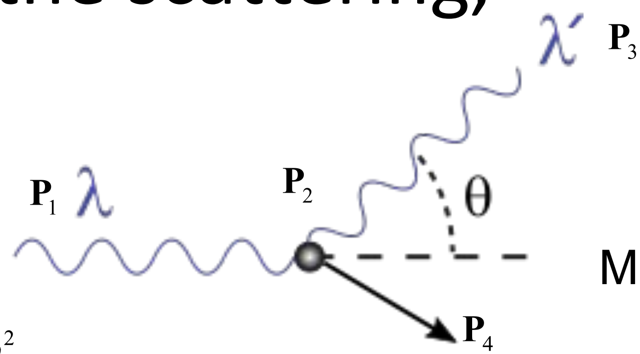
Lecture 34
(Pair Production & Uncertainty)

Physics 2310-01 Spring 2020
Douglas Fields

Compton Scattering Formula

- We know that four-momentum must be conserved in the scattering,



Energy Conservation

$$p_1^2 + p_3^2 - 2p_1p_3 + 2p_1m_ec - 2p_3m_ec = p_4^2$$

Momentum Conservation

$$p_4^2 = p_1^2 + p_3^2 - 2p_1p_3\cos\theta$$

Substitution

$$p_1^2 + p_3^2 - 2p_1p_3 + 2p_1m_ec - 2p_3m_ec = p_1^2 + p_3^2 - 2p_1p_3\cos\theta \Rightarrow$$

$$p_1m_ec - p_3m_ec = p_1p_3\left(1-\cos\theta\right) \Rightarrow$$

$$\frac{1}{p_3} - \frac{1}{p_1} = \frac{1}{m_ec}\left(1-\cos\theta\right) \Rightarrow \quad \text{with } p = \frac{h}{\lambda}$$

$$\lambda' - \lambda = \frac{h}{m_ec}\left(1-\cos\theta\right)$$

$$\frac{h}{m_ec} = 2.426\times10^{-12}\,m$$

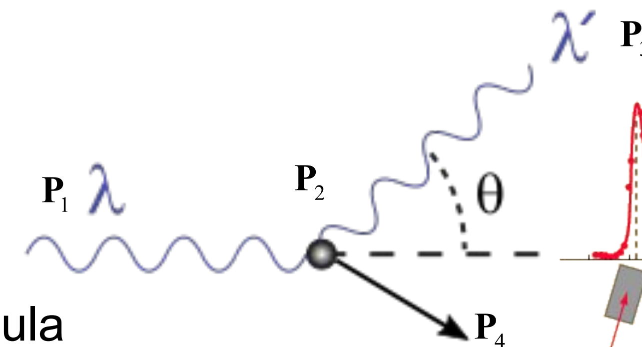
Compton Scattering Formula

Compton Scattering Results

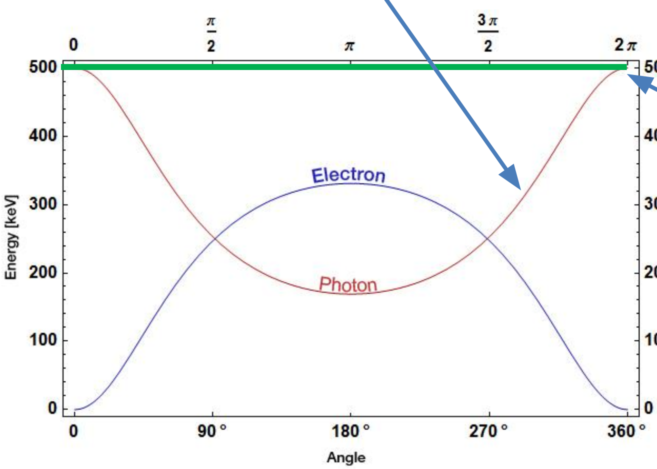
- Experimental results showed that the photon behaved like a scattered particle, not an E&M wave!

