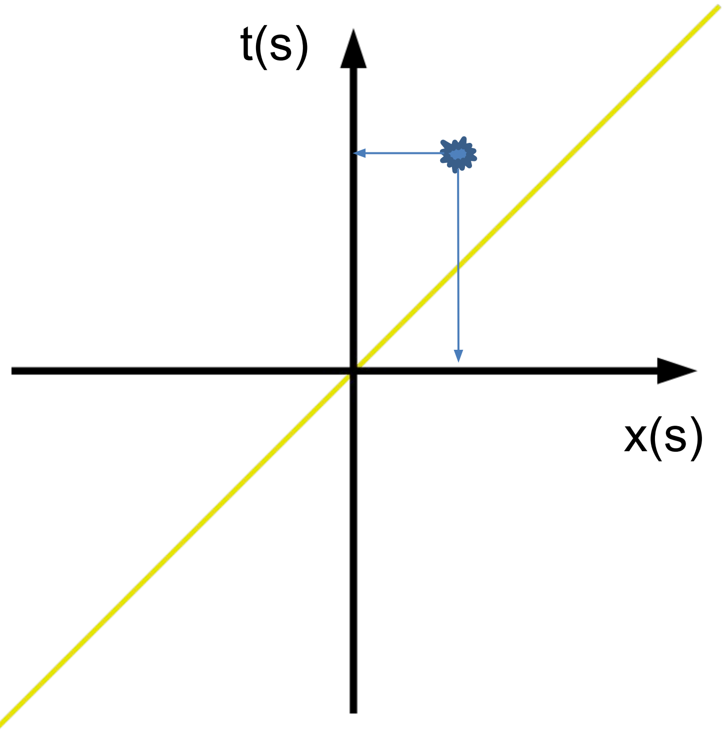
Spacetime Diagrams

- So, I will start off by drawing a basic spacetime diagram, using SR units.
- Since the spatial axis is in units of seconds (light-seconds, the distance light travels in one second), the yellow line represents the path travelled by a light beam emitted from the origin (x=0 and t=0).
- Note that when t is 1 s, the light beam will have gone one light-second, or just second in SR units.
- Then, events can be drawn on this diagram…



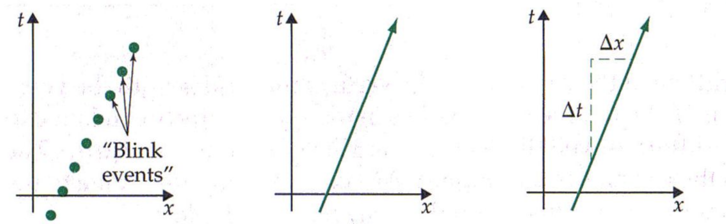
Events in Spacetime Diagrams

- Then one can read off the spacetime coordinates of an event directly.



Worldlines

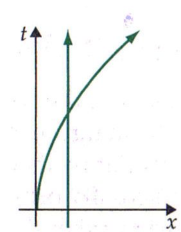
- For a moving particle, we can make a series of events that represent its position at different times.
- For infinitely small time differences, these become lines known as “worldlines”.
- Note that vertical lines represent an object at rest in this frame, i.e., v=0.
- The slope of the line, $\Delta t/\Delta x$, is the inverse of the velocity.
- I.e., a horizontal line has a slope of zero, but “infinite velocity”.
- We won’t see any horizontal worldlines…



(a) A sequence of events along an object’s path marks out its motion.

(b) As the time between the events goes to zero, the set of events becomes a curve.

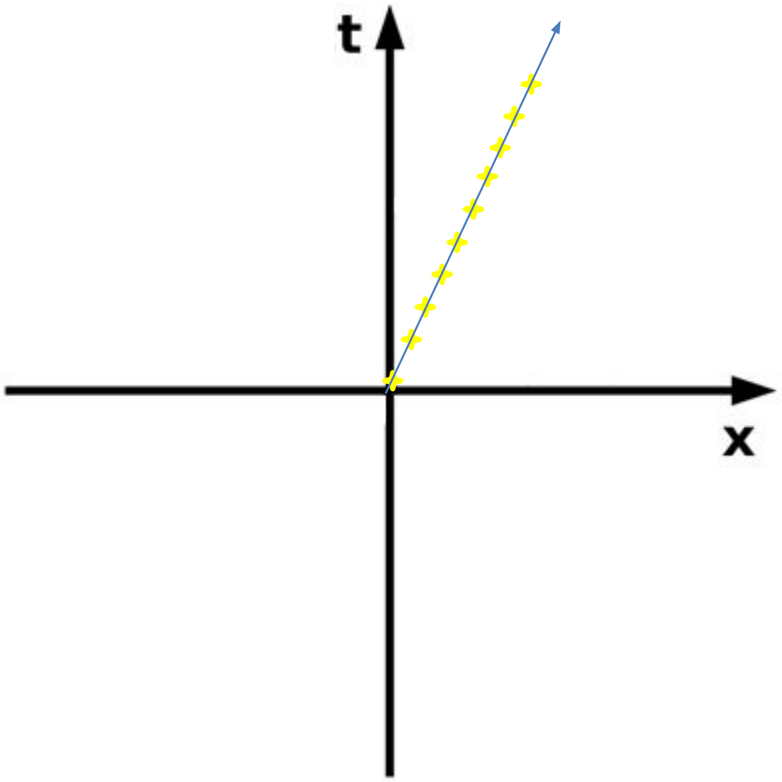
(c) The slope of this *worldline* is equal to the object’s inverse x-velocity ($1/v_x$).