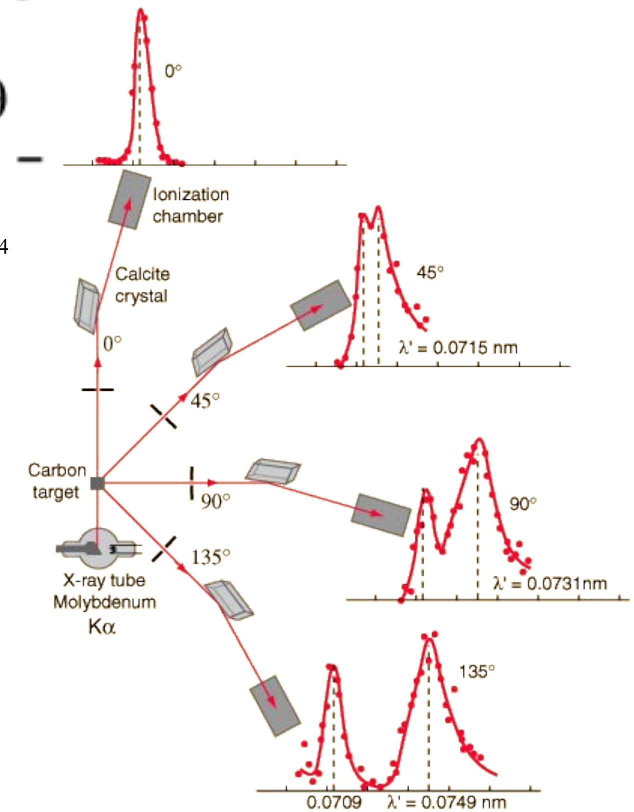
$$\lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$$

Compton Scattering Formula



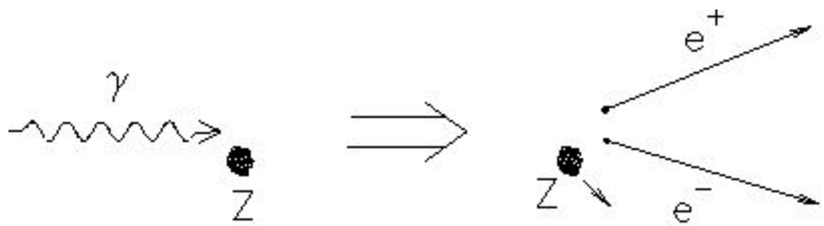
Classical wave result



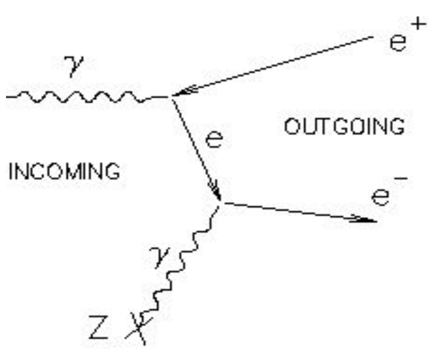


Pair Production

- Another effect that can only be explained through the quantized theory of light is pair production discovered in 1933.
- In this process, a high energy photon can (in the presence of an atomic nucleus) create an electron and an anti-electron (same mass as an electron, with positive charge called a positron) from the nothingness of the vacuum.
- However, the photon must have enough energy to account for both the rest masses of the particles and their kinetic energy.
- The atomic nucleus is needed to conserve both energy and momentum.

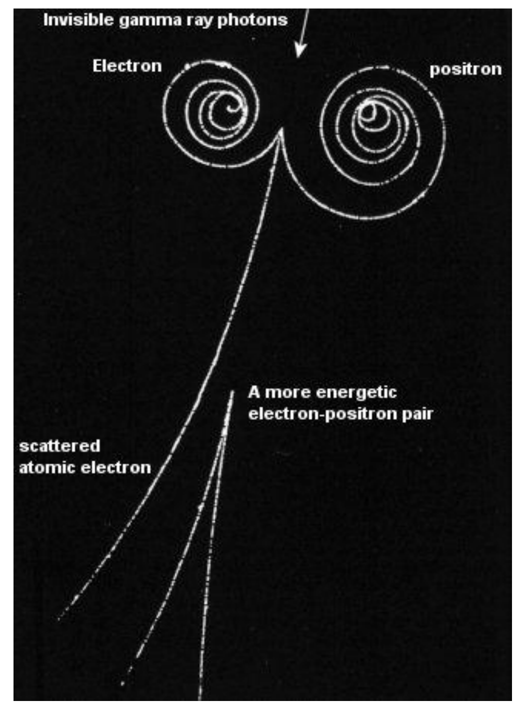


"Pair production Cartoon" by Jess H. Brewer - http://www.jick.net/~jess/hr/skept/EMC2/node9.html. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:Pair_production_Cartoon.gif#/media/File:Pair_production_Cartoon.gif



"Electron-Positron nuclear Pair production Feynman Diagram" by Jess H. Brewer - http://www.jick.net/~jess/hr/skept/EMC2/node9.html. Licensed under Public Domain via Commons - https://commons.wikimedia.org/wiki/File:Electron-Positron_nuclear_Pair_production_Feynman_Diagram.gif#/media/File:Electron-Positron_nuclear_Pair_production_Feynman_Diagram.gif

$$E_{\min} > 2m_e c = 1.022 MeV$$



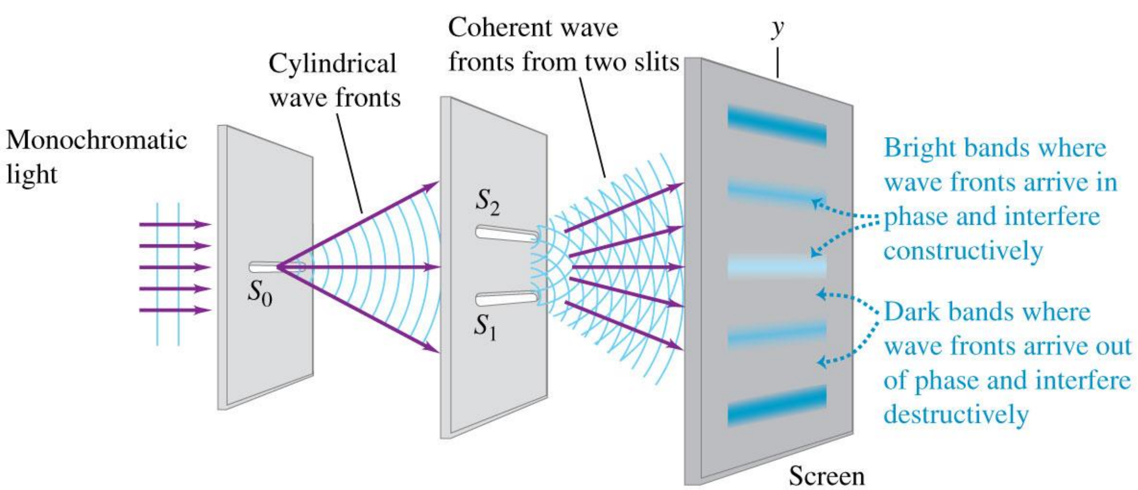
Wave Particle Duality

- So, we’ve already discussed the idea of wave-particle duality, but let’s explore this a little bit more.
- This concept was first stated by Niels Bohr in 1928 as the **principle of complementarity**.
- This just means that both the wave and particle natures are necessary for a full description of the behavior of light (just light?), but never at the same time.
- Either we see the wave nature, or the particle nature, but never both simultaneously.
- To illustrate what we mean, let’s revisit an old friend – the two-slit experiment.

Two-slit experiment revisited

- Let's revisit Young's two-slit experiment, but this time, thinking about photons.
- It is certainly true that if we take coherent monochromatic light hitting two slits, we will see an interference pattern (we actually saw this in class).
- But what happens if we turn down the intensity of the light so much, that we only have one photon pass through the slits at any one time?

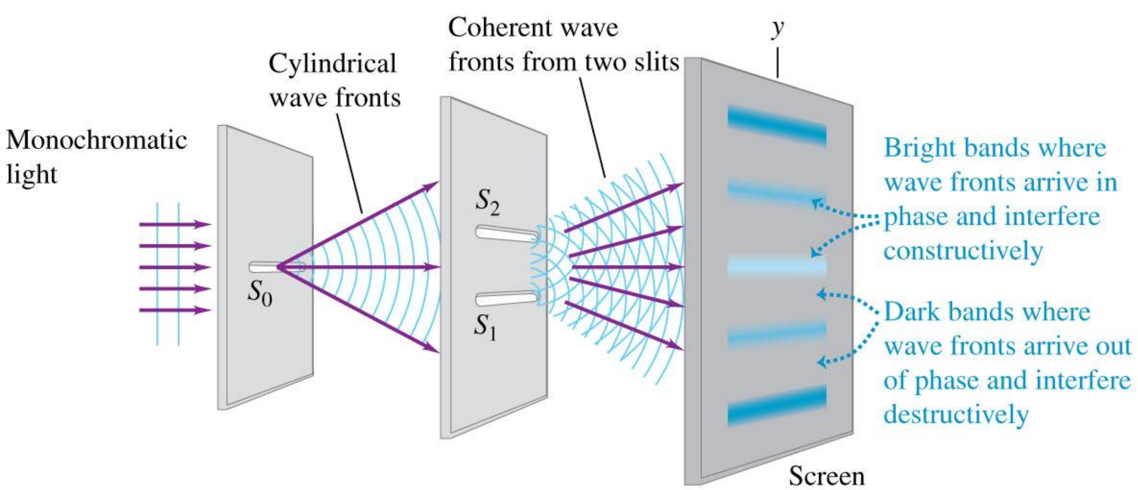
(a) Interference of light waves passing through two slits