(d)

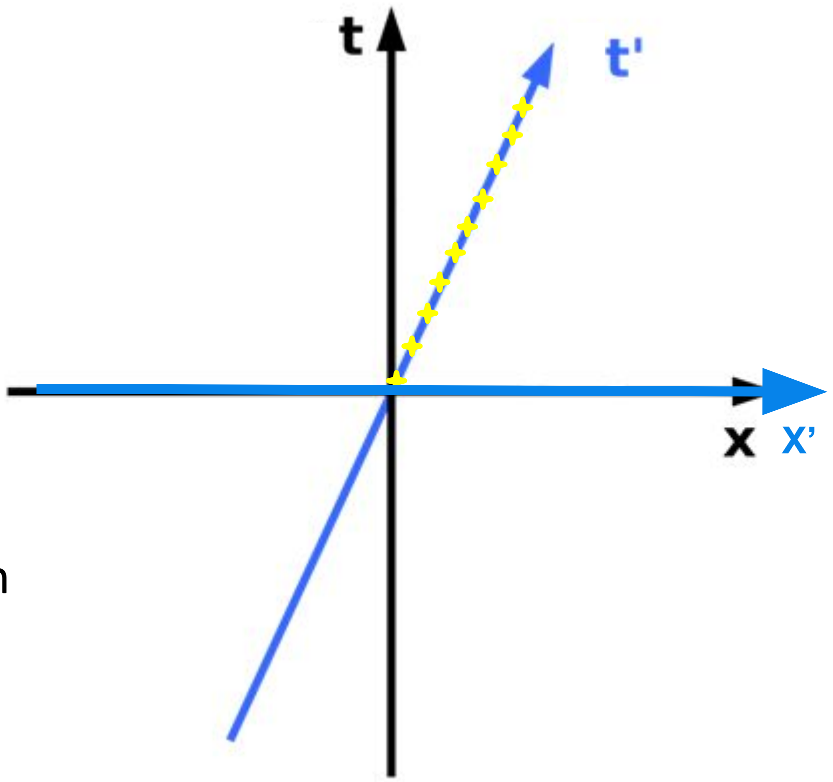
Spacetime Diagrams

- Now, let’s say that our x-t frame is fixed to a train station, and at t=0, the end of a (very fast) train passes by (at constant velocity).
- At the back of a train, a light blinks every so often.
- What do the events look like in our station frame?
- If the light was blinking at a very fast rate, we would get the worldline of the back of the train…



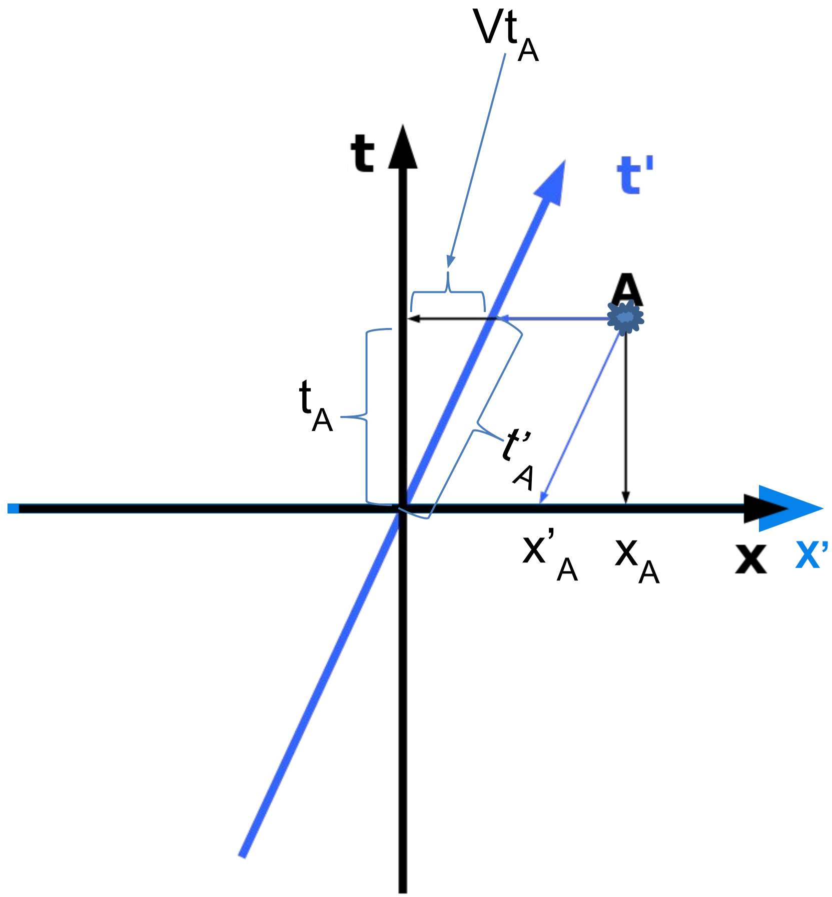
Two-frame Spacetime Diagrams

- Now, what if we want to draw the reference frame that is fixed on the back of the train, on the **SAME** diagram?
- Several things to note here:
  - The moving frame is no longer orthogonal, but this is no problem, since it **is** in it's own frame.
  - In Galilean Relativity, the x and the x' axes are the same.
  - Remember what the axis represents, the zero of the other axis – no matter where you are in x (or x'), t=0 is the same time everywhere (in Galilean Relativity).



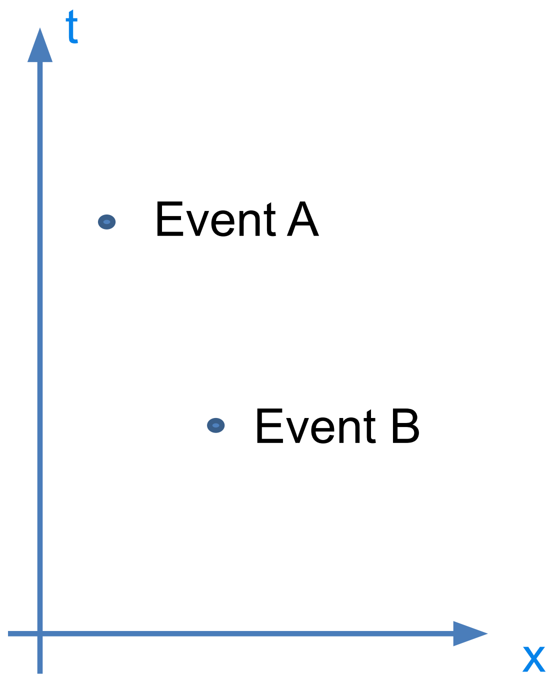
Two-frame Spacetime Diagrams

- Now, what if we want to draw an event on the spacetime diagram?
- How do we read the spacetime coordinates for both frames?
- Especially note that to read x' you have to go down to the x' axis ***PARALLEL*** to the t' axis!!
- Otherwise you are just reading x…
- Also Note: The scale on the t' axis is not the same as the scale on the t axis!
- $t_A = t'_A$, so the t' axis must have a different scale…
- Note that this is still Galilean relativity, so the fact that the axes have different scales is not a SR effect.