Two-slit experiment revisited

- We will use a modified experiment where we will employ photomultiplier tubes to detect the position of the photons at the plane of the screen.
- Note, that we are taking advantage of the particle nature of the photon to count them and detect precisely their position on the screen.
- But also remember that only one of them passes through the two-slits at a time, so with what are they interfering with?

(a) Interference of light waves passing through two slits



Two-slit experimental results

- So here is what we would see after a few minutes.
- And after a few more minutes.
- And a few more…
- And a few more…
- And a few more.

Wave-particle Duality Explained

- So, the photons are detected and emitted as particles, not as waves, as was demonstrated in the photoelectric effect and x-ray production experimental results.
- But they move as waves, not particles – their paths are indeterminate.
- Each photon must be thought of as interfering with itself – it must, in a sense, pass through both slits.
- What if we do something to see through which slit the photon passed?
  - Let's put an electron in one of the slits and then detect its recoil.
  - This will tell us exactly where the photon was at the plane of the slits.
  - But, it will also scatter the photon, thus destroying the interference pattern.
- If we try to make the photon behave as a particle at the slits (by trying to detect it), then the interference pattern (demonstration of its wave nature) will go away.
- We have always thought of particles as having a definite position and definite momentum…
- We now must examine that conception.

Is there a paradox?

What is sometimes called the **wave-particle paradox**/ puzzle/ mystery arises usually if we try to picture light or electrons or other tiny things in terms of macroscopic, familiar objects. If we imagine light as being like a water wave, it's impossible to picture how a photon is both dispersed enough to create an interference pattern and simultaneously localized enough to interact violently with a single electron. If we imagine light as being made of little particles, it's impossible to picture how one particle 'goes through' two slits and interferes with itself.

Light is not a water wave, and it's not a stream of little particles. Whenever we use one or other of these pictures (and they are very useful at times), we must be aware that they are also, at times, seriously misleading. Further, the conditions under which one picture is helpful are usually those under which the other is misleading. Consequently, using both pictures simultaneously leads to apparent paradoxes. So it's good to remember that the paradox lies in the use of inappropriate, imagined, macroscopic pictures, and does not come out of the laws of physics.

http://www.animations.physics.unsw.edu.au/jw/light/youngs-experiment-single-photons.html

Single-Slit Diffraction

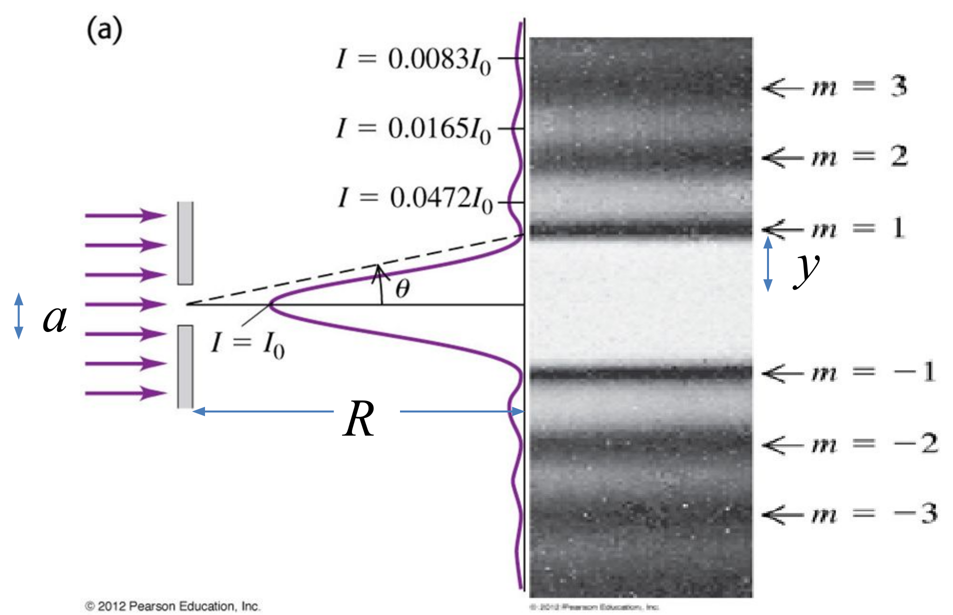
- To explore this further, let's look at single slit diffraction.
- The width of the central maximum can be defined by the positions of the first minima, related to the angle below.
- Now, if the photons had no momentum in the y-direction, the central spot would just be as wide as the slit, so there must be a y-momentum…

$$y = \frac{R\lambda}{a} \Rightarrow$$

$$\frac{y}{R} = \tan\theta \approx \sin\theta \approx \theta = \frac{\lambda}{a}$$

$$\frac{p_y}{p_x} = \tan\theta \approx \theta = \frac{\lambda}{a} \Rightarrow$$

$$p_y = p_x \frac{\lambda}{a}$$



Single-Slit Diffraction

- So, we can write the y-component of momentum as a function of the x-component, and then substitute with the relation between momentum and wavelength.
- Now, p_y and *a* both represent uncertainties about the y-momentum and y-position of the photon, so we can write this as an inequality:

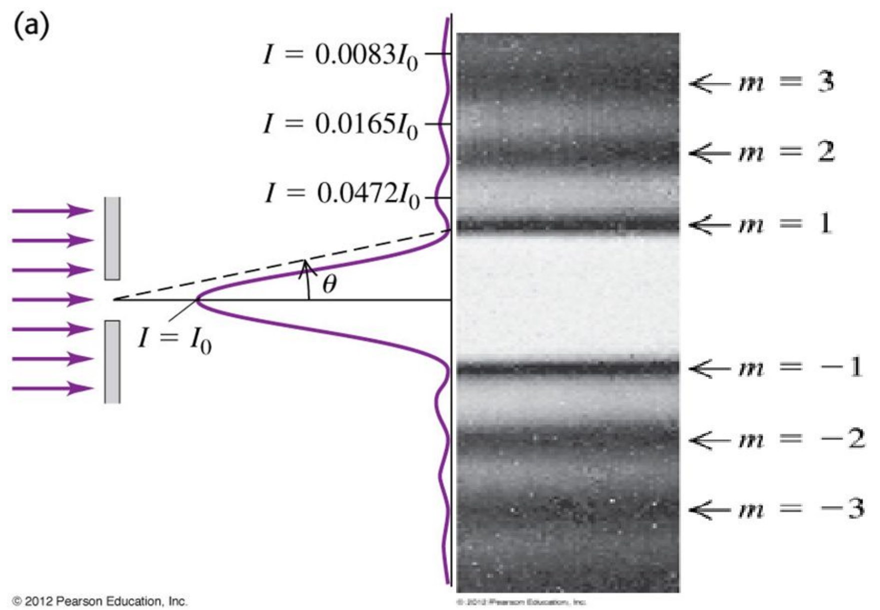
$$p_x = \frac{h}{\lambda} \Rightarrow$$

$$p_y = p_x \frac{\lambda}{a} = \frac{h}{\lambda}\frac{\lambda}{a} = \frac{h}{a} \Rightarrow$$

$$p_y a = h$$

$$\Delta p_y \Delta y \geq h$$

- In other words, if *a* gets smaller, p_y would get bigger (bigger spread in the central maximum) and vise versa.



Heisenberg Uncertainty Principle

- If we write this as a spread in both y-position and y-momentum, we get:

$$\Delta p_y \Delta y \geq \frac{h}{4\pi} \Rightarrow$$

$$\Delta p_y \Delta y \geq \frac{\hbar}{2}, \quad \hbar = \frac{h}{2\pi}$$

> We will return to where the 4π comes from…

- This is also true for the other coordinates such that the statement of the Heisenberg uncertainty principle is:

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}, \quad \Delta p_y \Delta y \geq \frac{\hbar}{2}, \quad \Delta p_z \Delta z \geq \frac{\hbar}{2}$$

- One cannot know both the position and momentum of a photon to better than this limit, even in principle.

Wave Nature Review

- Now, you might think that the uncertainty principle is something that is unique to quantum mechanics, but let’s take a few moments to review classical wave mechanics to see what it has to say about uncertainty in waves.