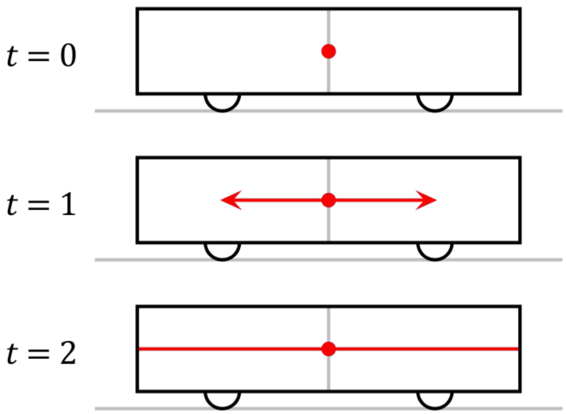
Three Definitions of Time Intervals

- Coordinate Time
  - The time difference, Δt, between two events as measured by two synchronized clocks within one inertial reference frame.
- Proper Time
  - The time difference, Δτ, between two events as measured by a single clock which is present at both events.
- Spacetime Interval
  - The time difference, Δs, between two events as measured by a single **inertial** clock present at both events.
- We will study each of these in turn and discover that they are very different.



Coordinate Time

- Coordinate time is just the time difference between two events (one might, or might not occur at t = 0) as measured in one particular inertial frame.
- Let's consider a particular case: a moving train has a radar system located in the center of the train.
- It fires out two light beams at t' = 0, one towards the front, and one towards the rear.
- In the t' frame, since light moves at velocity c, the events that mark the arrival of the light beam at the front (Event B) and rear (Event A) of the train happen at the same coordinate time (relative to the origin).
- The Coordinate Time for these events is Δt' = 0 in the train frame.



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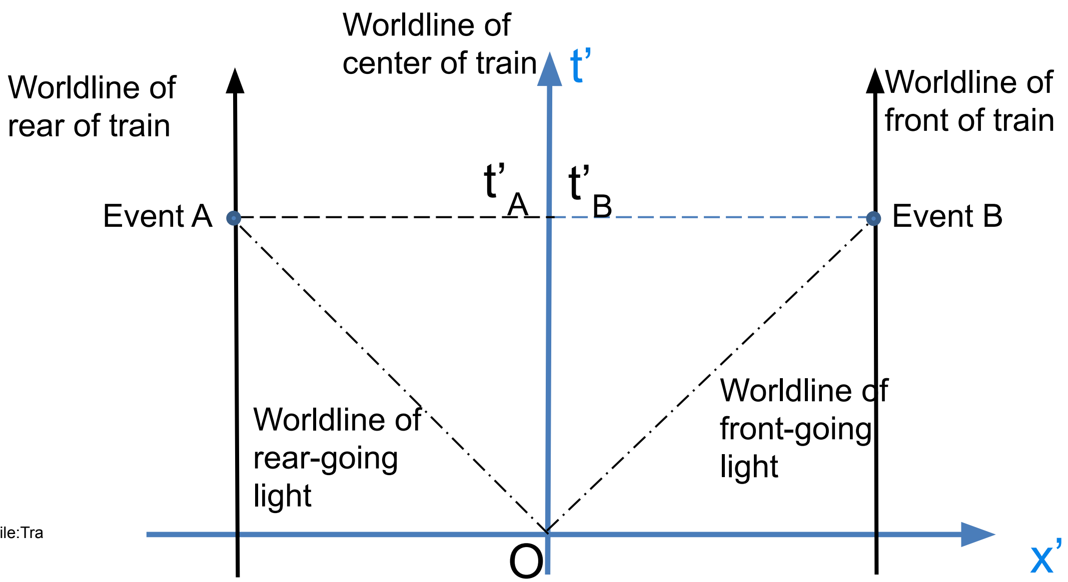
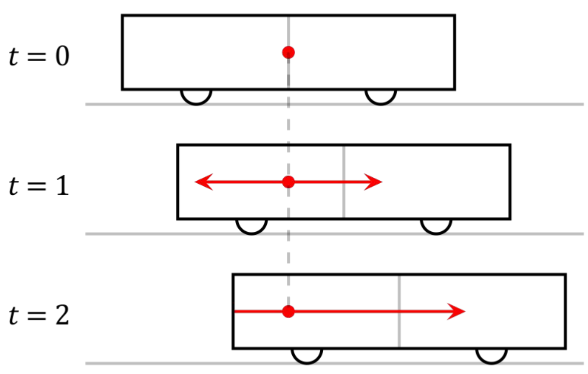


Coordinate Time *is Frame Dependent*

- Now, let's look at the same set of events in the frame of the train station.
- Since light travels at v = c in the x-t frame also, the worldlines of the flashes are still at 45 degrees in that frame (v=1), but the worldlines of the train (rear, center and front) are now tilted…
- So, in the x-t frame, the events that correspond to the intersection of the worldlines of the front and rear of the train with the worldlines of the light rays no longer have the same coordinate time!
- The Coordinate Time for these events is $\Delta t \neq 0$ in the station frame.