Superposition

- The wave equation, because it is linear, lends itself to superposition of solutions:
  - If y_1 is a solution of the wave equation:

$$y_1(x,t) = \cos(k_1 x - \omega_1 t)$$

  - And y_2 is also a solution of the wave equation:

$$y_2(x,t) = \cos(k_2 x - \omega_2 t)$$

  - Then $(Ay_1 + By_2)$ is also a solution of the wave equation.

Wave Pulses

- So, if our solution to the wave equation is:

$$y(x,t) = A\cos(kx - \omega t)$$

- How do we get a wave pulse???

$$y_1(x,t) = \sin(kx - \omega t)$$
$$y_2(x,t) = \sin((k + dk)x - (\omega + d\omega)t)$$

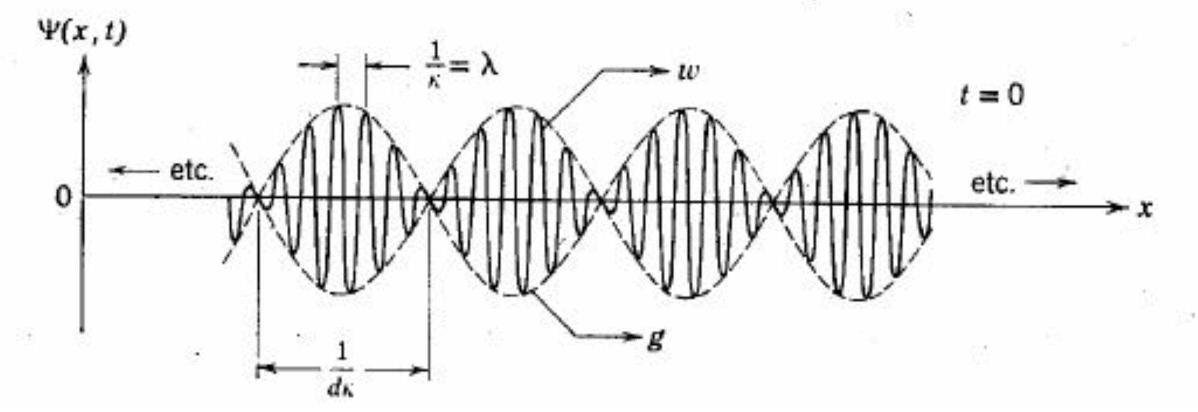
- Using sinA + sinB = 2cos[(A-B)/2]sin[(A+B)/2]

$$\begin{aligned} y(x,t) &= 2\cos\left(\frac{dk}{2}x - \frac{d\omega}{2}t\right)\sin\left(\frac{2k + dk}{2}x - \frac{2\omega + d\omega}{2}t\right) \\ &= 2\cos\left(\frac{dk}{2}x - \frac{d\omega}{2}t\right)\sin(kx - \omega t) \end{aligned}$$

Wave Pulses

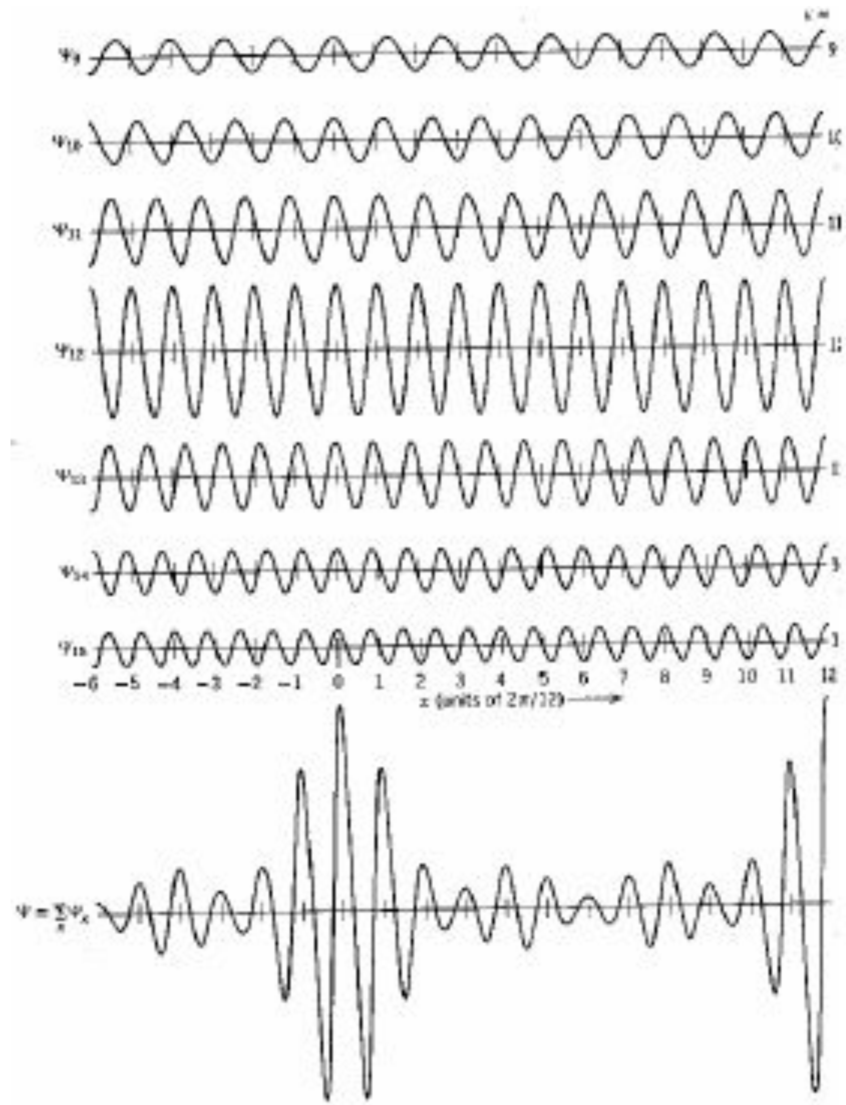
$$y(x,t) = 2\cos\left(\frac{dk}{2}x - \frac{d\omega}{2}t\right)\sin(kx - \omega t)$$

- At a fixed moment in time, the wave looks like this.
- Notice that there are really two frequencies, and the "position" of the wave is still not determined.



Wave Pulses

- In order to get a true wave packet, one must add many frequencies together (with different amplitudes).
- Even then, there is a repeating nature of the pulse.



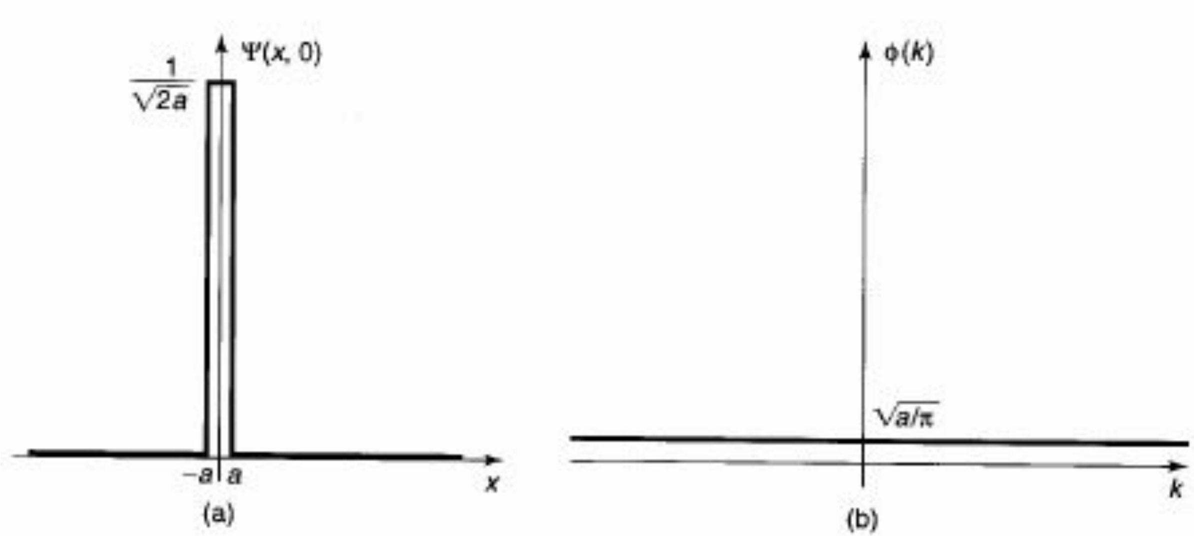
$$\Psi(x,t) = \sum_i A(k_i)\sin(k_i x - \omega t)$$