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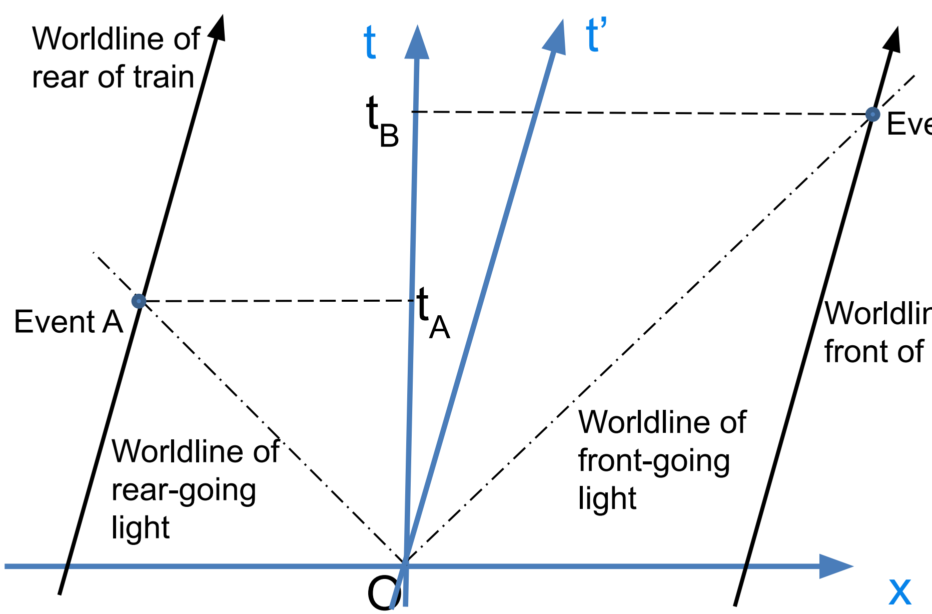
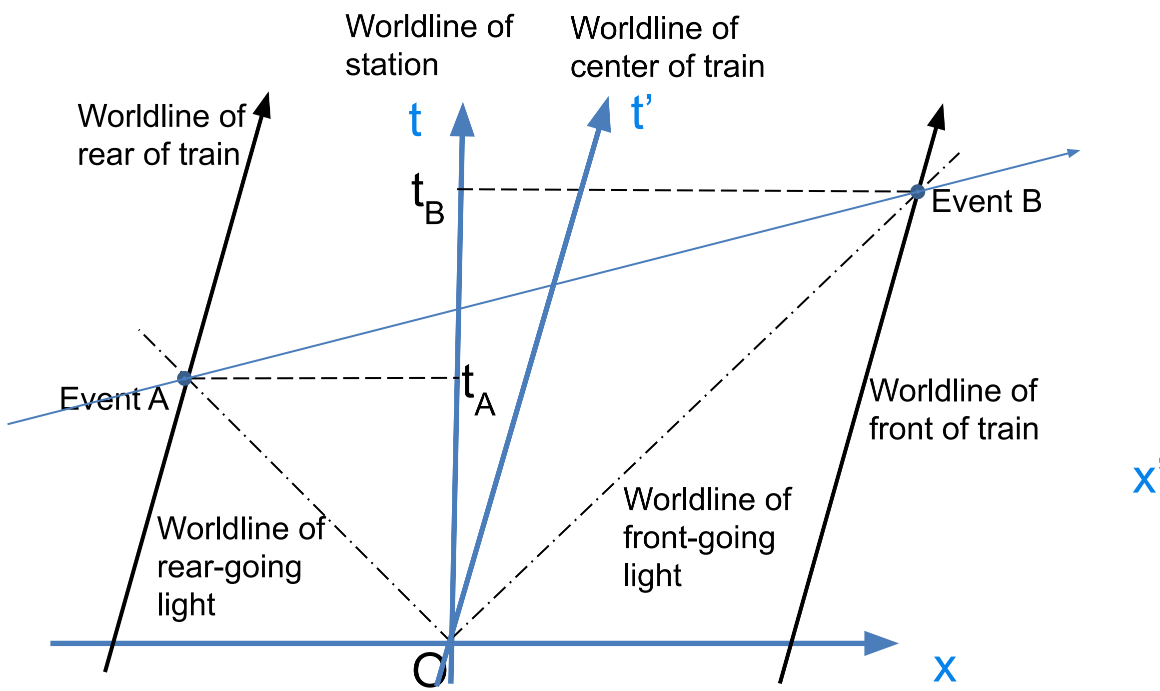


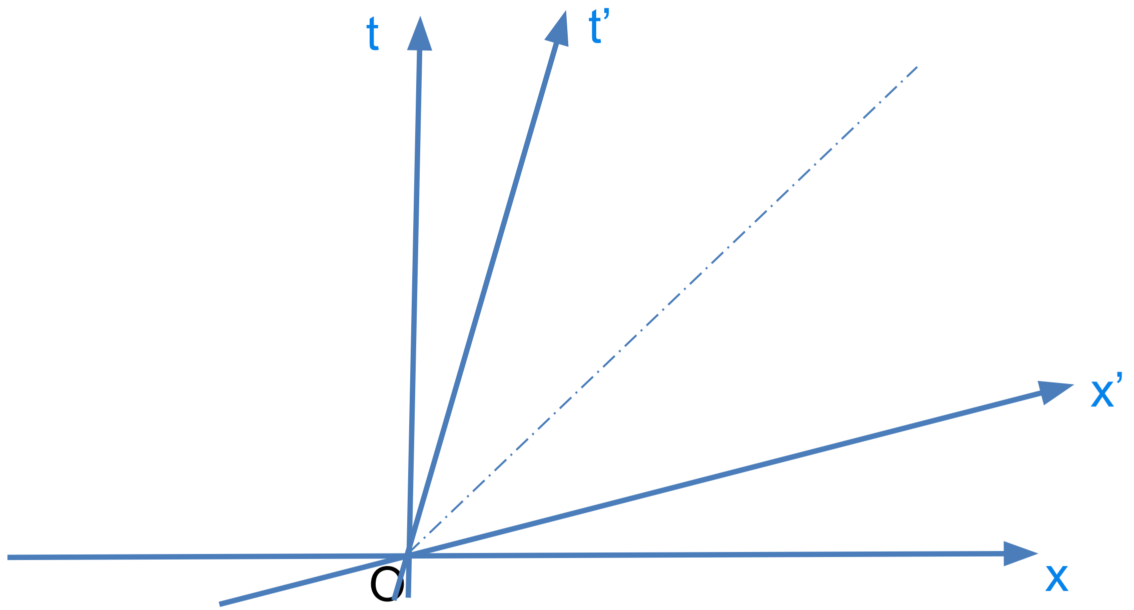
Simultaneity in the Moving Frame

- Now, if we want to represent an axis for the moving frame which represents simultaneous events in that frame, we now have two events which do exactly that!
- These events occur at the same time in the primed frame, so the line that connects them represents a set of events all with the same time.
- A line parallel to this line, but passing through the origin would then represent all events with t’ = 0 – the x’ axis.



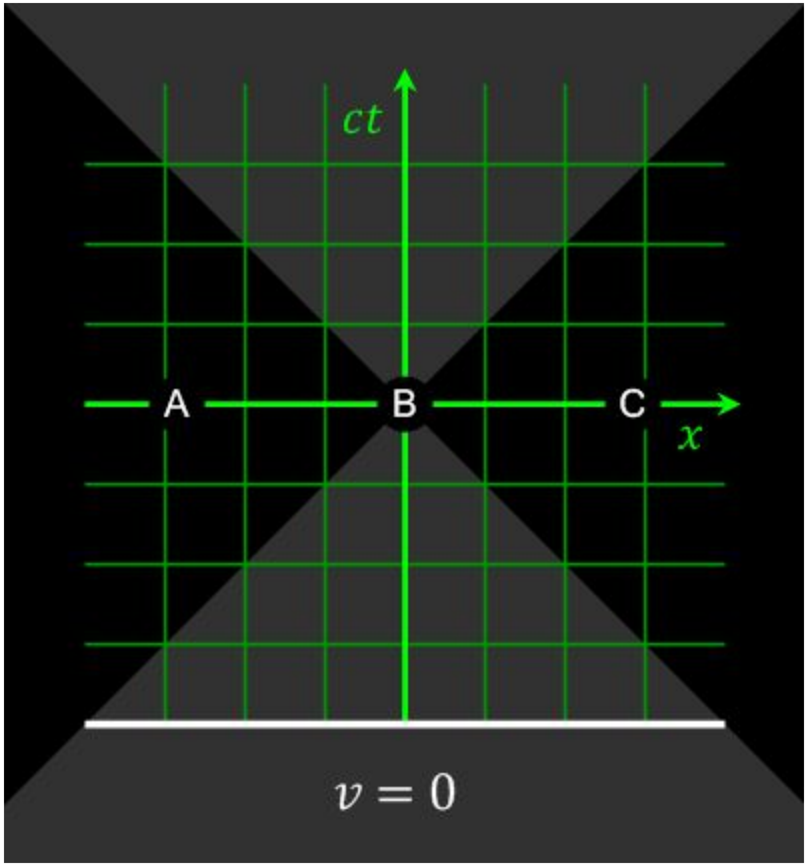
Two-Frame Graphs

- Now, we have the beginnings of a graphical way to see the relationship between spacetime coordinates as measured in one frame and spacetime coordinates as measured in another (both inertial!).
- We still need to determine what the relative scales are for the primed frame, since, as we have seen, they may not be the same as the unprimed frame.



Coordinate Time is Frame Dependent

- But, even without knowing the scales, we can see that the idea of simultaneity is now frame dependent.
- In fact, events that happen in one order in one frame can happen in the opposite order in another!



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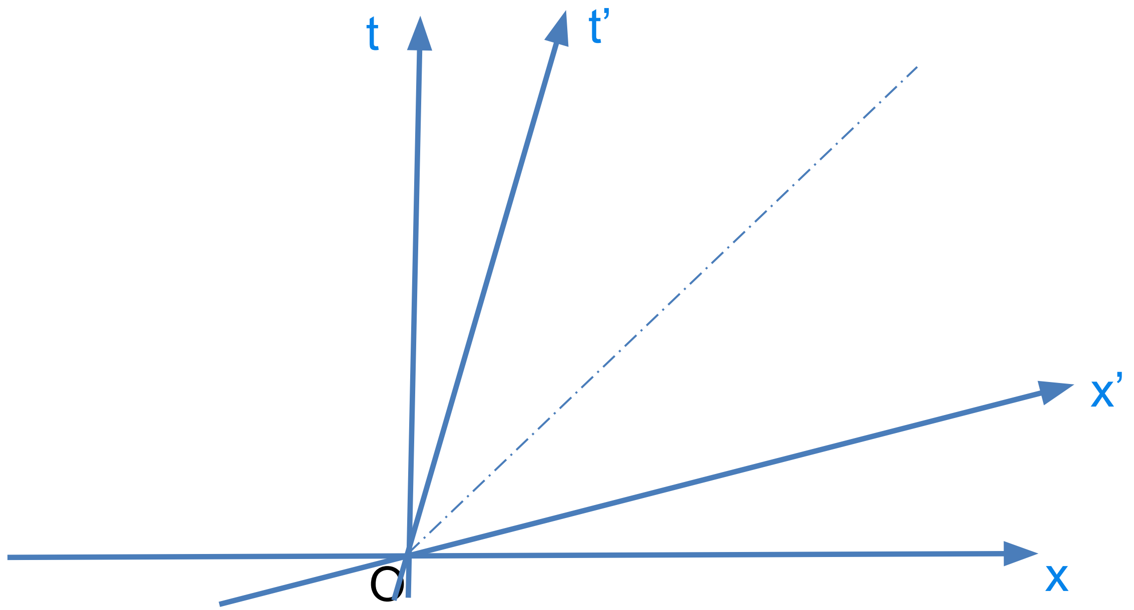
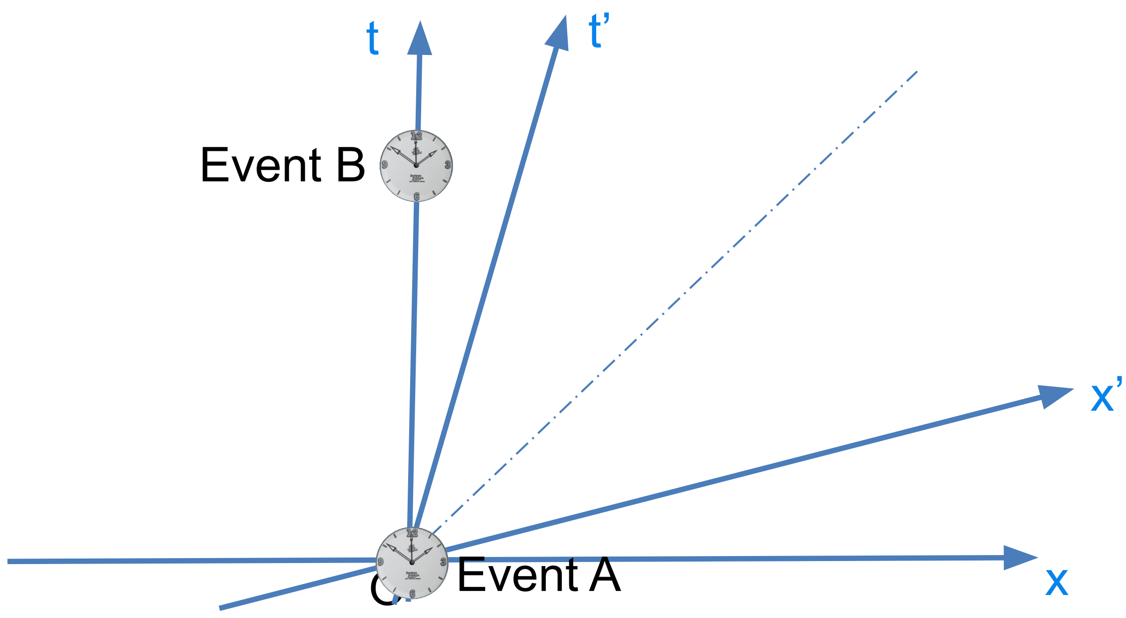
Proper Time