Wave Pulses

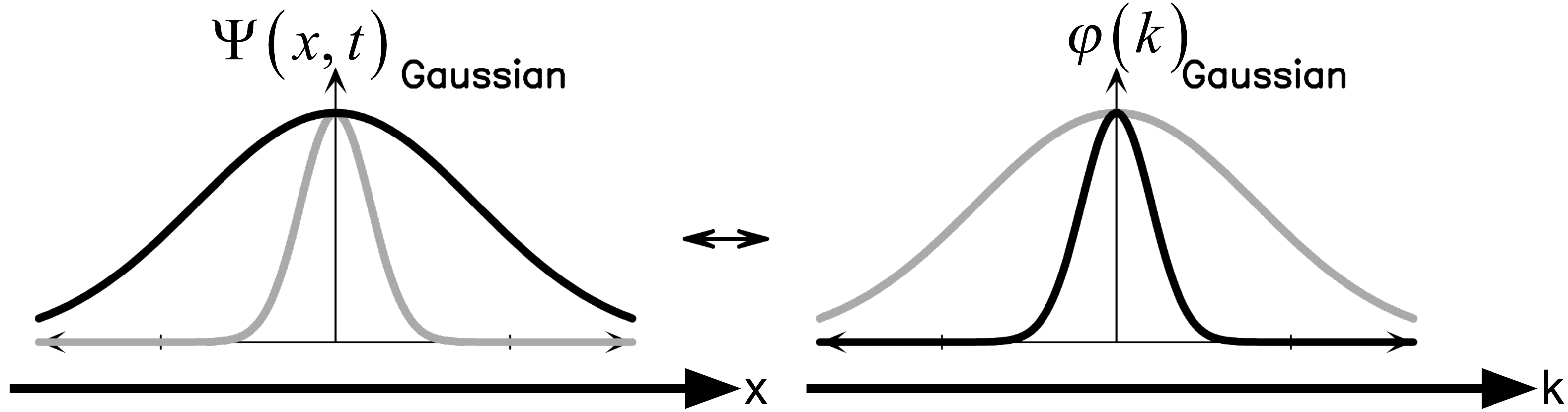
- In order to get a single pulse, you have to add ALL wavelengths with some strength, so we will have a continuous distribution in wave number.
- For a delta function, the strengths are all the same.

$$\Psi(x,t) = \sum_i A(k_i)\sin(k_i x - \omega t)$$

$$\Psi(x,t) = \int^{\infty} \phi(k)\sin(kx - \omega t)\,dk$$



Gaussian Wave Pulses



- For a Gaussian wave packet, the distribution of wavenumbers is also Gaussian and the *variance* of the Gaussian in space is related to the *variance* of the Gaussian in wavenumbers by: $\Delta x = \frac{1}{2\Delta k} \Rightarrow$

$$\Delta x \Delta k = \frac{1}{2}$$

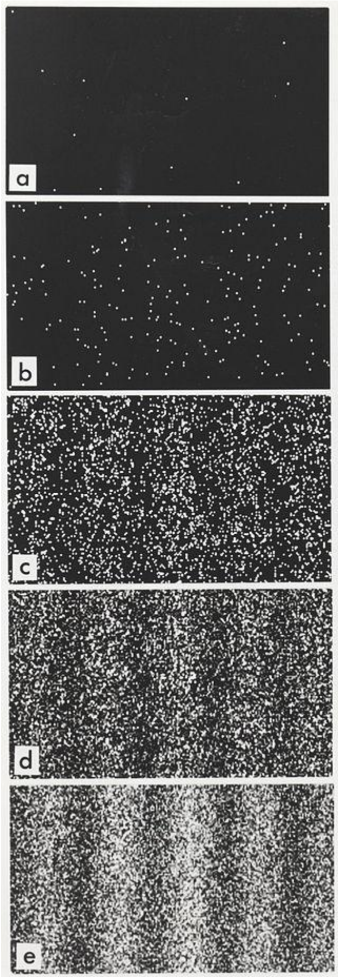
- It turns out that the Gaussian case represents the minimum of this product, so that for any wave structure:

$$\Delta x \Delta k \geq \frac{1}{2}$$

- Now, for our photons, $p = \frac{h}{\lambda} = \frac{hk}{2\pi} \Rightarrow k = \frac{2\pi p}{h} \Rightarrow \boxed{\Delta x \Delta p_x \geq \frac{h}{4\pi} = \frac{\hbar}{2}}$

Two-slit experiment revisited

- Want to be really shocked?
- This series of images was created by passing single ***electrons*** through a two-slit apparatus!
- Of course, it is the same for single photons as well.
- We will investigate the wave nature of particles in the next few lectures, but the uncertainty principle also applies to particles!



Feynman explains:
https://www.youtube.com/watch?v=2mIk3wBJDgE