- Another way to measure the time difference between two events is to take the same clock to both events – the proper time, Δτ.
- Since observers in both frame can "see" the clock face when it is at both events, this type of time difference should be frame independent.
- However, there is a problem with Proper Time…



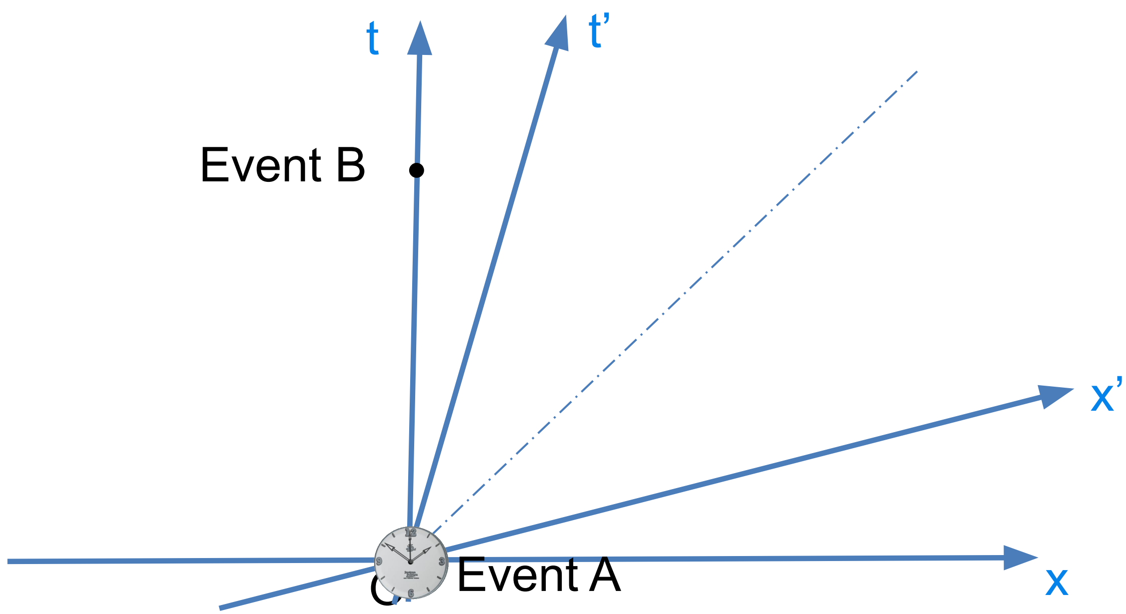
Proper Time

- Let's say that we have an event, Event A, somewhere on the earth's surface, call it the origin of the Home Frame.
- At a later time, but at the same place, we have another event, Event B.
- Let's consider two clocks:
- One which stays at x = 0 in the Home frame.
  - That clock will be at both events, and observers in both frames can read the face and get the proper time from that (stationary) clock.
- Another clock moves from Event A along some path (in x) and then arrives back at x = 0 to coincide with Event B.
  - That clock will also be at both events, and observers in both frames can read the face and get the proper time from that (moving) clock.



Proper Time is Path Dependent

- However, even though both frames agree on the Proper time given by each clock, the two answers from the two clocks will NOT necessarily be the same!
- This is analogous to the question of how far is it to Santa Fe from Albuquerque – the answer depends on the path!
  - (more on this in a bit)
- I chose the wording of the example for a reason…



Example

(https://en.wikipedia.org/wiki/Hafele%E2%80%93Keating_experiment)

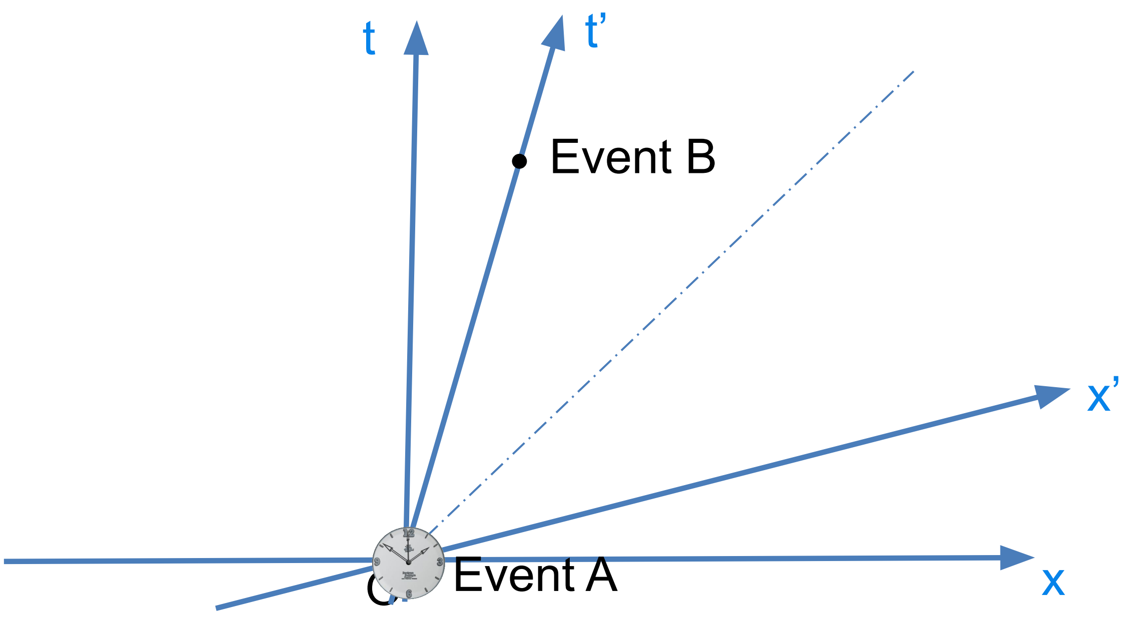
- The **Hafele–Keating experiment** was a test of the theory of relativity. In October 1971, Joseph C. Hafele, a physicist, and Richard E. Keating, an astronomer, took four cesium-beam atomic clocks aboard commercial airliners. They flew twice around the world, first eastward, then westward, and compared the clocks against others that remained at the United States Naval Observatory. When reunited, the three sets of clocks were found to disagree with one another, and their differences were consistent with the predictions of special and general relativity.
- The results were published in *Science* in 1972:[1][2]

	nanoseconds gained			
	predicted			measured
	gravitational(general relativity)	kinematic(special relativity)	total	
eastward	144±14	−184 ± 18	−40 ± 23	−59 ± 10
westward	179±18	96±10	275±21	273±7

- The published outcome of the experiment was consistent with special and general relativity. The observed time gains and losses were different from zero to a high degree of confidence, and were in agreement with relativistic predictions to within the ~10% precision of the experiment.

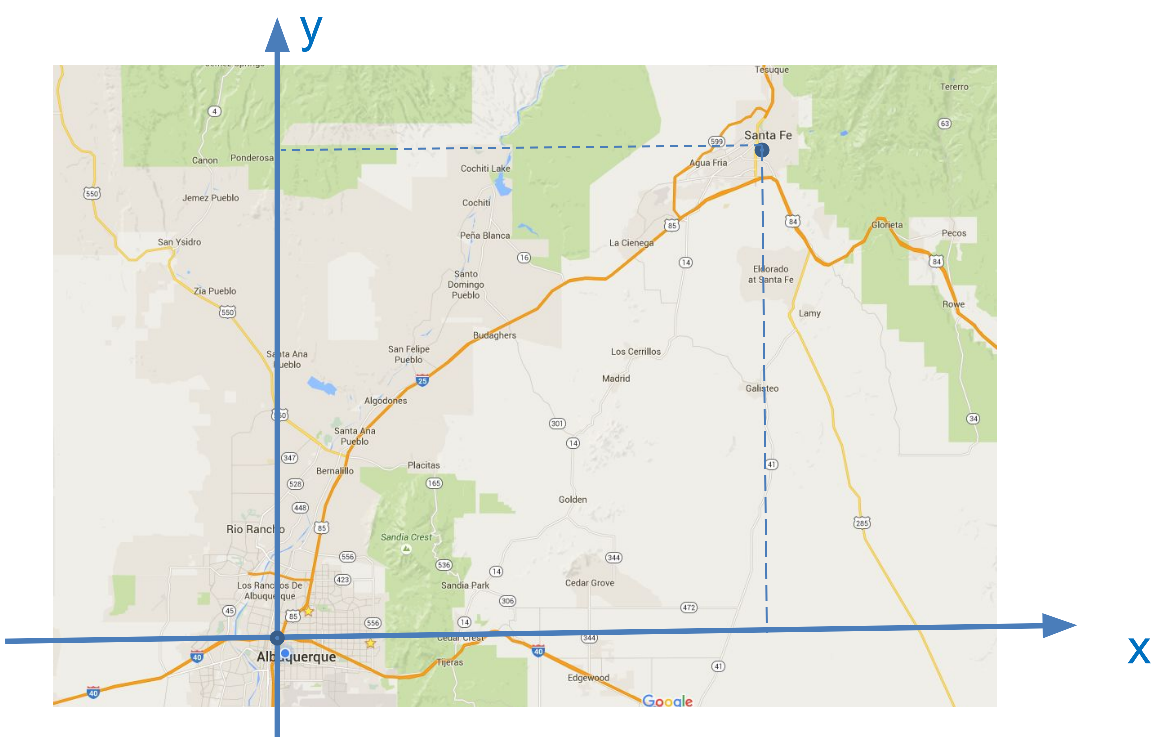
Spacetime Interval

- So, is there no time interval that we can agree on that is not dependent on frame or path?
- Yes, it is called the ***spacetime interval***, and it is the time difference, Δs, as measured by an ***inertial*** clock that is present at both events.
- Notice that only the first clock in our previous example is an inertial clock.
- In fact, between any two events, there is only one path that represents an inertial clock path: a straight-line path between the events – straight lines represent constant velocity.



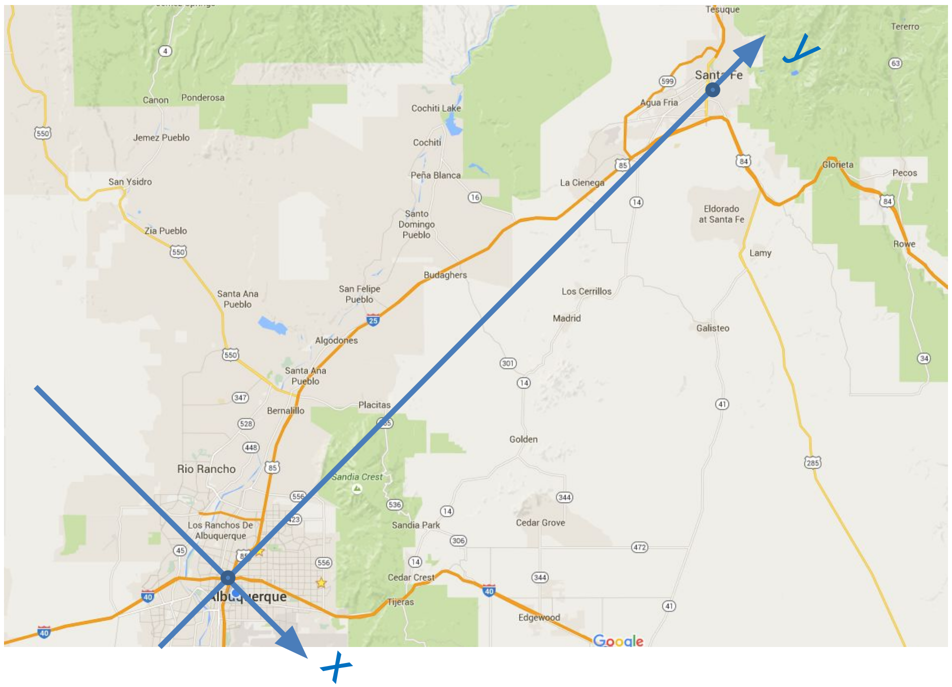
Geometry Analogy

- Here is an analogy to common geometry which you may or may not find useful.
- What is the coordinate differences between Albuquerque and Santa Fe?



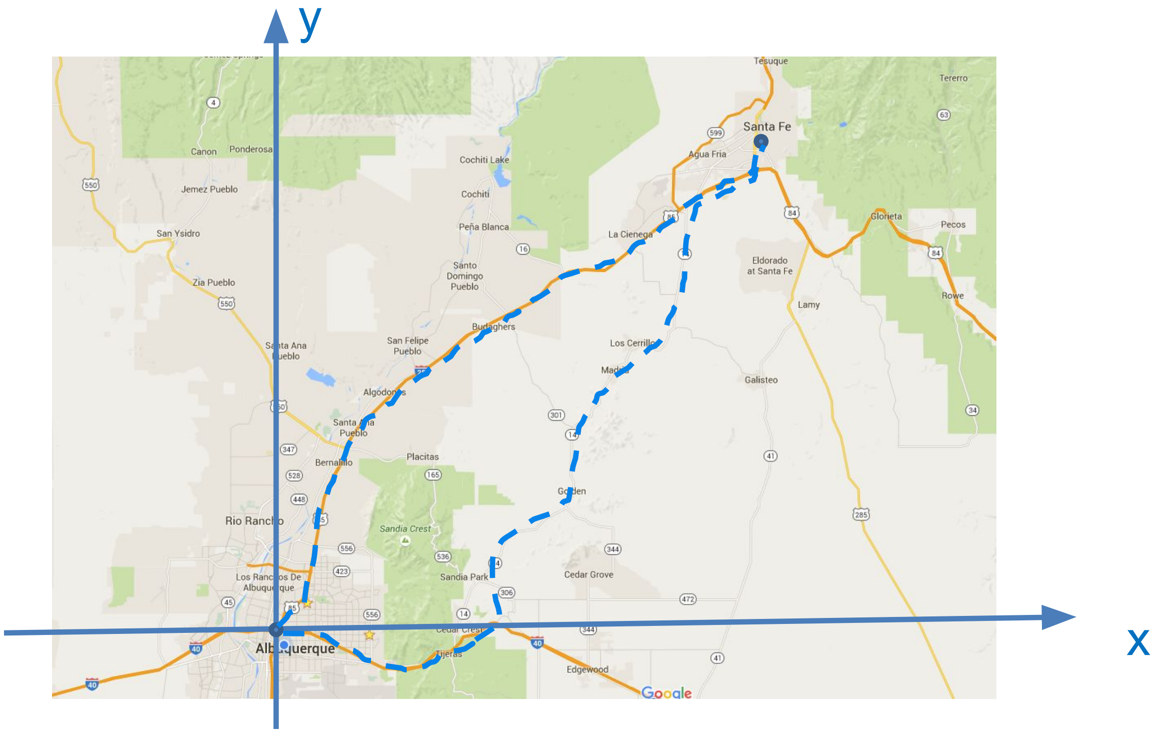
Geometry Analogy

- What about now?
- The coordinates (or coordinate differences) are relative to the choice of coordinate axes.
- These are analogous then to the Coordinate Time in SR.



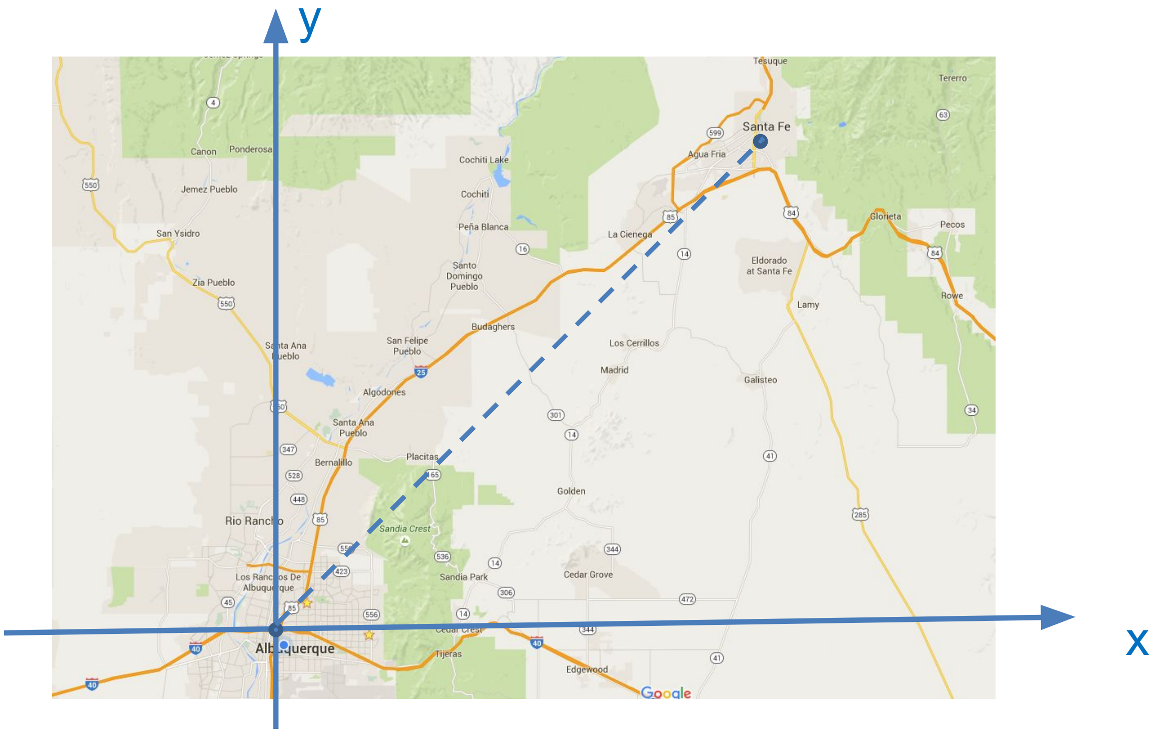
Geometry Analogy

- What about the path length from Albuquerque to Santa Fe?
- That depends on the path you take!
- But each path length doesn't depend on what coordinate system you use.



Geometry Analogy

- What if you took a straight-line path?
- That distance is unique, and also doesn't depend on the coordinate system.
- There is something REAL about that, it doesn't depend on how you measure it!



Geometry Analogy

Plane Geometry		Spacetime Geometry
Map	$\leftrightarrow$	Spacetime diagram
Points	$\leftrightarrow$	Events
Paths	$\leftrightarrow$	Worldlines
Coordinate systems	$\leftrightarrow$	Inertial reference frames
Relative rotation of coordinate Systems	$\leftrightarrow$	Relative velocity of inertial ref. frames
Differences between coordinate values	$\leftrightarrow$	Differences of spacetime coordinates
Path length along a path	$\leftrightarrow$	Proper Time along a worldline
Distance between points	$\leftrightarrow$	Spacetime interval between two events